# Problem 10.18

## Given:

Helical compression spring
B159 phosphor-bronze (Material)
Squared and ground ended

$d = 3.8$ mm
$OD = 31.4$ mm
$L_o = 71.4$ mm
$N_t = 12.8$ coils

## Find:

Investigate if the spring is solid-safe. If not find the largest free length with $n_s = 1.2$.

## Solution

Solid-safe means, to not have failure when the spring is compressed to it's solid length. We will check whether the shear stress ($\tau_s$) created in the spring when compressed solid is larger than the torsional yield strength $S_{sy}$ i.e. whether the factor of safety $n_s$ is larger than unity or not. If $n_s$ is larger than unity, the spring is solid-safe.

The mean coil diameter is;

$$D = OD - d = 31.4 - 3.8 = 27.6 \text{ mm}$$

The spring index C is $C = \frac{D}{d} = \frac{27.6}{3.8} = 7.263$

The Bergsträsser factor $K_B = \frac{4C+2}{4C-3}$ --- Eqn 10-5

$$K_B = \frac{4(7.263)+2}{4(7.263)-3} = \frac{31.053}{26.052} = 1.192$$

From Table 10.1, for squared and ground ended,

$$N_t = N_a + 2 \rightarrow N_a = N_t - 2 = 12.8 - 2 = 10.8 \text{ coils}$$

From Table 10-5 for phosphor bronze B159,

$G = 41.4$ GPa.

The spring-scale (or spring rate), from Eqn. 10-9,

$$k = \frac{d^4 G}{8 D^3 N_a} = \frac{(3.8)^4 (41.4 \times 10^3)}{8 (27.6)^3 (10.8)} = 4.752 \text{ N/mm}$$

Note that in the above equation G is used in terms of MPa not GPa.

From Table 10-1; $L_s = d N_t = (3.8)(12.8) = 48.64$ mm

The maximum deflection (or deflection to solid length) is,

$$y_s = L_o - L_s = 71.4 - 48.64 = 22.76 \text{ mm.}$$

The force needed to compress the spring to its solid length is;

$$F_s = k \cdot y_s = (4.752)(22.76) = 108.2 \text{ N}$$

The corresponding shear stress $\tau_s$ (Eqn. 10.7),

$$\tau_s = K_B \frac{8 F_s D}{\pi d^3} = 1.192 \frac{8 (108.2)(27.6)}{\pi (3.8)^3} = 165.2 \text{ MPa}$$

From Table 10-4 for phosphor-bronze B159:

To decide which range our diameter is, we are converting wire diameter to inches.

$$d = \frac{3.8}{25.4} = 0.15 \text{ in}$$

for this diameter, $A = 932$ MPa-mm$^m$ and $m = 0.064$

From Eqn. 10-14 $S_{ut} = \frac{A}{d^m} = \frac{932}{(3.8)^{0.064}} = 855.7$ MPa

The torsional yield strength $S_{sy}$, from Table 10-6 for nonferrous alloys, is:

$$S_{sy} = 0.35\ S_{ut} = 0.35\,(855.7) = 299.5\ \text{MPa}.$$

The factor of safety for shear;

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{299.5}{165.2} = 1.81$$

Since $n_s > 1$ or $\tau_s < S_{sy}$ the spring is said to be solid-safe.

if $n_s < 1$ was the result, then we would calculate the new $L_0 = L_s + y_s$ from

$$y_s = \frac{F_s}{k} \quad \text{by considering} \quad \tau_s = \frac{S_{sy}}{n_s}$$

i.e.

$$y_s = \frac{(S_{sy}/n_s)\,\pi d^3}{8 K_B D \cdot k}$$

# Problem 10-26

## Given:

$ID = 15$ mm

$C = 10$

$L_o = 125$ mm

$y_s = 75$ mm

Squared and ground end.

Unpeened

Material: HD A227

## Determine:

a) $d_w$

b) $N_t$

c) $k$

d) $n_s$

e) $n_f$

## Solution:

a) $ID = D - d_w = 15 \qquad C = \frac{D}{d_w} \rightarrow D = C \cdot d_w = 10 d_w$

$10 d_w - d_w = 15 \longrightarrow \boxed{d_w = 1.67 \text{ mm}}$

b) From Table 10-1 for square and ground

$L_s = d_w N_t \qquad L_s = L_o - y_s = 125 - 75 = 50$ mm.

$N_t = \frac{L_s}{d_w} = \frac{50}{1.67} \longrightarrow \boxed{N_t = 30 \text{ coils}}$

c) From Table 10-1 ; $N_t = N_a + 2 \longrightarrow N_a = N_t - 2$

$N_a = 30 - 2 = 28$ coils

From Table 10-5 for HD A227 spring and for the calculated diameter $G = 79.3$ GPa

From Eqn. 10-9 ;

$$k = \frac{d_w^4 \, G}{8 D^3 N_a} = \frac{(1.67)^4 \cdot (79.3 \times 10^3)}{8 (10 \times 1.67)^3 \cdot 28} \rightarrow \boxed{k = 0.591 \text{ N/mm}}$$

d) $n_s = \dfrac{S_{sy}}{\tau_s}$

From Table 10-4 for HD A227 ; $A = 1783$ MPa-mm$^m$

$m = 0.190$

Eqn. 10-14 $S_{ut} = \dfrac{A}{d_w^m} = \dfrac{1783}{(1.67)^{0.19}}$ $S_{ut} = 1617.5$ MPa.

From Table 10-6 for cold-drawn carbon steel;

$S_{sy} = 0.45\, S_{ut} = 0.45(1617.5) = 647$ MPa.

Force needed to bring the spring to it's solid length,

$F_s = k \cdot y_s = (0.591)(75) = 44.325$ N

Bergstässer factor, $K_B = \dfrac{4C+2}{4C-3} = \dfrac{42}{37} = 1.135$

From Eqn. 10-7;

$$\tau_s = K_B \frac{8 F_s D}{\pi d_w^3} = 1.135 \frac{8(44.325)\,16.7}{\pi\,(1.67)^3} = 459 \text{ MPa}$$

$n_s = \dfrac{647}{459} \longrightarrow \boxed{n_s = 1.41}$

e) When the load will repeatedly cycled from freelength to solid length:

$\tau_a = \tau_m = 0.5\,\tau_s = 0.5(459) = 229.5$ MPa.

and $r = \dfrac{\tau_a}{\tau_m} = 1.0$

Using the Gerber fatigue failure criterion with Zimmerli data for unpeened case:

$S_{sa} = 241$ MPa $S_{sm} = 379$ MPa --- Eqn. 10-28

Eqn. 10-30 $S_{su} = 0.67\, S_{ut} = 0.67(1617.5) = 970.5$ MPa.

The Gerber ordinate intercept for the Zimmerli data is

$$S_{se} = \frac{S_{sa}}{1-(S_{sm}/S_{su})^2} = \frac{241}{1-(379/970.5)} = 284.4\text{ MPa}$$

From Table 6-7, p.307

$$S_{sa} = \frac{r^2 S_{su}}{2 S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{rS_{su}}\right)^2}\right]$$

$$S_{sa} = \frac{(1)^2(970.5)^2}{2(284.4)}\left\{-1+\sqrt{1+\left[\frac{2(284.4)}{(1)(970.5)}\right]^2}\right\} = 263.4\text{ MPa.}$$

$$n_f = \frac{S_{sa}}{\tau_a} = \frac{263.4}{229.5} \longrightarrow \boxed{n_f = 1.15}$$