MENG541 Advanced Thermodynamics

CHAPTER 5 – ENTROPY GENERATION

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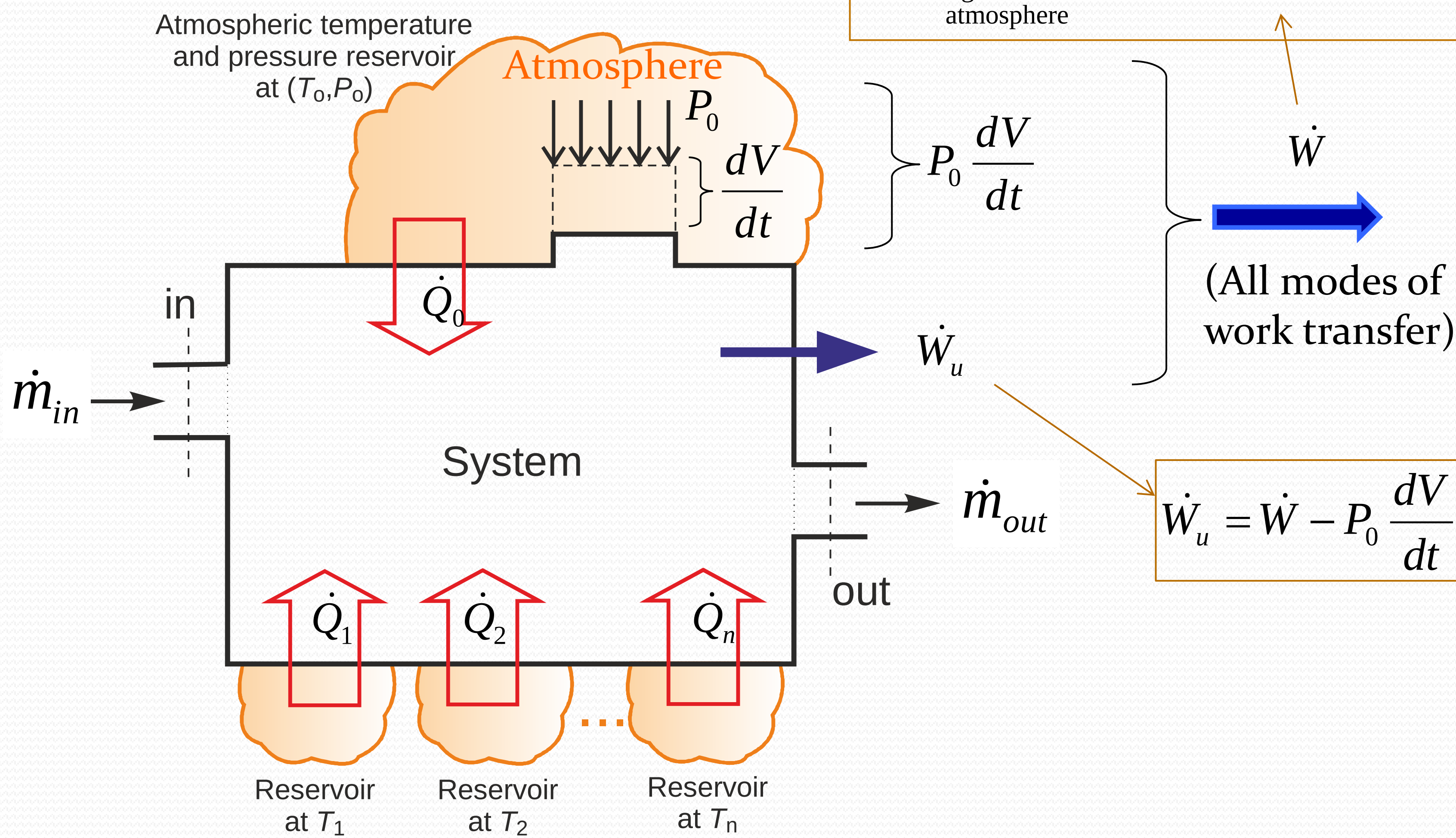
Chapter 5
Entropy Generation (Exergy Destruction)

Outline

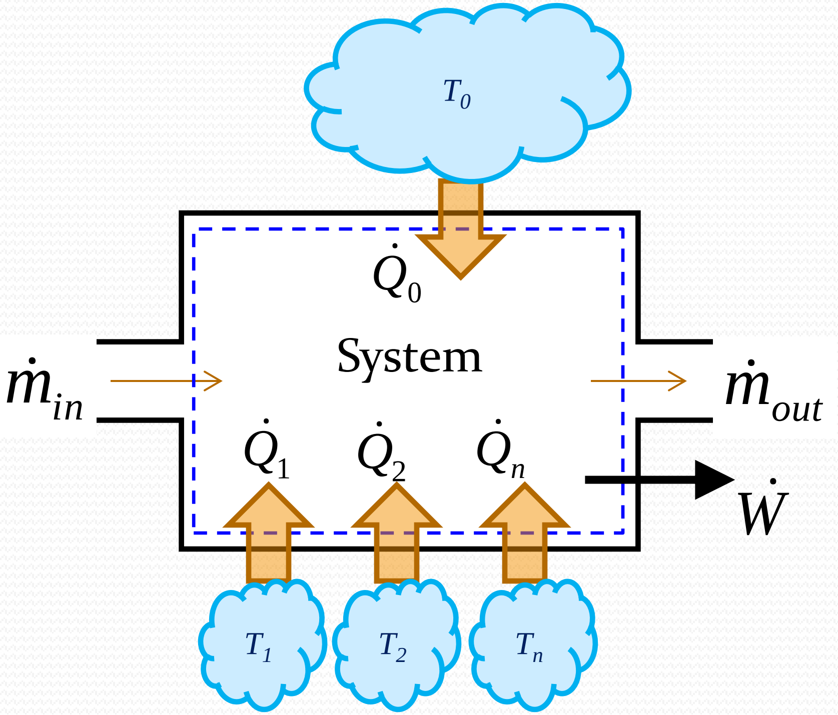
- Lost Available Work
- Cycles
  - Heat engine cycles
  - Refrigeration cycles
  - Heat pump cycles
- Nonflow Processes
- Steady-Flow Processes
- Exergy wheel diagrams

Lost Available Work

$$\dot{W} = \underbrace{P\frac{dV}{dt}}_{\text{Work done against the atmosphere}} + \dot{W}_{electrical} + \dot{W}_{shear} + \dot{W}_{magnetic}$$



Lost Available Work



<u>First law:</u>

$$\frac{dE}{dt} = \sum_{i=0}^{n} \dot{Q}_i - \dot{W} + \sum_{in} \dot{m} h^o + \sum_{out} \dot{m} h^o$$

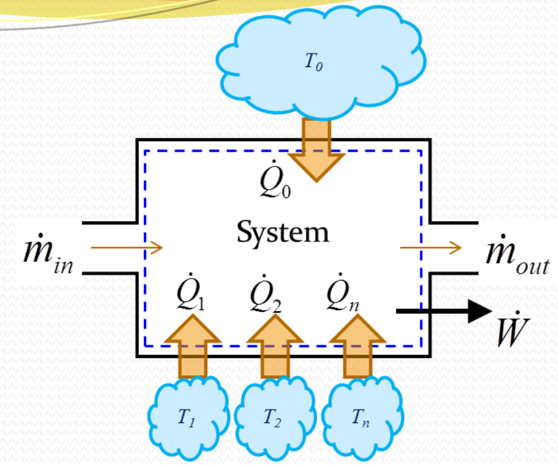
<u>Note</u>: h^o is known as *methalpy*, such that

$$h^o = h + \frac{V^2}{2} + gz$$

<u>Second law:</u>

$$\dot{S}_{gen} = \frac{dS}{dt} - \sum_{i=0}^{n} \frac{\dot{Q}_i}{T_i} - \sum_{in} \dot{m} s + \sum_{out} \dot{m} s \geq 0$$

Lost Available Work



$$\frac{dE}{dt} = \sum_{i=0}^{n} \dot{Q}_i - \dot{W} + \sum_{in} \dot{m}h^o + \sum_{out} \dot{m}h^o$$

$$\dot{S}_{gen} = \frac{dS}{dt} - \sum_{i=0}^{n} \frac{\dot{Q}_i}{T_i} - \sum_{in} \dot{m}s + \sum_{out} \dot{m}s \geq 0$$

Eliminate $\dot{Q}_0$ between the two equations:

$$\dot{W} = -\frac{d}{dt}(E - T_0 S) + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_i}\right) \dot{Q}_i + \sum_{in} \dot{m}(h^o - T_0 s) - \sum_{out} \dot{m}(h^o - T_0 s) - T_0 \dot{S}_{gen}$$

When reversible $\dot{S}_{gen}$ is zero, hence:

$$\dot{W}_{rev} = -\frac{d}{dt}(E - T_0 S) + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_i}\right) \dot{Q}_i + \sum_{in} \dot{m}(h^o - T_0 s) - \sum_{out} \dot{m}(h^o - T_0 s)$$

Therefore generally:

$\dot{W} = \dot{W}_{rev} - T_0 \dot{S}_{gen}$ however we know that $\dot{W}_{lost} = \dot{W}_{rev} - \dot{W}$

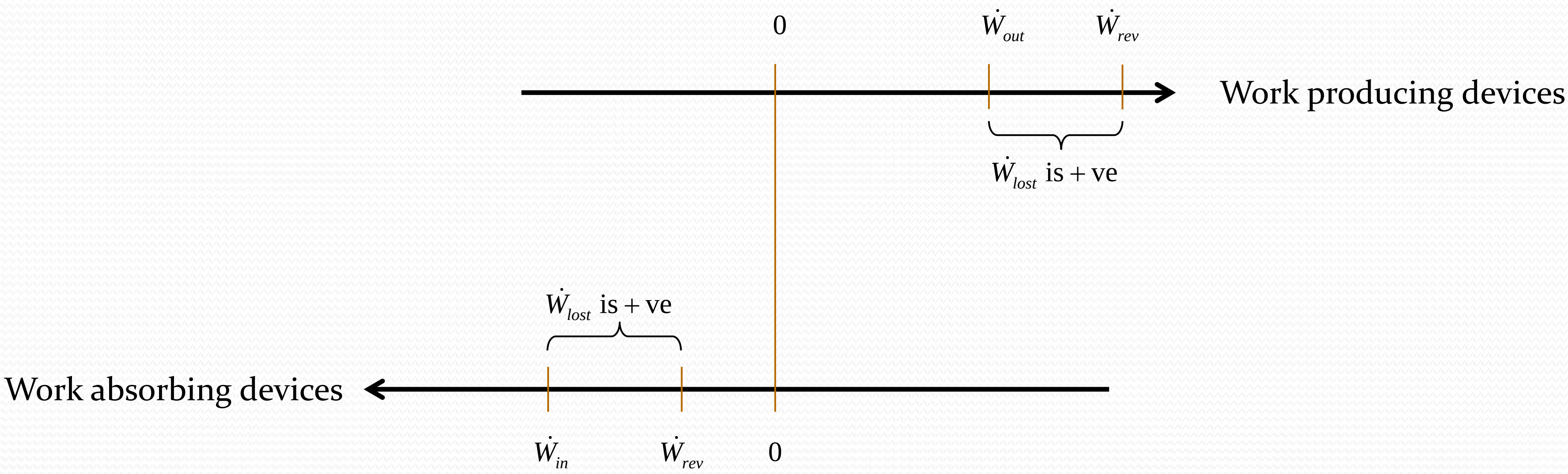
Hence

$$\dot{W}_{lost} = T_0 \dot{S}_{gen}$$

Also known as «exergy destruction X_{des}»
or «Irreversibility»

Lost Available Work

$\dot{W}_{lost}$ is always positive although $\dot{W}$ and $\dot{W}_{rev}$ can be either positive or negative (remember $\dot{W}_{lost} = \dot{W}_{rev} - \dot{W}$)



The main purpose of studying the lost available work is to diagnose the areas where irreversibilities are taking place in a prosess so that thermodynamic improvements can be made.

Lost Available Work

When the system is doing work against the atmosphere that has pressure P_0 then the atmosphere consumes a work rate of $P_0\, dV/dt$ such that :

$$\underbrace{\dot{X}_W}_{\substack{\text{Rate of} \\ \text{available work}}} = \dot{W} - P_0 \frac{dV}{dt}$$

In most flow systems $P_0\, dV/dt = 0$, therefore $\dot{X}_W = \dot{W}$ (i.e., exergy transfer by work is simply the work itself)

$$= -\frac{d}{dt}(E + P_0 V - T_0 S) + \sum_{i=1}^{n}\left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i$$

$$+ \sum_{in} \dot{m}(h^o - T_0 s) - \sum_{out} \dot{m}(h^o - T_0 s) - T_0 \dot{S}_{gen}$$

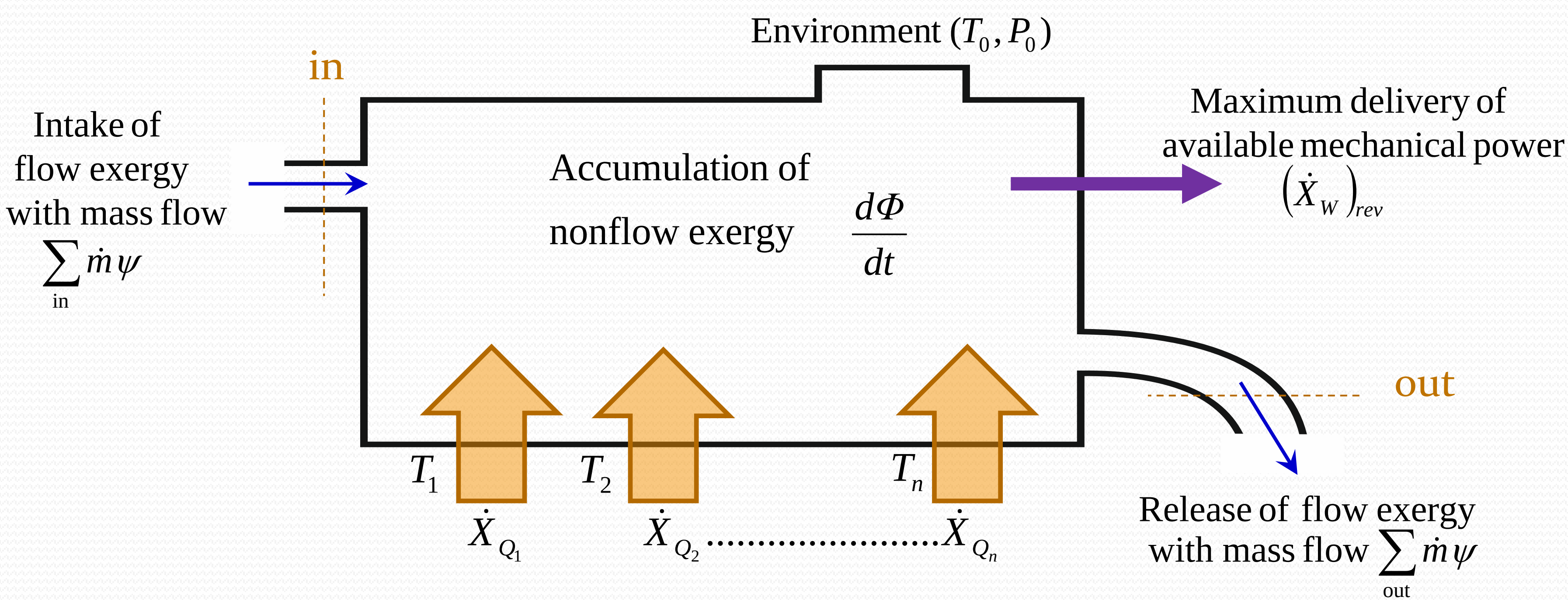
Lost Available Work

In the reversible limit : $\left(\dot{X}_W\right)_{rev} = \dot{W}_{rev} - P_0 \dfrac{dV}{dt}$

$$\underbrace{\left(\dot{X}_W\right)_{rev}}_{\text{Maximum delivery of available power}} = \underbrace{-\frac{d}{dt}(E + P_0V - T_0S)}_{\text{Accumulation of nonflow exergy } d\Phi/dt} + \underbrace{\sum_{i=1}^{n}\left(1-\frac{T_0}{T_i}\right)\dot{Q}_i}_{\text{Exergy tansfer with heat transfer}} + \underbrace{\sum_{in}\dot{m}(h^o - T_0 s)}_{\text{Intake of flow exergy with mass flow}} - \underbrace{\sum_{out}\dot{m}(h^o - T_0 s)}_{\text{Release of flow exergy with mass flow}}$$

Intake of flow exergy with mass flow: $\sum_{in}\dot{\Psi} = \sum_{in}\dot{m}\psi$

Release of flow exergy with mass flow: $\sum_{out}\dot{\Psi} = \sum_{out}\dot{m}\psi$



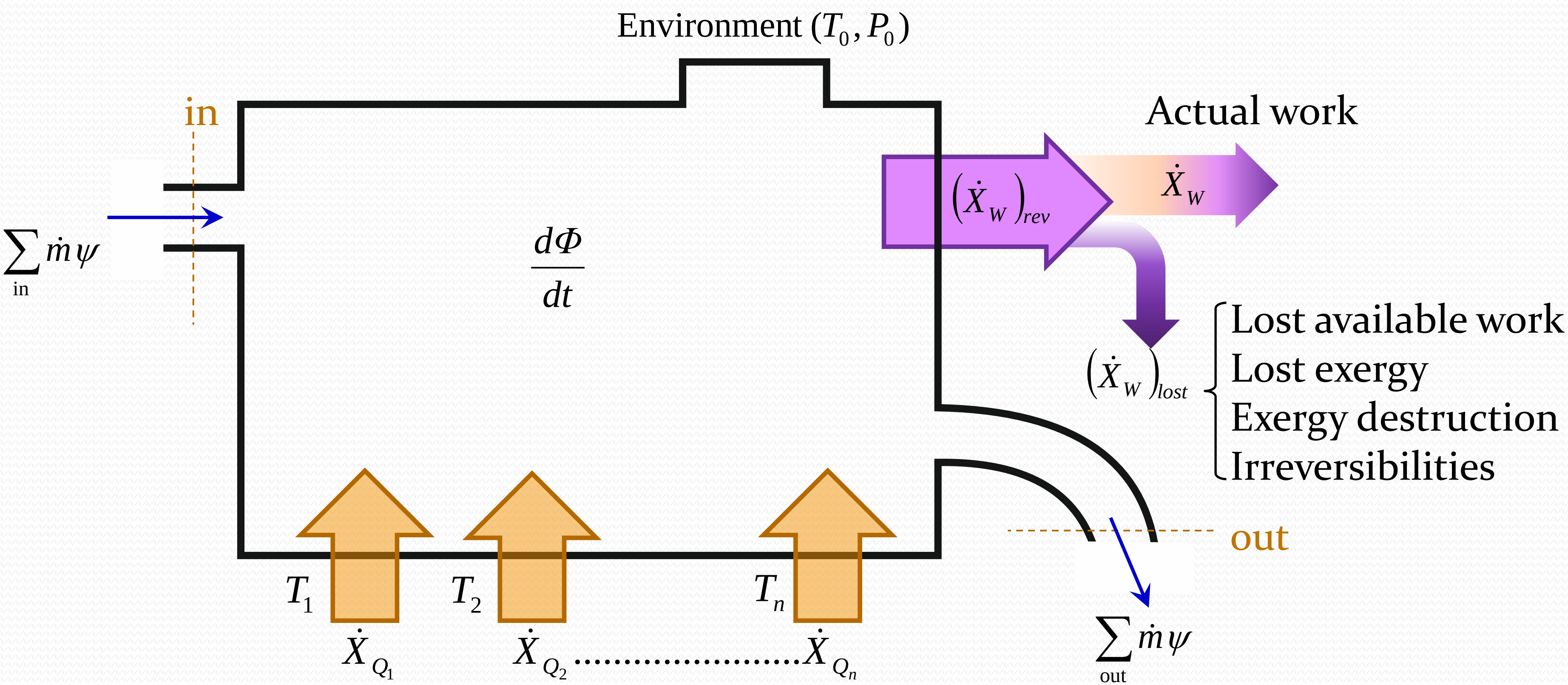
Lost Available Work

Lost available work is defined as the difference between the maximum available work W_{rev} and the actual work W. Alternatively it can be defined as:

Same as $\dot{W}_{lost}$

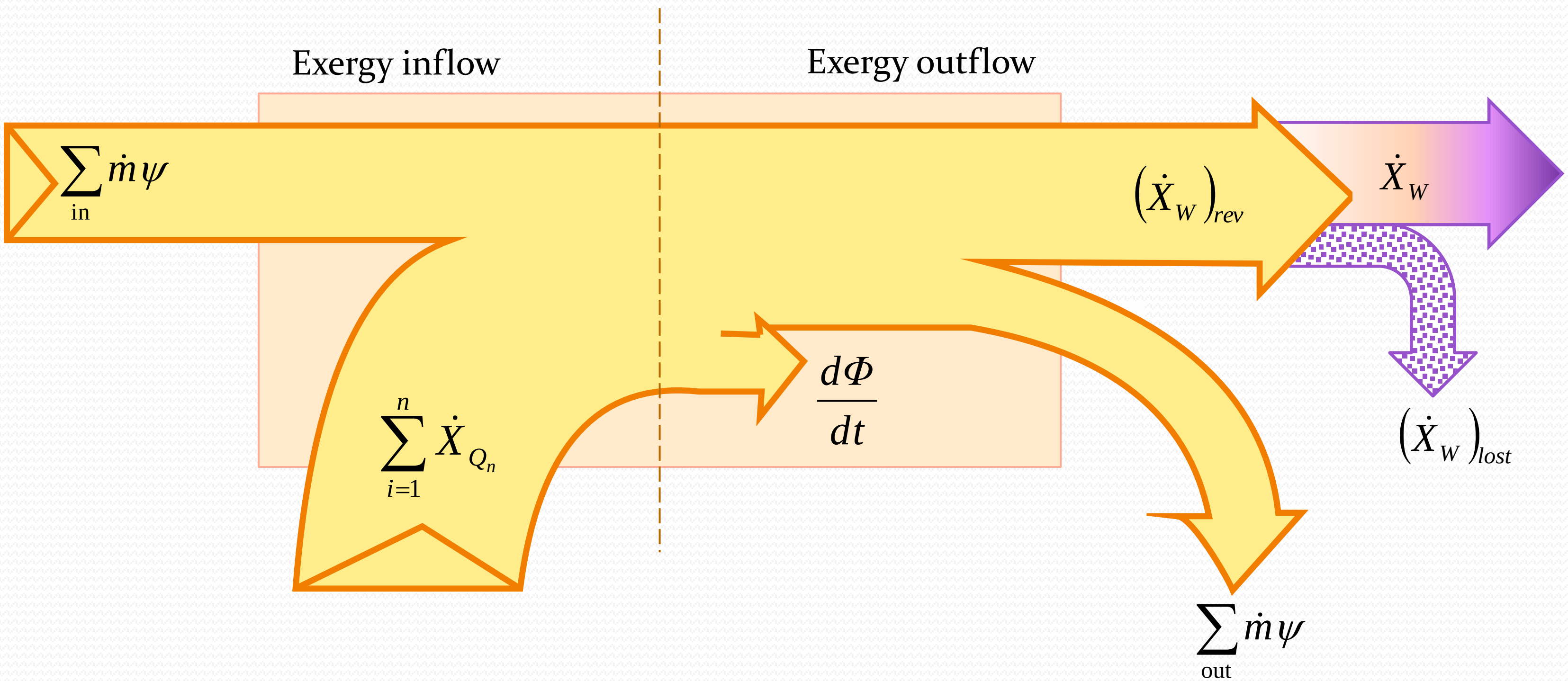
$$\dot{X}_{lost} = (\dot{X}_W)_{rev} - \dot{X}_W$$

Same as $\dot{X}_{des}$



Lost Available Work

Exergy balance of the open system discussed can be shown on a flow diagram as follows:



Lost Exergy in Cycles

Consider as closed systems that operate in an integral number of cycles.
The ceiling value for available power (maximum available power) is

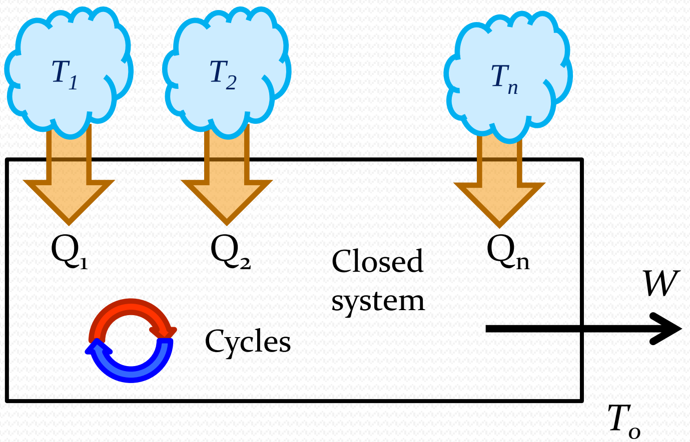
$$(\dot{X}_W)_{rev} = \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_i}\right) \dot{Q}_i$$

Exergy content of heat transfer $(\dot{Q}, T, T_0)$ can be expressed as

$$\dot{X}_Q = \dot{Q}\left(1 - \frac{T_0}{T}\right)$$

Therefore the lost available work for closed systems operating in cycles:

$$\dot{W}_{lost} = \underbrace{\sum_{i=1}^{n} (\dot{X}_Q)_i}_{(\dot{X}_W)_{rev}} - \dot{X}_W$$



Heat Engine Cycles

First and second laws state that :

$$Q_H - Q_L - W = 0$$

$$S_{gen} = \frac{Q_L}{T_L} + \frac{Q_H}{T_H} \geq 0$$

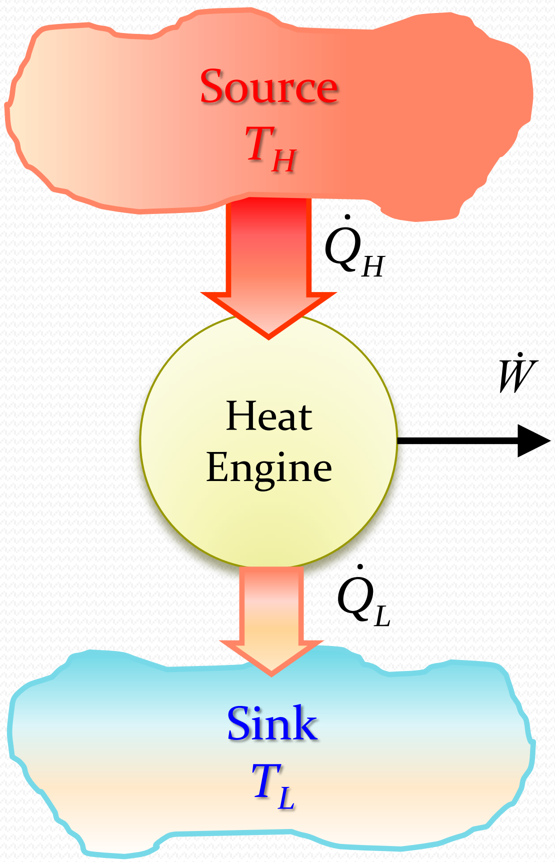
Obtained by applying the definition of entropy to the 2 reservoirs. Q_H is -ve

W_{lost} can be expressed as follows if temperature T_L is assumed to be T_0

$$W_{lost} = X_{Q_H} - X_W = Q_H\left(1 - \frac{T_L}{T_H}\right) - W$$

Also can be expressed as :

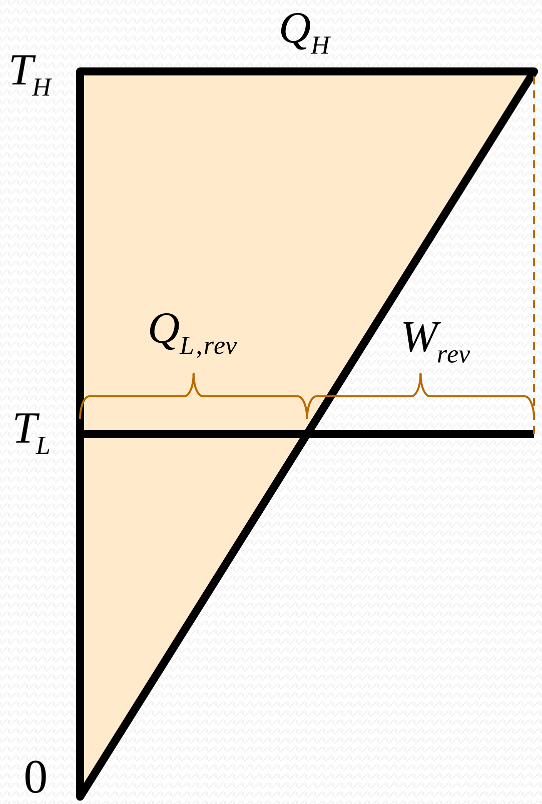
$$W_{lost} = T_L S_{gen}$$



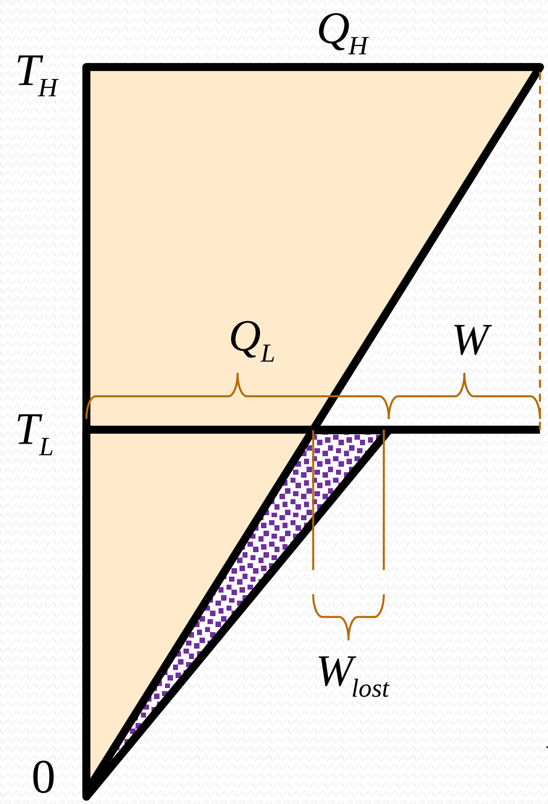
Heat Engine Cycles

Temperature -energy diagram for a heat engine cycle proposed by Adrian Bejan

Reversible



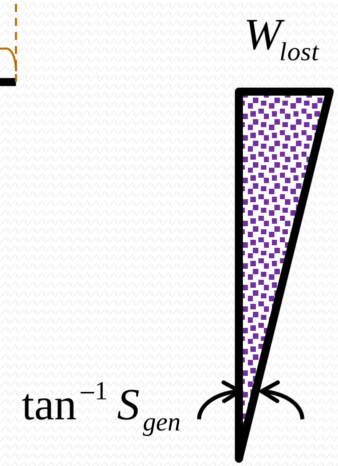
Irreversible



$$\tan\alpha = \frac{W_{lost}}{T_L}$$

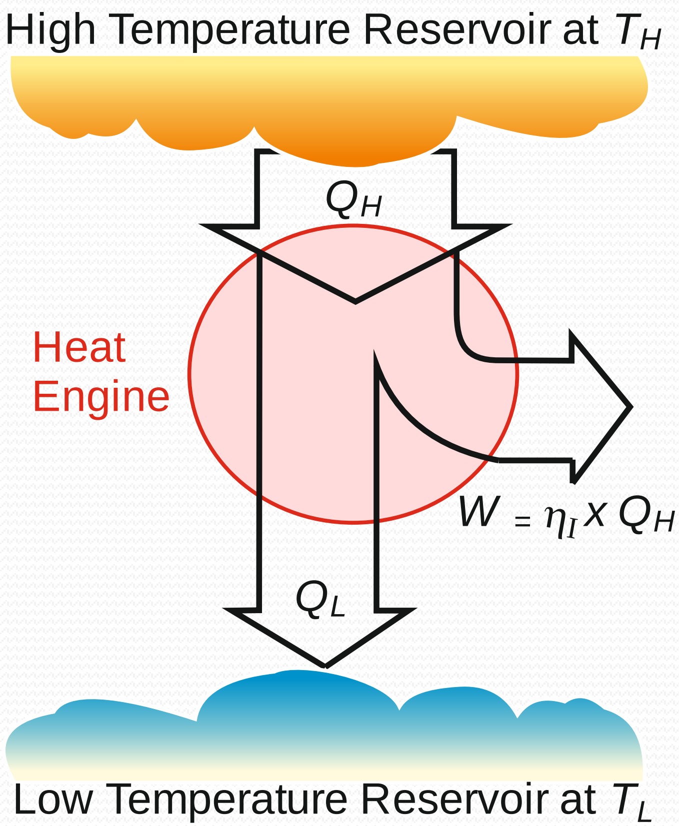
since $W_{lost} = T_L S_{gen}$,

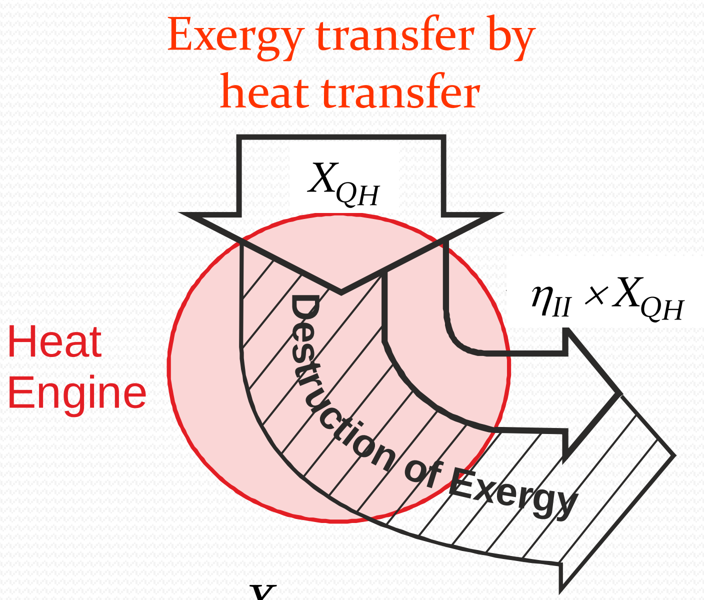
$\tan\alpha = S_{gen}$ or $\alpha = \tan^{-1} S_{gen}$



Heat Engine Cycles

Comparison between the first- and second-law efficiency of a heat-engine cycle





$$\eta_{II} = \frac{X_W}{X_{Q_H}}$$

$$\text{where } X_{Q_H} = (X_W)_{rev} = Q_H\left(1 - \frac{T_L}{T_H}\right)$$

Heat Engine Cycles

Second-law efficiency of a heat-engine cycle can also be expressed as follows:

$$\eta_{II} = \frac{X_W}{(X_W)_{rev}} = \frac{(X_W)_{rev} - W_{lost}}{(X_W)_{rev}} = 1 - \frac{T_L S_{gen}}{(X_W)_{rev}}$$

Relationship between first and second law efficiencies:

$$\eta_I = \frac{W}{Q_H} \quad \text{and} \quad \eta_{II} = \frac{X_W}{X_{Q_H}}$$

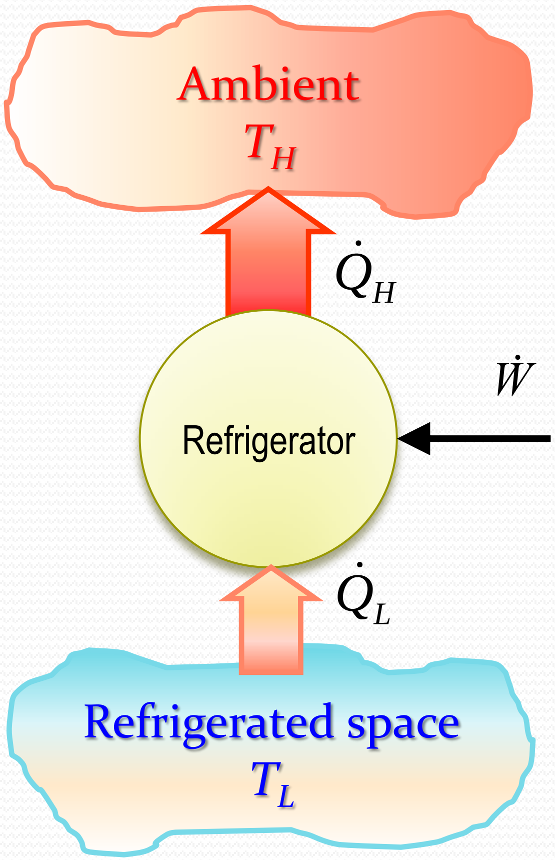
We know that work transfer is the same as the exergy transfer associated with it (i.e., $W = X_W$) Therefore,

$$\eta_I = \frac{\eta_{II} X_{Q_H}}{Q_H} = \frac{\eta_{II} \times \overbrace{Q_H (1 - T_L / T_H)}^{X_{Q_H}}}{Q_H}$$

$$\eta_I = \eta_{II} \left(1 - \frac{T_L}{T_H} \right)$$

Refrigeration Cycles

- They are closed systems in communication with two heat reservoirs
  - (1) the cold space (at T_L) from which refrigeration load Q_L is extracted
  - (2) the ambient (at T_H) to which heat Q_H is rejected



First and second laws state that :

$$Q_L - Q_H + W = 0$$

$$S_{gen} = \frac{Q_L}{T_L} + \frac{Q_H}{T_H} \geq 0$$

Obtained by applying the definition of entropy to the 2 reservoirs. Q_L is -ve

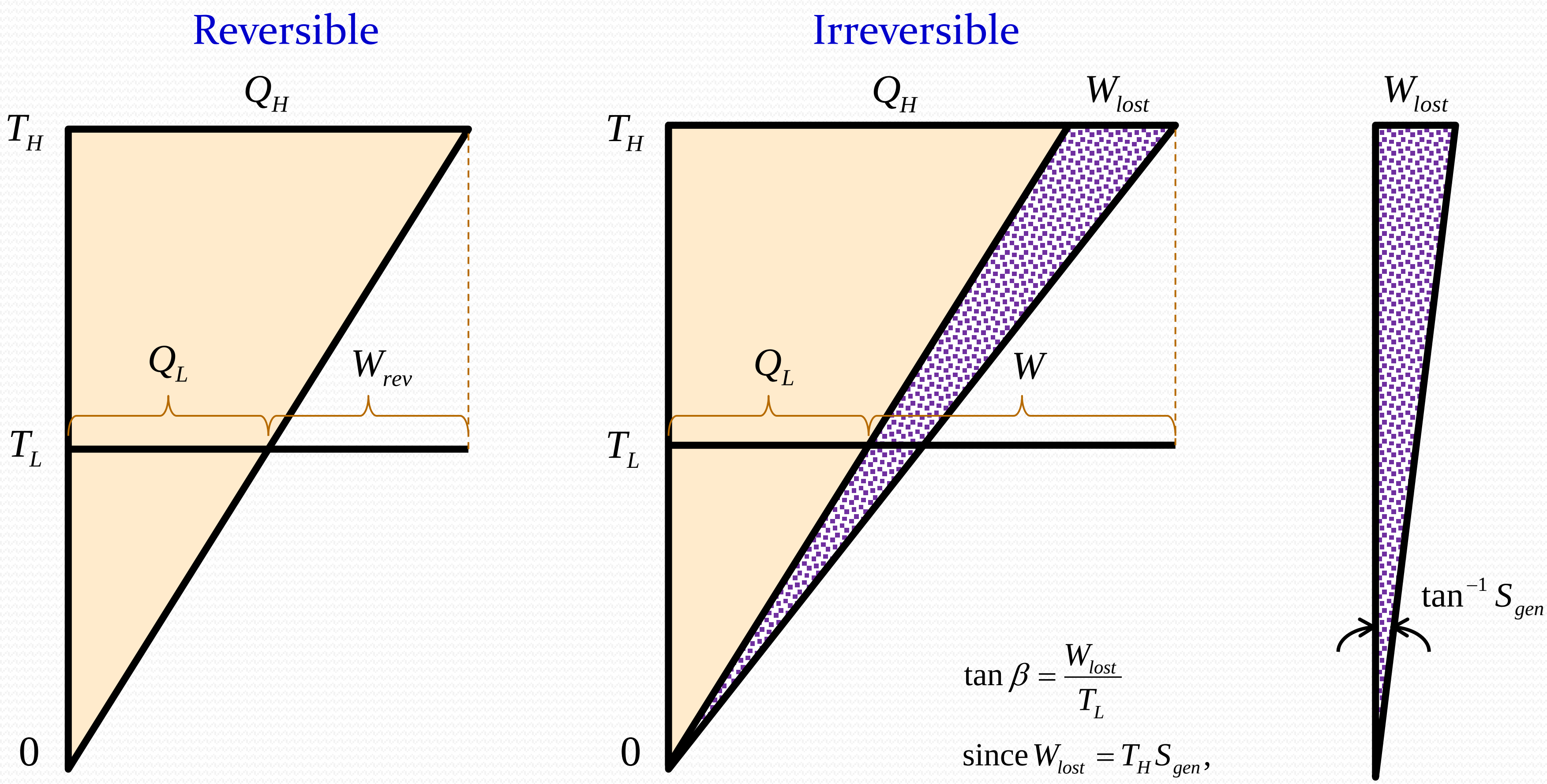
Here dead state - temperature T_0 is the temperature of the ambient, which is T_H. W_{lost} can be expressed as follows :

$$W_{lost} = \underbrace{X_{Q_L}}_{Q_L\left(1-\frac{T_H}{T_L}\right)} - \underbrace{X_W}_{\substack{\text{will be } W \\ \text{itself with a} \\ (-)\text{ve sign}}} = \underbrace{Q_L\left(1-\frac{T_H}{T_L}\right)}_{\substack{\text{This term will be} \\ \text{negative}}} - \underbrace{(-W)}_{\substack{\text{Work input} \\ \text{is a negative} \\ \text{number}}}$$

Rearranging $\Rightarrow W = Q_L\left(\frac{T_H}{T_L} - 1\right) + W_{lost}$

Refrigeration Cycles

Temperature -energy diagram for a refrigeration cycle proposed by Adrian Bejan



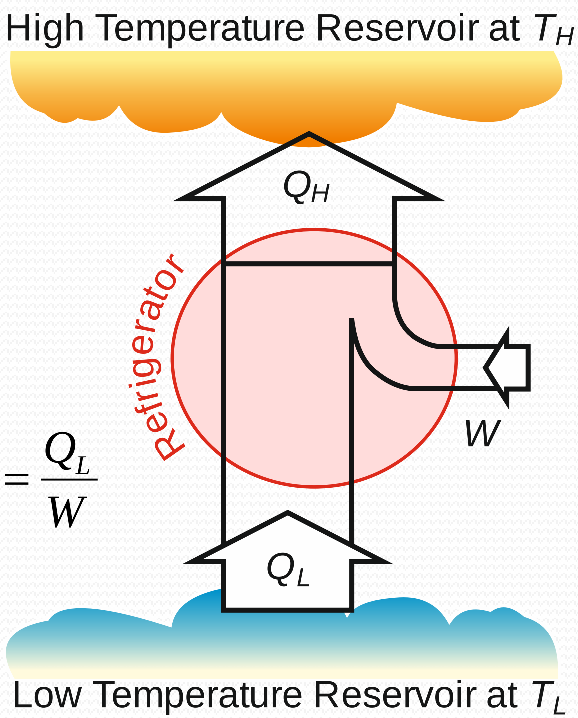
$$\tan\beta = \frac{W_{lost}}{T_L}$$

since $W_{lost} = T_H S_{gen}$,

$\tan\beta = S_{gen}$ or $\beta = \tan^{-1} S_{gen}$

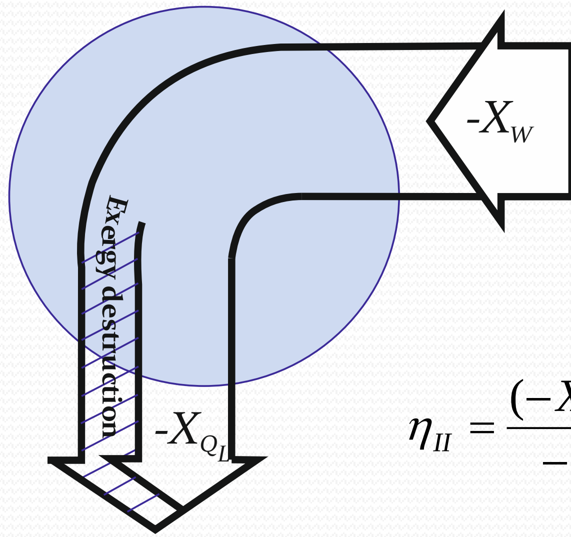
Refrigeration Cycles

Energy conversion vs exergy destruction during a refrigeration cycle



$$COP = \frac{Q_L}{W}$$

Refrigerator



Minimum work input when $W_{loss} = 0$ or when $-X_{Q_L} = (-X_W)_{rev}$

$$\eta_{II} = \frac{(-X_W)_{rev}}{-X_W} = \frac{-X_{Q_L}}{(-X_{Q_L}) + T_H S_{gen}}$$

$$COP = \frac{Q_L}{W} \quad \text{and} \quad \eta_{II} = \frac{COP}{COP_{rev}}$$

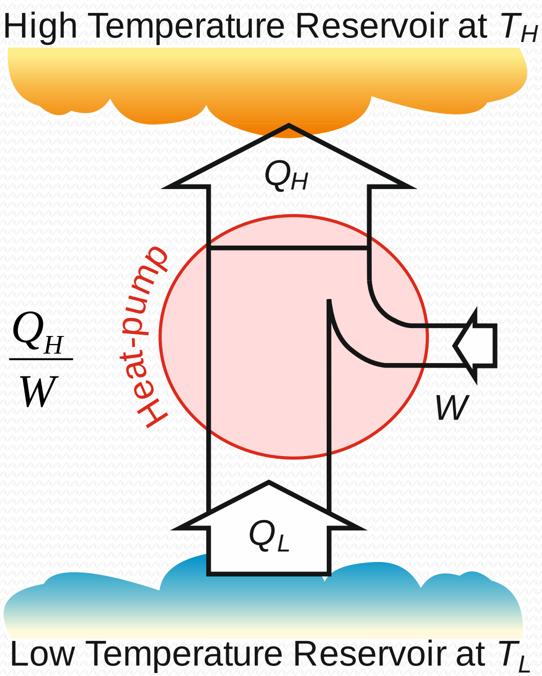
Noting that $COP_{rev} = \dfrac{1}{T_H/T_L - 1}$

$$\eta_{II} = COP\left(\frac{T_H}{T_L} - 1\right) \quad \text{or} \quad COP = \frac{\eta_{II}}{T_H/T_L - 1}$$

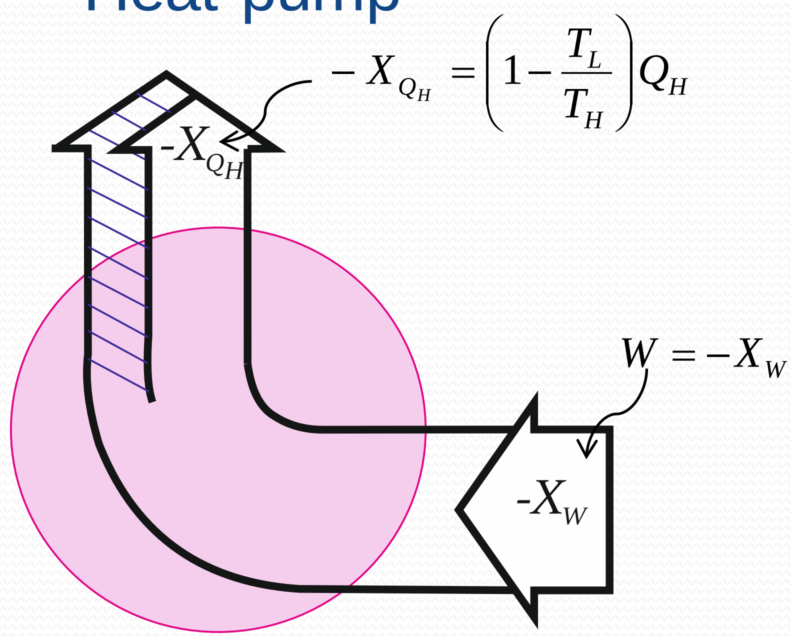
Heat-Pump Cycles

Energy conversion vs exergy destruction during a heat-pump cycle

$$COP = \frac{Q_H}{W}$$



Heat-pump



$$\underbrace{W_{lost}}_{\substack{\text{Exergy}\\ \text{destruction}\\ T_L S_{gen}}} = \underbrace{\left(1 - \frac{T_L}{T_H}\right)(-Q_H)}_{-W_{rev} \text{ or } -X_{Q_H}} - (-W)$$

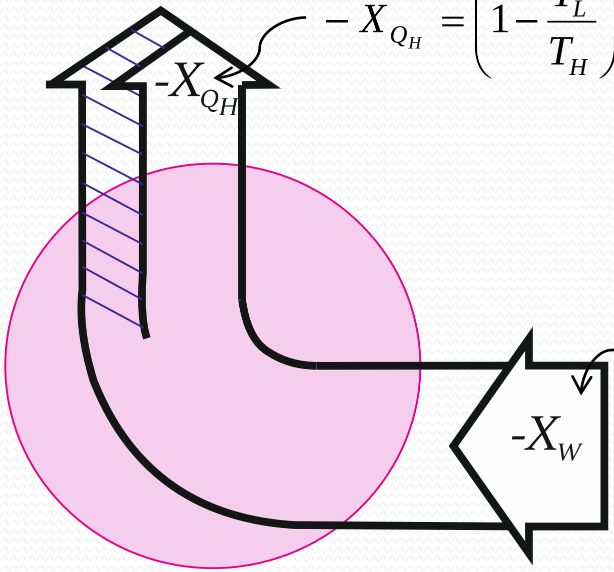
$$\text{or re-arranging} \Rightarrow W = \left(1 - \frac{T_L}{T_H}\right) Q_H + W_{lost}$$

Heat-Pump Cycles

The second-law efficiency of the heat-pump cycle is calculated by dividing the minimum work requirement by the actual work:

$$\eta_{II} = \frac{(-X_W)_{rev}}{-X_W} = \frac{-X_{Q_H}}{(-X_{Q_H}) + \underbrace{T_L S_{gen}}_{\text{Exergy destruction}}}$$

Heat-pump



$$-X_{Q_H} = \left(1 - \frac{T_L}{T_H}\right) Q_H$$

$$W = -X_W$$

$$COP = \frac{Q_H}{W} \quad \text{and} \quad \eta_{II} = \frac{COP}{COP_{rev}}$$

Noting that $COP_{rev} = \dfrac{1}{1 - T_L/T_H}$

$$\eta_{II} = COP\left(1 - \frac{T_L}{T_H}\right) \quad \text{or} \quad COP = \frac{\eta_{II}}{1 - T_L/T_H}$$

Nonflow Processes

General equation for available work :

$$\underbrace{\dot{X}_W}_{\text{Rate of available work}} = \dot{W} - P_0 \frac{dV}{dt}$$

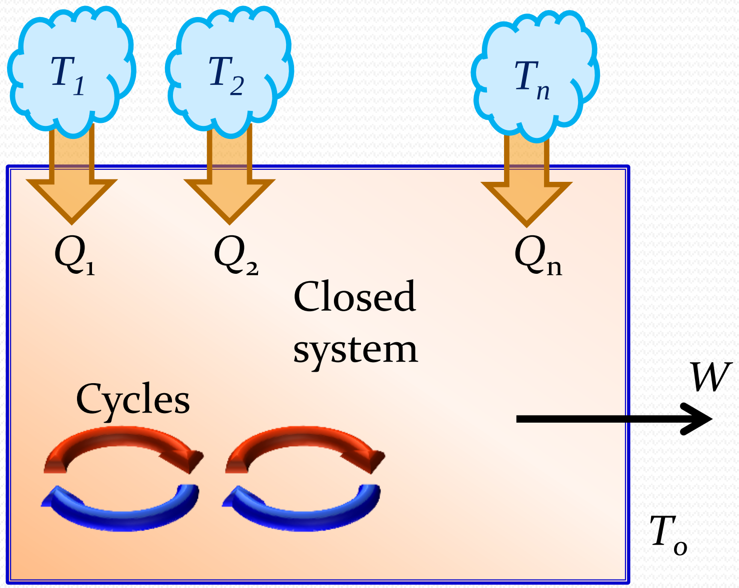
$$= -\frac{d}{dt}(E + P_0 V - T_0 S) + \sum_{i=1}^{n}\left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i + \sum_{in} \dot{m}(h^o - T_0 s) - \sum_{out} \dot{m}(h^o - T_0 s) - T_0 \dot{S}_{gen}$$

For the closed system shown consider a process $1 \rightarrow 2$ and integrate the above equation from $t = t_1$ to $t = t_2$:

$$X_W = A_1 - A_2 + \sum_{i=1}^{n} (X_Q)_i - T_0 S_{gen}$$

where $A = E - T_0 S + P_0 V$
or $a = e - T_0 s + P_0 v$
} Nonflow availability

A is a thermodynamic property of the system as long as T_0 and P_0 are fixed.



Nonflow Processes

$$X_W = A_1 - A_2 + \sum_{i=1}^{n} (X_Q)_i - T_0 S_{gen}$$

When the atmosphere is the only reservoir, the max work a closed system delivers can be expressed as :

$$(X_W)_{rev} = \underbrace{A - A_0}_{\text{This is known as the nonflow exergy}}$$

Note that the last two terms in the original equation drop out.
The nonflow exergy in full :

$$\Phi = A - A_0 = E - E_0 - T_0(S - S_0) + P_0(V - V_0)$$
$$\phi = a - a_0 = e - e_0 - T_0(s - s_0) + P_0(v - v_0)$$

The **nonflow exergy** is the reversible work delivered by a fixed-mass system during a process in which the atmosphere is the only reservoir.

Steady-flow Processes

General equation for available work :

$$\underbrace{\dot{X}_W}_{\text{Rate of available work}} = \dot{W} - P_0 \frac{dV}{dt}$$

Steady flow

$$= -\frac{d}{dt}(E + P_0 V - T_0 S) + \sum_{i=1}^{n} \underbrace{\left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i}_{(\dot{X}_Q)_i} + \sum_{in} \dot{m}\underbrace{(h^o - T_0 s)}_{b} - \sum_{out} \dot{m}\underbrace{(h^o - T_0 s)}_{b} - T_0 \dot{S}_{gen}$$

$$= \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m} b - \sum_{out} \dot{m} b - T_0 \dot{S}_{gen}$$

The flow availability at each port is defined as :

$$B = H^o - T_0 S$$

$$b = h^o - T_0 s$$

Steady-flow Processes

Consider multi-stream flow through devices where the streams do not mix. The equation obtained in the previous slide

$$\dot{X}_W = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m} b - \sum_{out} \dot{m} b - T_0 \dot{S}_{gen}$$

can be written as

$$\dot{X}_W = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{k=1}^{r} \left[(\dot{m} b)_{in} - (\dot{m} b)_{out} \right]_k - T_0 \dot{S}_{gen}$$

where k is the number of streams between 1 and r

Most popular examples would be single-stream devices and two-stream heat exchangers.

If the flow availability evaluated at standard environmental conditions (T_0, P_0) is b_0, such that

$$b_0 = h_0^o - T_0 s_0$$

then we can define flow exergy x_f as:

$$x_f = b - b_0$$

Hence

$$\dot{X}_W = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{k=1}^{r} \left[(\dot{m} x_f)_{in} - (\dot{m} x_f)_{out} \right]_k - T_0 \dot{S}_{gen}$$

Remember the flow exergy from Chp 4

The flow work is done against the fluid upstream in excess of the boundary work against the atmosphere such that exergy associated with this flow work:

$$x_{flow} = Pv - P_o v = (P - P_o) v$$

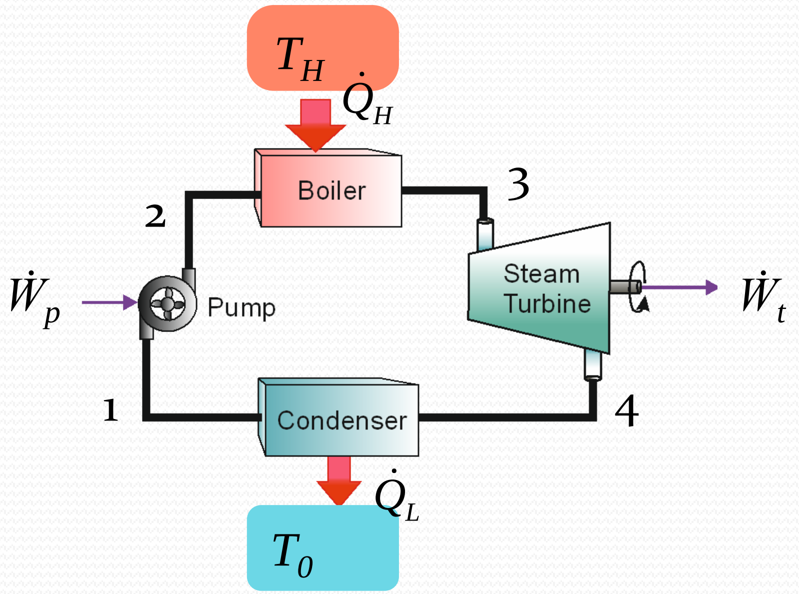
Steady-flow Processes

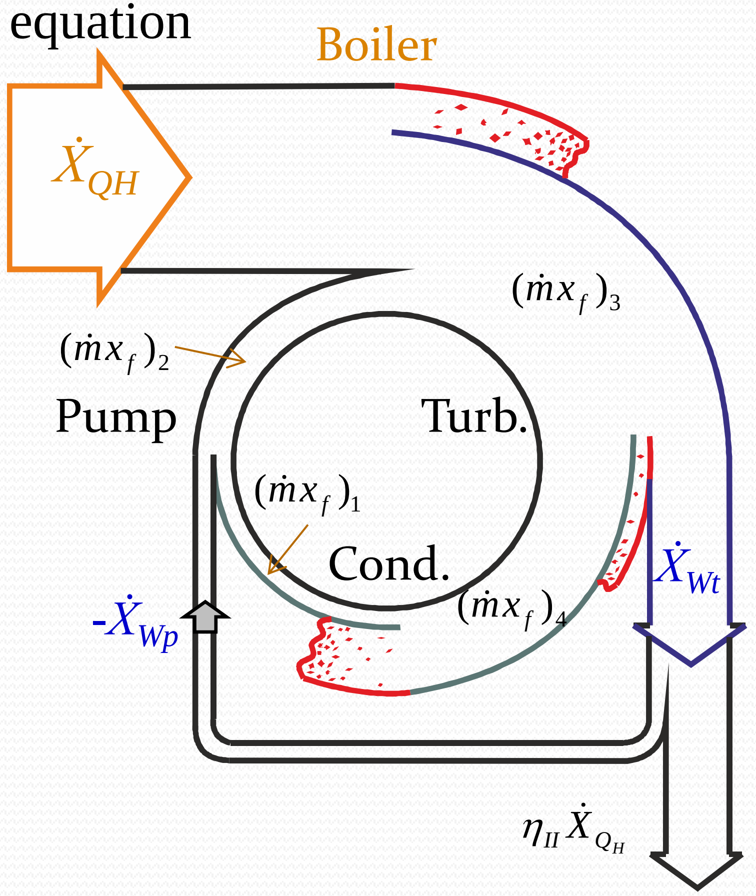
Consider a Rankine cycle operating between a high temperature T_H and the atmospheric reservoir temperature T_0. Using the equation

$$\dot{X}_W = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m} b - \sum_{out} \dot{m} b - T_0 \dot{S}_{gen}$$

it is possible to derive the following equation :

$$\dot{X}_W = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m} x_f - \sum_{out} \dot{m} x_f - T_0 \dot{S}_{gen}$$





Steady-flow Processes

$$\dot{X}_W = \sum_{i=1}^{n}(\dot{X}_Q)_i + \sum_{in}\dot{m}x_f - \sum_{out}\dot{m}x_f - T_0\dot{S}_{gen}$$

In the case of the boiler:

$$0 = \dot{X}_{Q_H} + \dot{m}(x_f)_2 - \dot{m}(x_f)_3 - T_0 S_{gen,boiler}$$

or

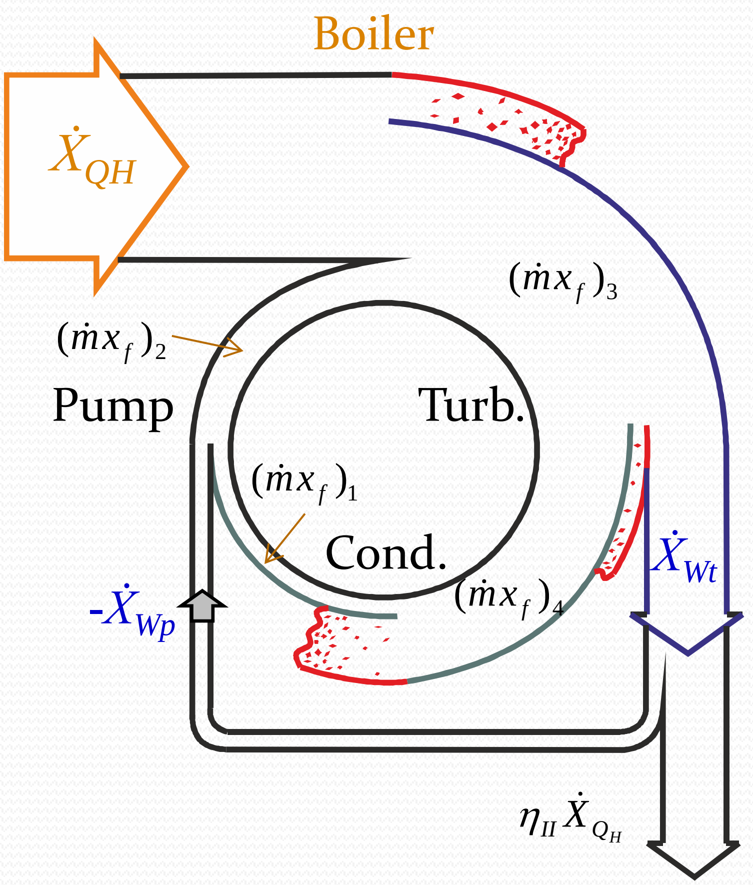
$$\underbrace{\dot{X}_{Q_H} + \dot{m}(x_f)_2}_{\text{Exergy inflow}} = \underbrace{\dot{m}(x_f)_3}_{\text{Exergy outflow}} + \underbrace{T_0 S_{gen,boiler}}_{\text{Exergy destroyed}}$$

In the case of the turbine $(\dot{X}_{W_t} = \dot{W}_t)$:

$$\dot{X}_{W_t} = 0 + \dot{m}(x_f)_3 - \dot{m}(x_f)_4 - T_0 S_{gen,turb}$$

or

$$\underbrace{\dot{m}(x_f)_3}_{\text{Exergy inflow}} = \underbrace{\dot{X}_{W_t} + \dot{m}(x_f)_4}_{\text{Exergy outflow}} + \underbrace{T_0 S_{gen,turb}}_{\text{Exergy destroyed}}$$



Steady-flow Processes

$$\dot{X}_W = \sum_{i=1}^{n} (\dot{X}_Q)_i + \sum_{in} \dot{m} x_f - \sum_{out} \dot{m} x_f - T_0 \dot{S}_{gen}$$

In the case of the condenser :

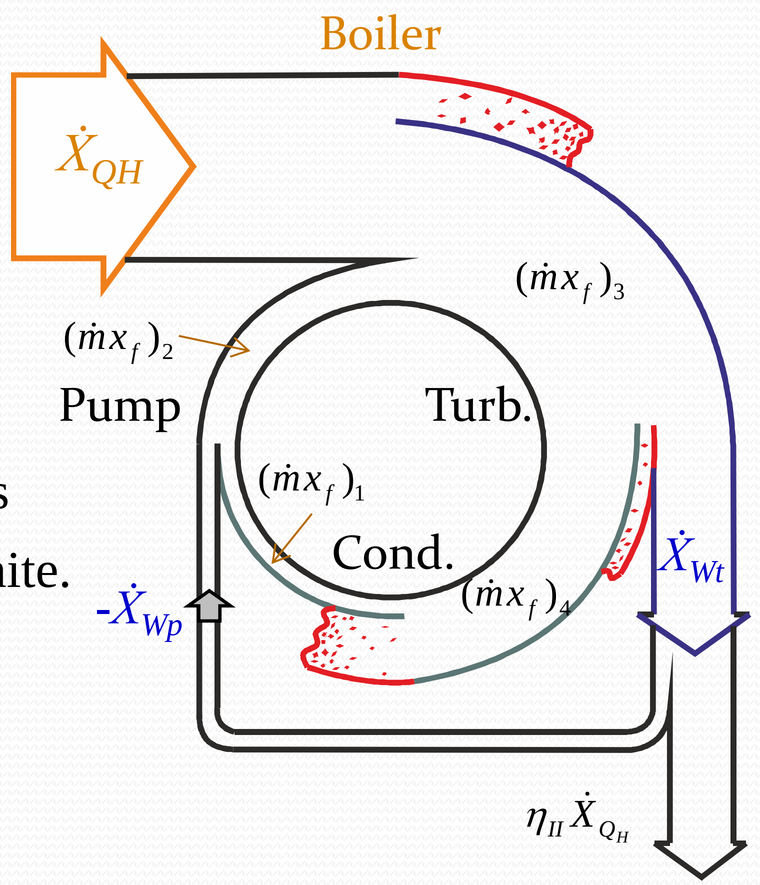
$$0 = \dot{m}(x_f)_4 - \dot{m}(x_f)_1 - \underbrace{T_0 S_{gen,condenser}}_{\text{A significant portion of stream exergy is destroyed due to heat transfer from condenser to the ambient}}$$

The exit temperature of the condenser is T_1, which is greater than T_0 and hence the exit exergy $(x_f)_1$ is finite.

In the case of the pump $(-\dot{X}_{W_p} = -\dot{W}_p)$:

$$-\dot{X}_{W_p} = 0 + \dot{m}(x_f)_1 - \dot{m}(x_f)_2 - T_0 S_{gen,\,pump}$$

$$\rightarrow \quad \dot{m}(x_f)_2 + \underbrace{T_0 S_{gen,\,pump}}_{\text{It is so small, it is not shown on the diagram}} = \dot{X}_{W_p} + \dot{m}(x_f)$$

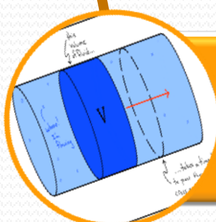


HOMEWORK

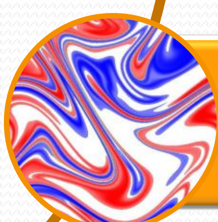
- Determine (by drawing an *exergy wheel* diagram) the exergy flow with the associated exergy destruction components of each component of a simple vapor-compression refrigeration cycle. Write down the exergy balance equations for each component and state any assumptions made.

Mechanisms of Entropy Generation or Exergy Destruction

- Heat Transfer across a Finite Temperature Difference



- Flow with Friction



- Mixing