3.1.3 *Manual Simulation Using Event Scheduling*

In conducting an event-scheduling simulation, a simulation table is used to record the successive system snapshots as time advances.

EXAMPLE 3.3 (Single-Channel Queue)

Reconsider the grocery store with one checkout counter that was simulated in Example 2.1 by an ad hoc method. The system consists of those customers in the waiting line plus the one (if any) checking out. The model has the following components:

System state $(LQ(t), LS(t))$, where $LQ(t)$ is the number of customers in the waiting line, and $LS(t)$ is the number being served (0 or 1) at time t.

Entities The server and customers are not explicitly modeled, except in terms of the state variables above.

Events

Arrival (A)

Departure (D)

Stopping event (E), scheduled to occur at time 60.

Event notices

(A, t), representing an arrival event to occur at future time t

(D, t), representing a customer departure at future time t

$(E, 60)$, representing the simulation-stop event at future time 60.

Activities

Interarrival time, defined in Table 2.6

Service time, defined in Table 2.7

Delay Customer time spent in waiting line.

The event notices are written as (event type, event time). In this model, the FEL will always contain either two or three event notices. The effect of the arrival and departure events was first shown in Figures 2.2 and 2.3 and is shown in more detail in Figures 3.5 and 3.6.

Interarrival Times	8	6	1	8	3	8	...
Service Times	4	1	4	3	2	4	...

Initial conditions are that the first customer arrives at time 0 and begins service.

Table 3.1. Simulation Table for Checkout Counter (Example 3.3)

	System State				Cumulative Statistics	
Clock	$LQ(t)$	$LS(t)$	Future Event List	Comment	B	MQ
0	0	1	$(D,4)$ $(A,8)$ $(E,60)$	First A occurs $(a^* = 8)$ Schedule next A $(s^* = 4)$ Schedule first D	0	0
4	0	0	$(A,8)$ $(E,60)$	First D occurs: $(D,4)$	4	0
8	0	1	$(D,9)$ $(A,14)$ $(E,60)$	Second A occurs: $(A,8)$ $(a^* = 6)$ Schedule next A $(s^* = 1)$ Schedule next D	4	0
9	0	0	$(A,14)$ $(E,60)$	Second D occurs: $(D,9)$	5	0
14	0	1	$(A,15)$ $(D,18)$ $(E,60)$	Third A occurs: $(A,14)$ $(s^* = 4)$ Schedule next D	5	0
15	1	1	$(D,18)$ $(A,23)$ $(E,60)$	Fourth A occurs: $(A,15)$ (Customer delayed)	6	1
18	0	1	$(D,21)$ $(A,23)$ $(E,60)$	Third D occurs: $(D,18)$ $(s^* = 3)$ Schedule next D	9	1
21	0	0	$(A,23)$ $(E,60)$	Fourth D occurs: $(D,21)$	12	1

EXAMPLE 3.4 (The Checkout-Counter Simulation, Continued)

Suppose that in the simulation of the checkout counter in Example 3.3 the simulation analyst desires to estimate mean response time and mean proportion of customers who spend 4 or more minutes in the system. A response time is the length of time a customer spends in the system. In order to estimate these customer averages, it is necessary to expand the model in Example 3.3 to explicitly represent the individual customers. In addition, to be able to compute an individual customer's response time when that customer departs, it will be necessary to know that customer's arrival time. Therefore, a customer entity with arrival time as an attribute will be added to the list of model components in Example 3.3. These customer entities will be stored in a list to be called "CHECKOUT LINE"; they will be called $C1, C2, C3, \ldots$ Finally, the event notices on the FEL will be expanded to indicate which customer is affected.

For example, $(D, 4, C1)$ means that customer $C1$ will depart at time 4. The additional model components are listed below:

Entities (Ci, t), representing customer Ci who arrived at time t

Event notices

(A, t, Ci), the arrival of customer Ci at future time t

(D, t, Cj), the departure of customer Cj at future time t

Set "CHECKOUT LINE," the set of all customers currently at the checkout counter (being served or waiting to be served), ordered by time of arrival

Three new cumulative statistics will be collected: S, the sum of customer response times for all customers who have departed by the current time; F, the total number of customers who spend 4 or more minutes at the checkout counter; and N_D the total number of departures up to the current simulation time. These three cumulative statistics will be updated whenever the departure event occurs; the logic for collecting these statistics would be incorporated into step 5 of the departure event in Figure 3.6.

The simulation table for Example 3.4 is shown in Table 3.2. The same data for interarrival and service times will be used again, so that Table 3.2 essentially repeats Table 3.1, except that the new components are included (and the comment column has been deleted). These new components are needed for the computation of the cumulative statistics S, F, and N_D. For example, at time 4 a departure event occurs for customer $C1$. The customer entity $C1$ is removed from the list called "CHECKOUT LINE"; the attribute "time of arrival" is noted to be 0, so the response time for this customer was 4 minutes. Hence, S is incremented by 4 minutes, and F and N_D are incremented by one customer. Similarly, at time 21 when the departure event $(D, 21, C4)$ is being executed, the response time for customer $C4$ is computed by

$$\begin{aligned}\text{Response time} &= \text{CLOCK TIME} - \text{attribute "time of arrival"}\\ &= \quad 21 \quad - \quad 15\\ &= 6 \text{ minutes}\end{aligned}$$

Then S is incremented by 6 minutes, and F and N_D by one customer.

For a simulation run length of 21 minutes, the average response time was $S/N_D = 15/4 = 3.75$ minutes, and the observed proportion of customers who spent 4 or more minutes in the system was $F/N_D = 0.75$. Again, this simulation was far too short to allow us to regard these estimates with any degree of accuracy. The purpose of Example 3.4, however, was to illustrate the notion that in many simulation models the information desired from the simulation (such as the statistics S/N_D and F/N_D) determines to some extent the structure of the model. ◀

Table 3.2. Simulation Table for Example 3.4

	System State		List	Future Event	Cumulative Statistics		
Clock	$LQ(t)$	$LS(t)$	"CHECKOUT LINE"	List	S	N_D	F
0	0	1	$(C1, 0)$	$(D, 4, C1)$ $(A, 8, C2)$ $(E, 60)$	0	0	0
4	0	0		$(A, 8, C2)$ $(E, 60)$	4	1	1
8	0	1	$(C2, 8)$	$(D, 9, C2)$ $(A, 14, C3)$ $(E, 60)$	4	1	1
9	0	0		$(A, 14, C3)$ $(E, 60)$	5	2	1
14	0	1	$(C3, 14)$	$(A, 15, C4)$ $(D, 18, C3)$ $(E, 60)$	5	2	1
15	1	1	$(C3, 14)$ $(C4, 15)$	$(D, 18, C3)$ $(A, 23, C5)$ $(E, 60)$	5	2	1
18	0	1	$(C4, 15)$	$(D, 21, C4)$ $(A, 23, C5)$ $(E, 60)$	9	3	2
21	0	0		$(A, 23, C5)$ $(E, 60)$	15	4	3

EXAMPLE 3.5 (The Dump Truck Problem)

Six dump trucks are used to haul coal from the entrance of a small mine to the railroad. Figure 3.7 provides a schematic of the dump truck operation. Each truck is loaded by one of two loaders. After loading, a truck immediately moves to the scale, to be weighed as soon as possible. Both the loaders and the scale have a first-come, first-served waiting line (or queue) for trucks. Travel time from a loader to the scale is considered negligible. After being weighed, a truck begins a travel time (during which time the truck unloads) and then afterward returns to the loader queue. The distributions of loading time, weighing time, and travel time are given in Tables 3.3, 3.4, and 3.5, respectively, together with the random digit assignment for generating these variables using random digits from Table A.1. The purpose of the simulation is to estimate the loader and scale utilizations (percentage of time busy). The model has the following components:

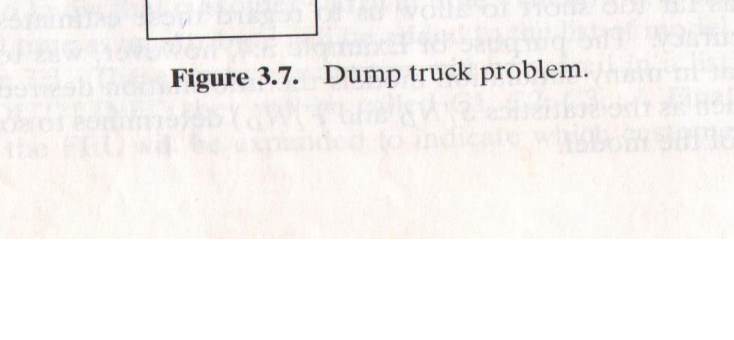

Figure 3.7. Dump truck problem.

Table 3.3. Distribution of Loading Time for the Dump Trucks

Loading Time	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
5	0.30	0.30	1–3
10	0.50	0.80	4–8
15	0.20	1.00	9–0

System state
$[LQ(t), L(t), WQ(t), W(t)]$, where

$LQ(t)$ = number of trucks in loader queue

$L(t)$ = number of trucks (0, 1, or 2) being loaded

$WQ(t)$ = number of trucks in weigh queue

$W(t)$ = number of trucks (0 or 1) being weighed, all at simulation time t

Event notices
(ALQ, t, DTi), dump truck i arrives at loader queue (ALQ) at time t
(EL, t, DTi), dump truck i ends loading (EL) at time t
(EW, t, DTi), dump truck i ends weighing (EW) at time t

Entities The six dump trucks ($DT1, \ldots, DT6$)

Lists
Loader queue, all trucks waiting to begin loading, ordered on a first-come, first-served basis

Weigh queue, all trucks waiting to be weighed, ordered on a first-come, first-served basis

Activities Loading time, weighing time, and travel time

Delays Delay at loader queue, and delay at scale

Table 3.4. Distribution of Weighing Time for the Dump Trucks

Weighing Time	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
12	0.70	0.70	1–7
16	0.30	1.00	8–0

Table 3.5. Distribution of Travel Time for the Dump Trucks

Travel Time	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
40	0.40	0.40	1–4
60	0.30	0.70	5–7
80	0.20	0.90	8–9
100	0.10	1.00	0

The simulation table is given in Table 3.6. It has been assumed that five of the trucks are at the loaders and one is at the scale at time 0. The activity times are taken from the following list as needed:

Loading Time	10	5	5	10	15	10	10
Weighing Time	12	12	12	16	12	16	
Travel Time	60	100	40	40	80		

When an end-loading (EL) event occurs, say for truck j at time t, other events may be triggered. If the scale is idle [$W(t) = 0$], truck j begins weighing and an end-weighing event (EW) is scheduled on the FEL; otherwise, truck j joins the weigh queue. If at this time there is another truck waiting for a loader, it will be removed from the loader queue and will begin loading by the scheduling of an end-loading event (EL) on the FEL. This logic for the occurrence of the end-loading event, as well as the appropriate logic for the other two events, should be incorporated into an event diagram as in Figures 3.5 and 3.6 of Example 3.3. The construction of these event logic diagrams is left as an exercise for the reader (Exercise 2).

As an aid to the reader, in Table 3.6 whenever a new event is scheduled, its event time is written as "$t+$ (activity time)." For example, at time 0 the imminent event is an EL event with event time 5. The clock is advanced to time $t = 5$, dump truck 3 joins the weigh queue (since the scale is occupied), and truck 4 begins to load. Thus, an EL event is scheduled for truck 4 at future time 10, computed by (present time) + (loading time) $= 5 + 5 = 10$.

In order to estimate the loader and scale utilizations, two cumulative statistics are maintained:

$$B_L = \text{total busy time of both loaders from time 0 to time } t$$

$$B_S = \text{total busy time of the scale from time 0 to time } t$$

Since both loaders are busy from time 0 to time 20, $B_L = 40$ at time $t = 20$. But from time 20 to time 24, only one loader is busy; thus, B_L increases by only 4 minutes over the time interval [20, 24]. Similarly, from time 25 to time 36, both loaders are idle ($L(25) = 0$), so B_L does not change. For the relatively

Table 3.6. Simulation Table for Dump Truck Operation (Example 3.5)

Clock	*System State*				*Lists*			*Cumulative Statistics*	
t	$LQ(t)$	$L(t)$	$WQ(t)$	$W(t)$	*Loader Queue*	*Weigh Queue*	*Future Event List*	B_L	B_S
0	3	2	0	1	$DT4$		$(EL, 5, DT3)$	0	0
					$DT5$		$(EL, 10, DT2)$		
					$DT6$		$(EW, 12, DT1)$		
5	2	2	1	1	$DT5$	$DT3$	$(EL, 10, DT2)$	10	5
					$DT6$		$(EL, 5+5, DT4)$		
							$(EW, 12, DT1)$		
[redacted]									
10	0	2	3	1		$DT3$	$(EW, 12, DT1)$	20	10
						$DT2$	$(EL, 20, DT5)$		
						$DT4$	$(EL, 10+15, DT6)$		
12	0	2	2	1		$DT2$	$(EL, 20, DT5)$	24	12
						$DT4$	$(EW, 12+12, DT3)$		
							$(EL, 25, DT6)$		
							$(ALQ, 12+60, DT1)$		
20	0	1	3	1		$DT2$	$(EW, 24, DT3)$	40	20
						$DT4$	$(EL, 25, DT6)$		
						$DT5$	$(ALQ, 72, DT1)$		
24	0	1	2	1		$DT4$	$(EL, 25, DT6)$	44	24
						$DT5$	$(EW, 24+12, DT2)$		
							$(ALQ, 72, DT1)$		
							$(ALQ, 24+100, DT3)$		
25	0	0	3	1		$DT4$	$(EW, 36, DT2)$	45	25
						$DT5$	$(ALQ, 72, DT1)$		
						$DT6$	$(ALQ, 124, DT3)$		
36	0	0	2	1		$DT5$	$(EW, 36+16, DT4)$	45	36
						$DT6$	$(ALQ, 72, DT1)$		
							$(ALQ, 36+40, DT2)$		
							$(ALQ, 124, DT3)$		
52	0	0	1	1		$DT6$	$(EW, 52+12, DT5)$	45	52
							$(ALQ, 72, DT1)$		
							$(ALQ, 76, DT2)$		
							$(ALQ, 52+40, DT4)$		
							$(ALQ, 124, DT3)$		

Table 3.6. Continued

Clock	System State				Lists		Future Event List	Cumulative Statistics	
t	$LQ(t)$	$L(t)$	$WQ(t)$	$W(t)$	Loader Queue	Weigh Queue		B_L	B_S
64	0	0	0	1			$(ALQ, 72, DT1)$ $(ALQ, 76, DT2)$ $(EW, 64+16, DT6)$ $(ALQ, 92, DT4)$ $(ALQ, 124, DT3)$ $(ALQ, 64+80, DT5)$	45	64
72	0	1	0	1			$(ALQ, 76, DT2)$ $(EW, 80, DT6)$ $(EL, 72+10, DT1)$ $(ALQ, 92, DT4)$ $(ALQ, 124, DT3)$ $(ALQ, 144, DT5)$	45	72
76	0	2	0	1			$(EW, 80, DT6)$ $(EL, 82, DT1)$ $(EL, 76+10, DT2)$ $(ALQ, 92, DT4)$ $(ALQ, 124, DT3)$ $(ALQ, 144, DT5)$	49	76

short simulation in Table 3.6, the utilizations are estimated as follows:

$$\text{average loader utilization} = \frac{49/2}{76} = 0.32$$

$$\text{average scale utilization} = \frac{76}{76} = 1.00$$

These estimates cannot be regarded as accurate estimates of the long-run "steady-state" utilizations of the loader and scale; a considerably longer simulation would be needed to reduce the effect of the assumed conditions at time 0 (five of the six trucks at the loaders) and to realize accurate estimates. On the other hand, if the analyst was interested in the so-called transient behavior of the system over a short period of time (say 1 or 2 hours), given the specified initial conditions, then the results in Table 3.6 can be considered representative (or constituting one sample) of that transient behavior. Additional samples can be obtained by conducting additional simulations, each one having the same initial conditions but using a different stream of random digits to generate the activity times.

Table 3.6, the simulation table for the dump truck operation, could have been simplified somewhat by not explicitly modeling the dump trucks as entities.