›es of Simulation Models

In this book we study two distinct types of simulation models: Monte Carlo simulation models and system simulation models. **Monte Carlo simulation** is basically a sampling experiment whose purpose is to estimate the distribution of an outcome variable that depends on several probabilistic input variables. For example, we might be interested in the distribution of profit for the financial model in Figure 1-2 when sales, costs, and inflation factors are random variables. The term *Monte Carlo simulation* was first used during the development of the atom bomb as a code name for computer simulations of nuclear fission. Researchers coined this term because of the similarity to random sampling in games of chance such as roulette in the famous casinos of Monte Carlo. Monte Carlo simulation is often used to evaluate the expected impact of policy changes and risk involved in decision making. **Risk** is often defined as the probability of occurrence of an undesirable outcome. Thus, we might be interested in the probability that 3-year profit will be less than a required amount. Monte Carlo simulation is the principal focus of Chapters 2 through 5.

Systems simulation, on the other hand, explicitly models sequences of events that occur over time. Thus, inventory, queueing, manufacturing, and material-handling problems are among the types of situations addressed with systems simulation. Systems simulation models are discussed in detail in Chapters 6, 7, and 8.

We illustrate a Monte Carlo simulation first using the following example.

EXAMPLE OF MONTE CARLO SIMULATION

Dave's Candies is a small family-owned business that offers gourmet chocolates and ice cream fountain service. For special occasions such as Valentine's Day, the store must place orders for special packaging several weeks in advance from their supplier. One product, Valentine's Day Chocolate Massacre, is bought for $7.50 a box and sells for $12.00. Any boxes that are not sold by February 14 are discounted by 50% and can always be sold easily. Historically, Dave's Candies has sold between 40 and 90 boxes each year with no apparent trend (either increasing or decreasing). Dave's dilemma is deciding how many boxes to order for the Valentine's Day customers. If demand exceeds the purchase quantity, Dave loses profit opportunity. On the other hand, if too many boxes are purchased, he will lose money by discounting them below cost.

We can easily develop an expression for Dave's profit if Q boxes are purchased and sales demand is D:

$$\text{profit} = \begin{cases} 12D - 7.50Q + 6(Q - D) & \text{if } D \le Q \quad \textbf{(1.1)} \\ 12Q - 7.50Q & \text{if } D > Q \quad \textbf{(1.2)} \end{cases}$$

In the first case, if demand is less than the amount ordered, Dave receives full revenue from the sales of D boxes, must pay for the Q boxes purchased, and receives half revenue for the surplus. In the second case, if demand exceeds the amount ordered, Dave can sell only Q boxes and makes a net profit of $12.00 – $7.50 = $4.50 per box.

The inputs to a simulation model of this situation would be:

1. The order quantity, Q (the decision variable)
2. The various revenue and cost factors (constants)
3. The demand, D (uncontrollable and probabilistic)

The model output we seek is the net profit.

If we know the demand, we can use equation (1.1) or (1.2) to compute the profit. Since demand is probabilistic, we need to be able to "sample" a value from the probability distribution of demand. For now, we simplify this problem by assuming that demand will be either 40, 50, 60, 70, 80, or 90 boxes with equal probability ($\frac{1}{6}$). This will allow us to generate samples by rolling a die. (In a later chapter we will see how to do this quite easily on a spreadsheet.) The following table associates the value of the roll of a die with one of the demand outcomes:

Roll of Die	*Demand*
1	40
2	50
3	60
4	70
5	80
6	90

We will perform a Monte Carlo simulation for an order quantity $Q = 60$. The simulation proceeds as follows:

1. Roll a die.
2. Determine the demand, D from the foregoing table.
3. Using $Q = 60$, compute the profit using equation (1.1) or (1.2).
4. Record the profit.

For example, suppose that the first roll of the die is 4. This corresponds to a demand of 70. Since $D = 70 > Q = 60$, we use equation (1.2) to compute the profit:

$$\text{profit} = 12(60) - 7.50(60) = \$270$$

By repeating the simulation, we can develop a distribution of profit and assess risk. Table 1-1 summarizes the results for 10 replications of this experiment. From the table, the average profit that Dave might expect using $Q = 60$ is \$246. We may

TABLE 1-1 Ten Replications of Dave's Candles Simulation Using $Q = 60$

Replication	*Role of Die*	*Demand*	*Profit*
1	5	80	\$270
2	3	60	270
3	2	50	210
4	4	70	270
5	1	40	150
6	3	60	270
7	5	80	270
8	6	90	270
9	2	50	210
10	3	60	270
		Average	\$246

also construct a frequency distribution of profits. We also see that 10% of the time profit is \$150, 20% of the time it is \$210, and 70% of the time it is \$270. This frequency distribution of profit provides an assessment of the risk involved in making the decision to order 60 boxes.

We might observe that if we repeated the simulation again, we could expect to roll different values of the die and will probably obtain a different value for the average profit as well as a different frequency distribution. This is an important insight into the nature of simulation: It is a sampling experiment that is itself uncertain. Therefore, we need to be able to quantify the uncertainty in our simulated results. Later in the book we shall see how to do this using basic statistical principles.

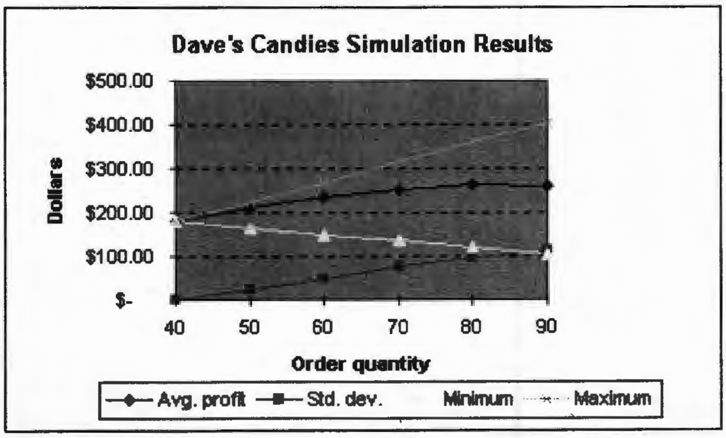
We might also observe that 10 replications will provide only limited results. For a larger number of replications, we would expect roughly an equal number of rolls of each value of the die. In this small experiment, we rolled a 2, 3, and 5 twice as often as a 1, 4, or 6. Thus, we might expect that our conclusions about the average profit and risk are somewhat biased and that the frequency distribution we obtained does not represent the true distribution of profit. We replicated this simulation 100 times and obtained the following frequency distribution:

Profit	*Frequency*
\$150	20
210	22
270	58

with an average profit of \$232.80. This average probably will be closer to the true expected value than what we obtained by using only 10 replications. Therefore, to obtain valid results with Monte Carlo simulation, we need to make a sufficiently large number of replications. Again, in a later chapter, we address this issue statistically.

Finally, the results in Table 1-1 are only descriptive; they do not tell us whether the order quantity $Q = 60$ is best. To find the best decision, we would have to experiment with different order quantities. Using a spreadsheet (which we describe in Chapter 2), we replicated the simulation 100 times for order quantities of 40, 50, 60, 70, 80, and 90. The summary results are shown in Figure 1-3. We see that the order quantity that maximizes the average profit is $Q = 80$, yielding an average profit of

FIGURE 1-3 Summary Results of 100 Replications



$251.40. Although the optimal order quantity can easily be determined analytically (see Problem 7 at the end of the chapter), simulation provides insights that an analytic model cannot. From Figure 1-3 we also see that as the order quantity increases, the standard deviation of profit, as well as the range of profits, also increases. This suggests that ordering high quantities, while providing more opportunity to realize higher profits, also increases the risk of obtaining a much lower profit. An order quantity of $Q = 80$, for example, might result in a profit as high as $360 or as low as $120. If Dave requires a certain contribution to profit to meet other expenses, the risk of gaining only a $120 profit by ordering 80 boxes might not seem so attractive. Simulation helps to provide an assessment of such risks.

This example showed the nature of Monte Carlo simulation: repeated sampling from probability distributions to develop the distribution of an output variable. The next example illustrates the nature of a systems simulation model, one that depends on the sequence of prior events and the passage of time.

EXAMPLE OF SYSTEMS SIMULATION

Mantel Manufacturing supplies various automotive components to major automobile assembly divisions on a just-in-time basis. The company has received a new contract for water pumps. Planned production capacity for water pumps is 100 units per shift. Because of fluctuations in customers' assembly operations, demand fluctuates and is historically between 80 and 130 units per day. To maintain sufficient inventory to meet its just-in-time commitments, Mantel's management is considering a policy to run a second shift if inventory falls to 50 or below. For the annual budget planning process, managers need to know how many additional shifts will be needed.

We may use simulation to analyze this situation. In this case, however, the inventory level depends on prior events, and we must simulate the passage of time in order to answer the question. The fundamental equation that governs this process each day is

$$\text{ending inventory} = \text{beginning inventory} + \text{production} - \text{demand} \qquad \textbf{(1.3)}$$

Suppose that we begin with an inventory of 100 units. As in the preceding example, we simplify the problem by assuming that demand occurs in increments of 10 so that we may use the roll of a die to randomly generate the demand each day. Thus, we associate the demand with the roll of a die as follows:

Roll of Die	*Demand*
1	80
2	90
3	100
4	110
5	120
6	130

The simulation process would proceed as follows:

1. Begin a new day.
2. Set the beginning inventory equal to the ending inventory from the previous day.
3. Determine the demand by rolling the die.
4. If the beginning inventory is 50 or less, the day's production is 200 units; otherwise, production is 100 units.

TABLE 1-2 Results of Five Simulated Days for Mantel Manufacturing Example

Day	*Beginning Inventory*	*Roll of Die*	*Demand*	*Production*	*Ending Inventory*
1	100	5	120	100	80
2	80	4	110	100	70
3	70	6	130	100	40
4	40	6	130	200	110
5	110	1	80	100	130

5. Use equation (1.3) to compute the ending inventory.
6. Stop if enough days have been simulated; otherwise, return to step 1.

Table 1-2 shows five simulated days. We see that the inventory falls below 50 on day 4, so a second shift is run, increasing the day's production to 200. Of course, five simulated days provide little meaningful information. Figure 1-4 shows the result of simulating this process for 100 days. The graph shows that six additional shifts were needed over the 100 days to maintain the desired inventory level. Extrapolating this to a 250-day working year, the company should expect to need about 2.5(6) = 15 additional shifts. As we noted in the preceding example, we should expect some variability in this result if we repeated the simulation or ran it for a longer period. Statistical methods will help us to quantify this variation. Management may also wish to experiment with different overtime policies to weigh the risks of running out of stock versus the costs of additional shifts.

The Simulation Process

Using simulation effectively requires careful attention to the modeling and implementation process. The simulation process consists of five essential steps:

1. ***Develop a conceptual model of the system or problem under study.*** This step begins with understanding and defining the problem, identifying the goals and objectives of the study, determining the important input variables, and defining output measures. It might also include a detailed logical description of the system that is being studied. Simulation

FIGURE 1-4 100-Day Production and Inventory Simulation