Design and Analysis of Experiments

INTRODUCTION

Experimental Design

Experiments are performed by investigators in virtually all fields of inquiry, usually to discover something about a particular process or system.

Experiment is a test.

Designed experiment is a test or series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes in the output response.

The process or system under study can be represented by the model shown in the figure below:

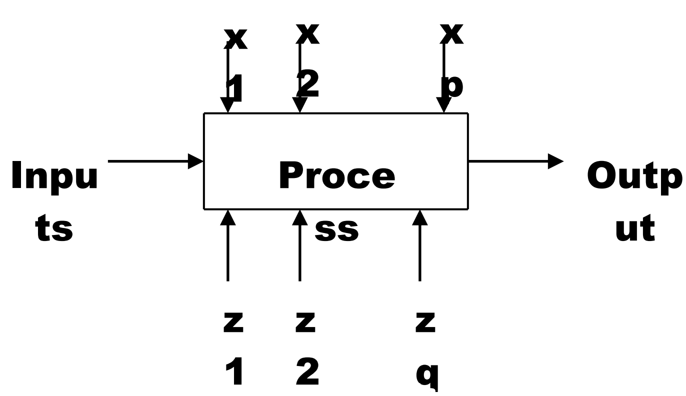

Figure: General model of a process or system

We can visualize the process as a combination of machines, methods, people, and other resources that transforms some input (often a material) into an output that has one or more observable responses.

Some of the process variables $x_1, x_2, \ldots, x_p$ are controllable, whereas other variables $z_1, z_2, \ldots, z_q$ are uncontrollable (although they may be controllable for purposes of a test).

The **objectives of the experiment** may include the following:

1. Determining which variables are most influential on the response y
2. Determining where to set the influential x's so that y is almost always near the desired nominal value
3. Determining where to set the influential x's so that variability in y is small.
4. Determining where to set the influential x's so that the effects of the uncontrollable variables $z_1, z_2, \ldots, z_q$ are minimized.

Experimental design methods **play an important role** in **process development** and **process troubleshooting** to **improve performance**.
The **objective** in many cases may be **to develop a robust process**, that is, a process affected minimally by external sources of variability (the z'_s).

*****In any experiment, **the results** and **conclusions** that can be drawn **depend** to a large extent **on the manner in which the data were collected**.

Applications of Experimental Design:

Experimental design methods, have found broad application in many disciplines. In fact we may view experimentation as part of the scientific process and as one of the ways we learn about how systems or processes work.

Generally, we learn through a series of activities in which we make conjectures about a process, perform experiments to generate data from the process, and then use the information from the experiment to establish new conjectures, which lead to new experiments, and so on.

Experimental design is a critically important tool in the engineering world for improving the performance of a manufacturing process.
It also has extensive application in the development of new processes.

The application of experimental design techniques **early in process development** can **result in**:

1. Improved process yields
2. Reduced variability and closer conformance to nominal or target requirements.
3. Reduced development time.
4. Reduced overall costs.

Experimental design also **plays a major role in engineering design activities**, **where new products are developed** and **existing ones are improved**.

Some **applications of Experimental design** are:

1. Evaluation and comparison of basic design configurations.
2. Evaluation of material alterations.
3. Selection of design parameters so that the product will work well under a wide variety of field conditions, that is, so that the product is robust.
4. Determination of key product design parameters that impact product performance.

The use of experimental design in these areas can result in:

- products that are easier to manufacture,
- products that have enhanced field performance and reliability,
- lower product cost,
- and shorter product design and development time.

Basic Principles of Experimental Design

If any **experiment** is to **be performed most efficiently**, then a **scientific approach** to **planning the experiments** must **be employed**.

By the **statistical design of experiments**, we refer to the process of planning the experiment so that **appropriate data** that can be analyzed by statistical methods will be collected, **resulting in valid** and **objective conclusions**.

*****The **statistical approach** to experimental design is **necessary** if we wish to draw **meaningful conclusions** from the data.

When the problem involves data that are subject to experimental errors, **statistical methodology is the only objective approach to analysis**.

Thus, there are two aspects to any experimental problem:

- The **design of the experiment** and
- the **statistical analysis** of the data.

These **two subjects** are **closely related** since the method of analysis depends directly on the design employed.

The three basic principles of experimental design are:

- **Replication**,
- **Randomization**, and
- **Blocking**.

Replication has two important properties:

First, it allows the experimenter to obtain an estimate of the experimental error. This estimate of error becomes a basic unit of measurement for determining whether observed differences in the data are really statistically different.

Second, if the sample mean (e.g. ÿ) is used to estimate the effect of a factor in the experiment, then replication permits the experimenter to obtain a more precise estimate of this effect.

e.g. If σ^2 is the variance of the data, and there are n replicates, then the variance of the sample mean is:

$$\sigma^2_{\ddot{y}} = \sigma^2/n$$

- If n=1, the experimental error will be high and we will be unable to make satisfactory inferences, since the observed difference in the sample mean could be the result of experimental error.

- If n is reasonably large, the experimental error will be sufficient small and we would be reasonably safe in conclusion.

Randomization is the **cornerstone** underlying the **use of statistical methods** in experimental design.

By **randomization** we mean that **both, the allocation of the experimental material and the order in which the individual runs** or trials of the experiment **are to be performed,** are **randomly** determined.

Statistical methods require that the observations (or errors) be independently distributed random variables. Randomization usually makes this assumption valid.

By properly **randomizing the experiment**, we also **assist in "averaging out"** the **effects** of **extraneous factors** that may be present.

Blocking is a technique used **to increase the precision of an experiment**.

A block is a portion of the experimental material that should be more homogeneous than the entire set of material.

Blocking involves making **comparisons among the conditions** of interest in the experiment **within each block**.

Guidelines for Designing Experiments:

To use the statistical approach in designing and analyzing, **it is necessary that everyone involved in the experiment have** a **clear idea in advance of:**

- **Exactly what is to be studied**,
- **How the data are to be collected**, **and**
- **At least a qualitative understanding of how these data are to be analyzed**.

An outline of the recommended procedure is as follows:

1. Recognition and statement of the problem

In practice it is often not simple neither to realize that a problem requiring experimentation exists, nor is it simple to develop a clear and generally accepted statement of this problem.

It is necessary to develop all ideas about the objectives of the experiment.

Usually, it is important to solicit input from all concerned parties:

- Engineering,
- Quality assurance,
- Manufacturing,
- Marketing,
- Management,
- The customer, and operating personnel
- Etc….

A clear statement of the problem often contributes substantially to a better understanding of the phenomena and the final solution of the problem.

2. Choice of factors and levels

The experimenter must choose the **factors to be varied in the experiment**, the **ranges** over which these factors will be varied, and the **specific levels** at which runs will be made. **Thought must** also **be given to how these factors** are **to be controlled at** the **desired values and how** they are **to be measured**.

3. Selection of the response variable

In selecting **the response variable**, the **experimenter should be certain** that **this variable** really **provides useful information about the process** under study. Most often, the **average or standard deviation** (or both) of the measured characteristic **will be** the **response variable**.

Multiple responses are not unusual.

4. Choice of experimental design

If the first three steps are done correctly, this step is relatively easy.

Choice of design **involves:**

- T**he consideration of sample size** (number of replicates),
- **The selection of a suitable run order** for the experimental trials,
- And **the determination of whether or not blocking** or other randomization restrictions are involved.

In selecting the design, it is important to keep the experimental objectives in mind.

5. Performing the experiment

When running the experiment, **it is vital to monitor the process carefully to ensure that everything is being done according to plan**.

Errors in experimental procedure at this stage will usually destroy experimental validity.

Up-front planning is crucial to success.

6. Data analysis

Statistical methods should be used to analyze the data so that results and conclusions are objective rather than judgmental in nature.

If the experiment has been designed correctly and if it has been performed according to the design, then the statistical methods required are not elaborate.

There are many excellent software packages designed to assist in data analysis, and simple graphical methods play an important role in data interpretation.

Residual analysis and model adequacy checking are also important analysis techniques.

Note: **Remember that:**

- **Statistical methods cannot prove that a factor (or factors) has a particular effect. They only provide guidelines as to the reliability and validity of results.**
- **Properly applied, statistical methods do not allow anything to be proved, experimentally, but they do allow us to measure the likely error in a conclusion or to attach a level of confidence to a statement.**
- **The primary advantage of statistical methods is that they add objectivity to the decision-making process.**

- **Statistical techniques coupled with good engineering or process knowledge and common sense will usually lead to sound conclusions.**

7. Conclusions and recommendations

Once the data have been analyzed,

- The experimenter must draw practical conclusions about the results and recommend a course of action.
- Graphical methods are often useful in this stage, particularly in presenting the results to others.
- Follow-up runs and confirmation testing should also be performed to validate the conclusions from the experiment.

Throughout this entire process, it is important to keep in mind that experimentation is an important part of the learning process, where;

- **We tentatively formulate hypotheses about a system,**
- **Perform experiments to investigate these hypotheses, and**
- **On the basis of the results formulate new hypotheses, and so on.**

This suggests that experimentation is iterative.

Using statistical techniques in experimentation

Much of the research in engineering, science and industry is empirical and makes extensive use of experimentation. Statistical methods can greatly increase the efficiency of these experiments and often strengthens the conclusions so obtained.

The intelligent use of statistical techniques in experimentation requires that the experimenter keep the following points in mind:

1. **Use your non-statistical knowledge of the problem.**
2. **Keep the design and analysis as simple as possible.**
3. **Recognize the difference between practical and statistical significance.**
4. **Experiments are usually iterative**

<u>Choice of Sample Size</u>:

The choice of sample size and the probability of type II error β are closely connected. Suppose that we are testing the hypothesis

Ho: $\mu_1 = \mu_2$

H_1: $\mu_1 \neq \mu_2$

And the means are not equal so that $\partial = \mu_1 - \mu_2$.
Since Ho is not true, we are concerned about wrongly failing to reject Ho. The probability of type II error depends on the true difference in means ∂.

A graph of β versus ∂ for a particular sample size is called the operating characteristic curve (or O. C. curve) for the test.

If β error is also a function of sample size, generally, for a given value of ∂, the β error decreases as the sample size increases.

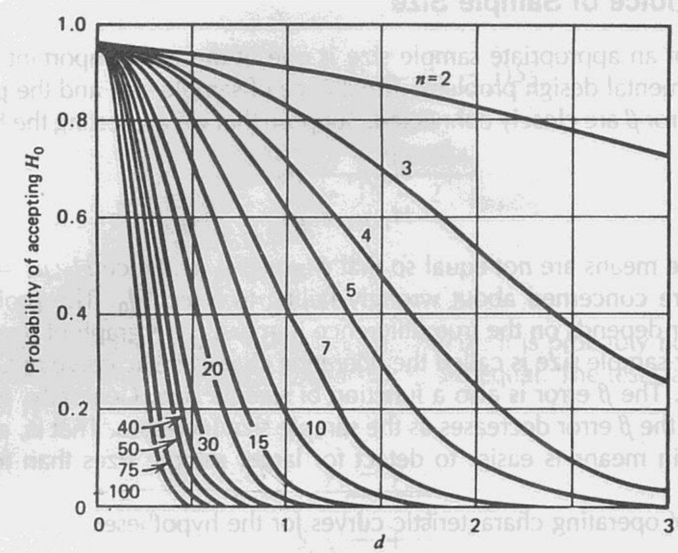

Figure 2-9. Operating characteristic curves for the two-sided *t* test with $\alpha = .05$. (Reproduced with permission from "Operating Characteristics for the Common Statistical Tests of Significance," C. L. Ferris, F. E. Grubbs, and C. L. Weaver, *Annals of Mathematical Statistics*, June 1946.)