Solution-Key for selected exercises from chapter-7

Exercise 3.

How could random numbers that are uniform on the interval [0,1] be transformed into random numbers that are uniform on the interval [-11,17]?

Solution:

Let $X = -11 + 28R$ Where: $R \sim U[0, 1]$

Exercise 4.

Use the linear congruential method to generate a sequence of three two-digit random integers. Let Xo=27, a=8, c=47, and m=100.

Solution:

$X_o=27$, $a=8$, $c=47$, $m=100$
$X_1=(8x27+47) \bmod 100 = 63$, → $R_1=63/100=0.63$
$X_2=(8x63+47) \bmod 100 = 51$, → $R_2=51/100=0.51$
$X_3=(8x51+47) \bmod 100 = 55$, → $R_1=55/100=0.55$

Exercise 7.

The sequence of numbers 0.54, 0.73, 0.98, 0.11, and 0.68 has been generated. Use the Kolmogorov-Smirnov test with $\alpha=0.05$ to determine if the hypothesis that the numbers are uniformly distributed on the interval [0,1] can be rejected.

Solution:

i	1	2	3	4	5
R_i	0.11	0.54	0.68	0.73	0.98
i/N	0.20	0.40	0.60	0.80	1.0
$(i/N)-R_i$	0.09	----	----	0.07	0.02
$R_i-[(i-1)/N]$	0.11	0.34	0.28	0.13	0.18

Therefore: $D^+=0.09$ and $D^-=0.34$
→ $D=0.34$ and $D_{0.05} = 0.565$ → Numbers are uniformly distributed [0,1].

Exercise 8.

Consider the following 50 two-digit values:

0.34	0.90	0.25	0.89	0.87	0.44	0.12	0.21	0.46	0.67
0.83	0.76	0.79	0.64	0.70	0.81	0.94	0.74	0.22	0.74
0.96	0.99	0.77	0.67	0.56	0.41	0.52	0.73	0.99	0.02
0.47	0.30	0.17	0.82	0.56	0.05	0.45	0.31	0.78	0.05
0.79	0.71	0.23	0.19	0.82	0.93	0.65	0.37	0.39	0.42

Use the chi-square test, with α=0.05, to determine if the hypothesis that the numbers are uniformly distributed on the interval [0, 1] can be rejected.

Solution:

Let's have 10 intervals → E_i=50/10=5.

Interval	O_i	Ei	$[(O_i-E_i)^2]/E_i$
[0-0.1)	3	5	0.8
[0.1-0.2)	3	5	0.8
[0.2-0.3)	4	5	0.2
[0.3-0.4)	5	5	0
[0.4-0.5)	6	5	0.2
[0.5-0.6)	3	5	0.8
[0.6-0.7)	4	5	0.2
[0.7-0.8)	10	5	5
[0.8-0.9)	6	5	0.2
[0.9-1)	6	5	0.2
Total:	50	50	8.4

Since $\chi^2_o = 8.4 < \chi^2_{.05,9}=16.9$ → We fail to reject the null hypothesis of no difference between the sample distribution and the uniform distribution.

Exercise 9.

Consider the following 50 two-digit values:

0.34	0.90	0.25	0.89	0.87	0.44	0.12	0.21	0.46	0.67
0.83	0.76	0.79	0.64	0.70	0.81	0.94	0.74	0.22	0.74
0.96	0.99	0.77	0.67	0.56	0.41	0.52	0.73	0.99	0.02
0.47	0.30	0.17	0.82	0.56	0.05	0.45	0.31	0.78	0.05
0.79	0.71	0.23	0.19	0.82	0.93	0.65	0.37	0.39	0.42

Based on runs up and runs down, determine whether the hypothesis of independence can be rejected, where $\alpha=0.05$.

Solution:

Runs up and Runs down→

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a=27, N=50

Therefore; $\mu_a= (2N-1)/3=33$, and $\sigma^2_a=(16N-29)/90=8.57$

$Z_o= (a- \mu_a)/ \sigma_a = -2.05$, $Z_{.025}=1.96$

Reject the Null Hypothesis of Independence.

Exercise 10. Consider the following 50 two-digit values:

0.99	0.17	0.99	0.46	0.05	0.66	0.10	0.42	0.18	0.49
0.37	0.51	0.54	0.01	0.81	0.28	0.69	0.34	0.75	0.49
0.72	0.43	0.56	0.97	0.30	0.94	0.96	0.58	0.73	0.05
0.06	0.39	0.84	0.24	0.40	0.64	0.40	0.19	0.79	0.62
0.18	0.26	0.97	0.88	0.64	0.47	0.60	0.11	0.29	0.78

Determine whether there is an excessive number of runs above or below the mean. Use $\alpha=0.05$.

Sloution: Runs above or below the mean→

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b=31 runs, $n_1=24$, $n_2=26$, $N=n_1+n_2=50$

$\mu_b= [(2 n_1 n_2)/N] +1/2=25.46$

$\sigma^2_b= [2 n_1 n_2(2 n_1 n_2 -N)]/[N^2(N-1)] =12.21$

$Z_o= (b- \mu_b)/ \sigma_b =1.59$, $Z_{.025}=1.96$

Therefore; we fail to reject the Null Hypothesis of Independence.

Exercise 11. Consider the 50 two-digit values below. Can the hypothesis that the numbers are independent be rejected on the basis of the length of runs up and down when $\alpha=0.05$?

0.34, 0.90, 0.25, 0.89, 0.87, 0.44, 0.12, 0.21, 0.46, 0.67
0.83, 0.76, 0.79, 0.64, 0.70, 0.81, 0.94, 0.74, 0.22, 0.74
0.96, 0.99, 0.77, 0.67, 0.56, 0.41, 0.52, 0.73, 0.99, 0.02
0.47, 0.30, 0.12, 0.82, 0.56, 0.05, 0.45, 0.31, 0.78, 0.95
0.79, 0.71, 0.23, 0.19, 0.82, 0.93, 0.65, 0.37, 0.39, 0.42

Solution:

+, -, +, -, -, -, +, +, +, +,
-, +, -, +, +, +, -, -, +, +,
+, -, -, -, -, +, +, +, -, +,
-, -, +, -, -, +, -, +, -, +,
-, -, -, +, +, -, -, +, +,

Therefore, the length of runs up and down→

1, 1, 1, 3, 4, 1, 1, 1, 3, 2, 3, 4, 3, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 3, 2, 2, 2

$E(y_i) =$ { (2/(i+3)!)[N(i2+3i+1)-(i3+3i2-i-4)], for all $i \le N-2$ }
{ 2/N!, for $i=N-1$ }

Therefore; $E(y_1) = (2/24)\,[50(5) -(-1)] = 20.92$
$E(y_2) = 8.93$, $E(y_3) = 2.51$

$\mu_a = (2N-1)/3 = 99/3 = 33$

$E(y_{i\geq4}) = \mu_a - E(y_1) - E(y_2) - E(y_3) = 0.64$

Run Length (i)	Observed Runs (O_i)	Expected Runs $E(y_i)$	$\frac{[O_i-E(y_i)]^2}{E(y_i)}$
1	14	20.92	2.29
2	6	8.93	
3	5	2.51	0.07
≥4	2	0.64	
		$\chi^2_o =$	2.36

Since $\chi^2_{.05,1} = 3.84$→ Independence Null Hypothesis cannot be rejected.

Exercise 12. Consider the 50 two-digit values in exercise-11 (above). Can the hypothesis that the numbers are independent be rejected on the basis of the length of runs above and below the mean, when $\alpha=0.05$?

Solution: Length of runs above and below the mean→

1, 1, 1, 2, 4, 9, 1, 6, 1, 3, 4, 2, 3, 1, 1, 2, 2, 3, 3

Therefore; $n_1=29$, $n_2=21$, $b=19$ runs, $N=50$

Run Length (i)	1	2	3	≥4
Observed Runs (O_i)	7	4	4	4

w_1= 2(29/50) (21/50) = 0.24
w_2= $(29/50)^2(21/50)$ + (29/50) $(21/50)^2$= 0.24
w_3= $(29/50)^3(21/50)$ + (29/50) $(21/50)^3$= 0.12
E(I) =29/21 + 21/29 = 2.11, E(A)= 50/2.11 = 23.7,
$E(y_1)$ = [50(0.24)/2.11] = 5.68, $E(y_2)$ = [50(0.24)/2.11] =5.69,
$E(y_3)$ = [50(0.12)/2.11] = 2.84
EY) ≥4 = 23.7- (5.68+5.69+2.84) =9.49

Run Length (i)	Observed Runs (O_i)	Expected Runs ($E(y_i)$)	$\frac{[O_i-E(y_i)]^2}{E(y_i)}$
1	19	5.68	3.41
2	8	5.69	0.5
3	2	2.84	
≥4	2	9.49	0.83
		χ^2_o =	4.74

Run Length (i)	Observed Runs (O_i)	Expected Runs ($E(y_i)$)	$\frac{[O_i-E(y_i)]^2}{E(y_i)}$
1	19	5.68	3.41
2	8	5.69	0.5
≥3	4	12.33	5.63
		χ^2_o =	9.54

Since $\chi^2_{.05,2}$=5.99→ Independence Null Hypothesis should be rejected.

Exercise 15 (b). Develop the poker test for five-digit numbers.

Solution:

P(5 different digits)=0.9x0.8x0.7x0.6=0.3024
P(exactly one triplet)= $\binom{5}{3}$(0.1)(0.1)(0.9(0.8)=0.072
P(triplet and a pair)= $\binom{5}{3}$(0.1)(0.1)(0.9)(0.1)=0.009
P(4 like digits)= $\binom{5}{4}$(0.1)(0.1)(0.1)(0.9)=0.0045
P(5 like digits)= (0.1)(0.1)(0.1)(0.1)=0.0001
P(exactly one pair)= $\binom{5}{2}$(0.1)(0.9)(0.8)(0.7)=0.504
P(2 different pairs)=1-(0.3024+0.072+0.009+0.0045+0.0001+0.504)=0.108