Solved Exercises from textbook edition 5, Chapter 2

Problem 2-1(Similar to Problem 2-16 Edition 8 of the Textbook): The mean breaking strength of a fiber is required to be at least 150 psi.

Given: σ=3 psi, n=4 and $\bar{Y}$=148.75

a)- State the hypotheses that you think should be tested?

Ho: $\mu=150$, H_1: $\mu<150$

b)- Test these hypotheses using $\alpha=0.05$. What are your conclusions?

$Z_o= (\bar{Y}-\mu_o)/(\sigma/\sqrt{n})= (148.75 -150)/(3/2)=-0.83$
From standard normal distribution table we read $Z_\alpha=1.65$.
As a rule, we should reject Ho if $Z_o < -Z_\alpha$
Therefore, we fail to reject Ho. This implies the mean breaking strength of the fiber is at least 150 psi.

d)- Construct a 95% Confidence Interval on the mean breaking strength of the fiber?

$\bar{Y} \pm Z_{\alpha/2} *(\sigma/\sqrt{n})$ Therefore: 148.75 ± (1.96*3)/2 →
$145.81 \leq \mu \leq 151.69$.

Problem 2-2(Similar to Problem 2-17 Edition 8 of the Textbook): The viscosity of a liquid detergent is supposed to average 800 centistokes at 25°C. A random sample of 16 batches of detergent is collected, and the average viscosity is 812. Suppose we know that the standard deviation of viscosity is 25 centistrokes.

Therefore; n=16, $\bar{Y}$=812, σ=25
a)- State the hypotheses that should be tested?
Ho: $\mu=800$, H_1: $\mu\neq800$

b)- Test these hypotheses using $\alpha=0.05$. What are your conclusions?

Given $\alpha=0.05$ → $Z_o= (\bar{Y}-\mu_o)/(\sigma/\sqrt{n})= (812 -800)/(25/4)=1.92$
From standard normal distribution table we read $Z_{\alpha/2}=1.96$.
Since $-Z_{\alpha/2}=-1.96<Z_o=1.92< Z_{\alpha/2}=1.96$→
We fail to reject Ho. This implies the viscosity of a liquid detergent can be 800 centistokes at 25_o C.

d)- Construct a 95% Confidence Interval on the mean viscosity of the liquid detergent?

$\bar{Y} \pm Z_{\alpha/2} *(\sigma/\sqrt{n})$ → Therefore: 812 ± (1.96*25)/4 →
$799.75 \leq \mu \leq 824.25$

Problem 2-3(Similar to Problem 2-19 Edition 8 of the Textbook): The diameters of steel shafts produced by a certain manufacturing process should have a mean diameter of 0.255 inches. The diameter is known to have a standard deviation of 0.0001 inches. A random sample of 10 shafts has an average diameter of 0.2545 inch.

Therefore: μ_o = 0.255 inches, σ = 0.0001 inches, n = 2410 and $\bar{Y}$ = 0.2545 inches.

a)- Set up appropriate hypotheses on the mean μ.

Ho: μ=0.255, H_1: μ≠0.255

b)- Test these hypotheses using α=0.05. What are your conclusions?

$Z_o= (\bar{Y}-\mu_o)/(\sigma/\sqrt{n})= (0.2545 -0.255)/(0.0001/\sqrt{10})= -15.81$
From standard normal distribution table we read $Z_{\alpha/2}=1.96$.
As a rule, we should reject Ho if $|Z_o| > Z_{\alpha/2}$.
Therefore, we reject Ho. This implies the mean diameter of steel shafts is different than 0.255 inches.

d)- Construct a 95% Confidence Interval on the mean diameter of steel shafts?

$\bar{Y} \pm Z_{\alpha/2} *(\sigma/\sqrt{n})$ Therefore: $0.2545 \pm (1.96*0.0001)/ \sqrt{10} \rightarrow$
$0.2544 \leq \mu \leq 0.2546$

Problem 2-5(**Similar to Problem 2-20 Edition 8 of the Textbook**)**:** The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days: 108, 124, 124, 106, 115, 138, 163, 159, 134, 139.

Therefore; n=10, S=19.54, $\bar{Y}$=131

a)- We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

Ho: μ = 120 and H_1: μ > 120

b)- Test these hypotheses using α=0.01. What are your conclusions?

Since σ is unknown→ $t_o= (\bar{Y}-\mu_o)/(S/\sqrt{n})= (131 - 120)/(19.54/\sqrt{10})= 1.78$

From t-distribution table we read $t_{\alpha, n-1} = t_{0.01,9} = 2.821$

Since $t_o=1.78< t_{0.01,9}=2.821$→ We fail to reject Ho. → We could not prove the mean shelf life to be larger than 120 days.

d)- Construct a 99% Confidence Interval on the mean shelf life?

$\bar{Y} \pm t_{\alpha/2, n-1} *(S/\sqrt{n})$; Therefore: $131 \pm (3.25*19.54)/ \sqrt{10} \rightarrow 110.91 \leq \mu \leq 151.09$

Problem 2-7(**Similar to Problem 2-22 Edition 8 of the Textbook**): The time to repair an electronic instrument is a normally distributed random variable in hours. The repair times for 16 such instruments chosen at random are as follows:

Hours: 159, 224, 222, 149, 280, 379, 362, 260, 101, 179, 168, 485, 212, 264, 250, 170

Therefore; $\bar{Y}$ =241.5 and S=98.7259

a)- You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

Ho: μ = 225 and H_1: μ > 225

b)- Test the hypotheses you formulated in part (a). What are your conclusions? Use α=0.05.

Since б is unknown→ $t_o = (\bar{Y}-\mu_o)/(S/\sqrt{n}) = (241.5 - 225)/(98.7259/\sqrt{16}) = 0.6685$
From t-distribution table we read $t_{\alpha, n-1} = t_{0.05,15} = 1.753$.
Since; $t_o=0.6685 < t_{0.05,15}=1.753$→ We fail to reject Ho. → Mean repair time does not exceeds 225 hours.

d)- Construct a 95% Confidence Interval on the mean shelf life?
$\bar{Y} \pm t_{\alpha/2, n-1} *(S/\sqrt{n})$ → therefore: $241.5 \pm (2.131*98.7259)/\sqrt{16}$ →
$188.9 \leq \mu \leq 294.1$

Problem 2-9(Similar to Problem 2-24 Edition 8 of the Textbook): Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The filling processes can be assumed to be normal, with standard deviations of $б_1$ = 0.015 ounces and $б_2$ = 0.018 ounces. The quality engineering department suspects that both machines fill to the same net volume, whether or not this volume is 16.0 ounces. An experiment is performed by taking a random sample from the output of each machine.

Machine 1		Machine 2	
16.03	16.01	16.02	16.03
16.04	15.96	15.97	16.04
16.05	15.98	15.96	16.02
16.05	16.02	16.01	16.01
16.02	15.99	15.99	16.00

Therefore: $б_1$ = 0.015, $б_2$ = 0.018 and μ_o= 16.0 Ounces.

a) State the hypotheses that should be tested in this experiment?

Ho: $\mu_1 = \mu_2$, H_1: $\mu_1 \neq \mu_2$

b)- Test the hypotheses using α=0.05. What are your conclusions?

α=0.05, $\bar{Y}_1$ = 16.015, $\bar{Y}_2$ = 16.005, S_1= 0.03 and S_2 = 0.025

$Z_o = (\bar{Y}_1 - \bar{Y}_2)/\sqrt{((б_{12}/n_1)+(б_{22}/n_2))} = (16.02-16.01)/\sqrt{((0.015)^2/10)+(0.018)^2/10)} = 1.35$
$Z_{0.025}$=1.96 From Standard Normal Distribution Table.

Since $-Z_{0.025}=-1.96<Z_o=1.35<Z_{0.025}=1.96$→ So, we fail to reject Ho Therefore; both machines can fill to the same net volume.

d)- Find a 95 % Confidence Interval on the difference in mean fill volume for the two machines.

$(\bar{Y}_1 - \bar{Y}_2) \pm Z_{\alpha/2} \sqrt{((б_{12}/n_1)+(б_{22}/n_2))}$ →
$-0.005 \leq \mu_1-\mu_2 \leq 0.025$

Problem 2-10(Similar to Problem 2-26 Edition 8 of the Textbook):

Two types of plastics are suitable for use by an electronic calculator manufacturer. The breaking strength of this plastic is important. It is known $\sigma_1 = \sigma_2 = 1.0$ psi. From random samples of n1=10 and n2=12 we obtain $\bar{Y}_1=162.5$ and $\bar{Y}_2=155.0$. The company will not adopt plastic-1 unless its breaking strength exceeds that of plastic-2 by at least 10 psi. Based on the sample information, should they use plastic 1? In answering this question, set up and test appropriate hypotheses using $\alpha=0.01$. Construct a 99 percent confidence interval on the true mean difference in breaking strength.

Ho: $\mu_1 = \mu_2 + 10$ and H_1: $\mu_1 < \mu_2 + 10$

$Z_o = (\bar{Y}_1 - (\bar{Y}_2+10))/\sigma\sqrt{((1/n_1)+(1/n_2))} = (162.5-(155+10)/1*\sqrt{((1/10)+(1/12))} = -5.8387$
$Z_{0.01} = 2.33$ From Standard Normal Distribution Table.
Since $Z_o=-5.83874 < -Z_{0.01}=-2.33 \rightarrow$ we reject Ho Therefore; they cannot adopt plastic-1.

A 99 % Confidence Interval on the mean difference: Given $Z_{\alpha/2}=2.58 \rightarrow$
$(\bar{Y}_1 - \bar{Y}_2) \pm Z_{\alpha/2}\sqrt{((\sigma_1^2/n_1)+(\sigma_2^2/n_2))} \rightarrow$
$6.395 \le \mu_1-\mu_2 \le 8.605$

Problem 2-11(Similar to Problem 2-25 Edition 8 of the Textbook):

The following are the burning times of chemical flares of two different formulations. The design engineers are interested in both the mean and variance of the burning times.

Type 1		Type 2	
65	82	64	56
81	67	71	69
57	59	83	74
66	75	59	82
82	70	65	79

(a)- Test the hypothesis that the two variances are equal. Use $\alpha=0.05$.

Ho: $\sigma^2_1=\sigma^2_2$ and H1: $\sigma^2_1 \neq \sigma^2_2$

$F_o = S^2_1/S^2_2 = 85.82/87.73 = 0.98$ Note: $F_{(1-\alpha),m_1,m_2} = \frac{1}{F_{\alpha,m_2,m_1}}$
Since; $F_{0.025,9,9} = 4.03$ and $F_{0.975,9,9} = 1 / F_{0.025,9,9} = 1 / 4.03 = 0.248$; Therefore, we failed to reject Ho

(b)- Using the results of (a), test the hypothesis that the mean burning times are equal.

Ho: $\mu_1 = \mu_2$ and H_1: $\mu_1 \neq \mu_2$
$t_o = (\bar{Y}_1 - (\bar{Y}_2))/S_p * \sqrt{((1/n_1)+(1/n_2))}$, Where $S^2_p = 86.78 \rightarrow t_o = 0.048$
$t_{0.025,18} = 2.101$
Therefore, we failed to reject Ho.

Problem 2-12(Similar to Problem 2-27 Edition 8 of the Textbook):

An article in Solid State Technology, "Orthogonal Design for Process Optimization and Its Application to Plasma Etching" by G. Z. Yin and D. W. Jillie (May, 1987) describes an experiment to determine the effect of the C2F6 flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. Data for two flow rates are as follows:

C_2F_6 Flow (SCCM)	Uniformity Observation					
	1	2	3	4	5	6
125	2.7	4.6	2.6	3	3.2	3.8
200	4.6	3.4	2.9	3.5	4.1	5.1

a)Does the C_2F_6 flow rate affect average etch uniformity? Use $\alpha=0.05$.

Ho: $\mu_1= \mu_2$ H1: $\mu_1 \neq \mu_2$

Since; σ_1^2 & σ_2^2 are unknown, so we should first test $\sigma_1^2 = \sigma_2^2$?

Ho: $\sigma_1^2 = \sigma_2^2$ and H1: $\sigma_1^2 \neq \sigma_2^2$

Fo= S_1^2/S_2^2= 0.5777/0.6747=0.856
As a rule we should reject Ho if Fo > $F_{\alpha/2, n1-1,n2-1}$ or Fo < $F_{1-\alpha/2, n1-1,n2-1}$.
As a hint; $F_{1-\alpha/2, n1-1,n2-1}= 1/ F_{\alpha/2, n2-1,n1-1}$.
Since $F_{0.975, 5,5}$=0.14< Fo=0.856 < $F_{0.025, 5,5}$=7.15→ Variances can be equal.

Then: Ho: $\mu_1= \mu_2$ H1: $\mu_1 \neq \mu_2$
to= $(\bar{Y}_1- (\bar{Y}_2))/S_p * \sqrt{((1/n_1)+ (1/n_2))}$, Where S_{p2}= 0.6262→
to= -1.3498
Since ltol= 1.3498 < $t_{0.025,10=}$ 2.228
So, we fail to reject Ho.

Problem 2-13(Similar to Problem 2-29 Edition 8 of the Textbook):

A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of imourity: $\bar{Y}_1$= 12.5, S^2_1=101.17 and n_1=8. After installation, a random sample yielded $\bar{Y}_2$= 10.2, S^2_2=94.73 and n_2=9.

a)- Can you conclude that the two variances are equal? Use $\alpha=0.05$.

Ho: $\sigma^2_1=\sigma^2_2$ and H1: $\sigma^2_1 \neq \sigma^2_2$

Fo = S^2_1/S^2_2 = 101.17/94.73= 1.068
Since; $F_{0.025,7,8}$ = 4.53 and $F_{0.975,7,8}$ = 1 / $F_{0.025,8,7}$ = 1 / 4.9 = 0.204; Therefore, we failed to reject Ho.

b)- Has the filtering device reduced the percentage of impurity significantly? Use $\alpha=0.05$.

Ho: $\mu_1= \mu_2$ and H1: $\mu_1 > \mu_2$
to= $(\bar{Y}_1- (\bar{Y}_2))/S_p * \sqrt{((1/n_1)+ (1/n_2))}$= 0.47, Where S^2_p= 97.74→ S_p= 9.89
$t_{0.05, 15}$ = 1.753,
Therefore, we failed to reject Ho.

Therefore, the filtering device does not reduce the impurity significantly.

Problem 2-14(Similar to Problem 2-30 Edition 8 of the Textbook):

Twenty observations on etch uniformity on silicon wafers are taken during qualification experiment for a plasma etcher. The data are as follows:

5.34, 6.65, 4.76, 5.98, 7.25, 6.00, 7.55, 5.54, 5.62, 6.21, 5.97, 7.35, 5.44, 4.39, 4.98, 5.25, 6.35,

4.61, 6.00, 5.32

Therefore; $\bar{y}$= 5.83, S^2=0.79 and S=0.889

a)- Construct a 95 percent confidence interval estimate of σ^2.

$\chi^2_{\alpha/2,\ n-1} = \chi^2_{0.025/2,\ 19}$ =32.85 and $\chi^2_{1-\alpha/2,\ n-1} = \chi^2_{0.975,19}$=8.91

$[(n-1)\ S^2]/\ \chi^2_{\alpha/2,\ n-1} \le \sigma^2 \le [(n-1)\ S^2]/\ \chi^2_{1-\alpha/2,\ n-1}$ → $(19x0.79)/32.85 \le \sigma^2 \le (19x0.79)/8.91$ →

$0.46 \le \sigma^2 \le 1.68$

b)- Test the hypothesis that σ^2=1.0. Use α=0.05. What are your conclusions?

Ho: σ^2=1.0 and H1: $\sigma^2 \ne 1.0$

$\chi^2_0 = [(n-1)\ S^2]/\ \chi^2_0 = (19x0.79)/1$= 15.01 Since it is not larger than 32.85 and not smaller than 8.91, we would fail to reject Ho.

c)- Discuss the normality assumption and its role in the problem.

The test on Variances is very sensitive to normality assumption.