$m = x^5 + 2x^3 - 1$ , $b = x^3 - 4x + 1$ , (1)

$b^{-1} \bmod m$ – ? in $Z_7$

1) $A = (1, 0, m)$ , $B = (0, 1, b)$

$q = quo(A3/B3) \rightarrow$

$$\begin{array}{r|l} x^5 + 2x^3 - 1 & x^3 - 4x + 1 \\ \hline x^5 - 4x^3 + x^2 & x^2 + 6 - quo \\ \hline 6x^3 - x^2 - 1 & \\ 6x^3 - 24x + 6 & \\ \hline -x^2 + 23x - 7 \bmod 7 = & \\ -x^2 + 3x - rem & \end{array}$$

$T = A - qB$

$T = (1, -x^2 - 6, -x^2 + 3x)$

2) $A = (0, 1, x^3 - 4x + 1)$ , $B = (1, -x^2 - 6, -x^2 + 3x)$

$q = quo(A3/B3)$

$$\begin{array}{r|l} x^3 - 4x + 1 & -x^2 + 3x \\ \hline x^3 - 3x^2 & -x - 3 - quo \\ \hline 3x^2 - 4x + 1 & \\ 3x^2 - 9x & \\ \hline 5x + 1 - rem & \end{array}$$

$T = (x + 3, 1 - (x+3)(x^2+6), 5x+1) =$

$= (x+3, 1 - x^3 - 6x - 3x^2 - 18, 5x+1) =$

$= (x+3, -x^3 - 3x^2 - 6x - 17, 5x+1) =$

$= (x+3, -x^3 - 3x^2 - 6x - 3, 5x+1)$

3) $A = (1, -x^2 - 6, -x^2 + 3x)$ , $B = (x+3, -x^3 - 3x^2 - 6x - 3, 5x+1)$

$q = quo(A3/B3)$

$$\begin{array}{r|l} -x^2 + 3x & 5x + 1 \\ \hline -15x^2 - 3x & -3x - 3 \\ \hline 6x & \\ -15x - 3 & \\ \hline 21x + 3 \bmod 7 = 3 & \end{array}$$

$T = (1 + (3x+3)(x+3), -x^2 - 6 - (3x+3)(x^3 + 3x^2 + 6x + 3), 3)$

$= (1 + 3x^2 + 3x + 9x + 9, -x^2 - 6 - 3x^4 - 3x^3 - 9x^3 - 9x^2 - 18x^2 - 18x - 9x - 9, 3) =$

$= (3x^2 + 12x + 10, -3x^4 - 12x^3 - 28x^2 - 27x - 15, 3)$ (2)

$= (3x^2 + 12x + 10, -3x^4 - 5x^3 - 6x - 1, 3)$

$B3 = T3 = 3 \rightarrow$

inverse $= B2/B3 \; T2/T3 = -\frac{3x^4 + 5x^3 + 6x + 1}{3} =$

$= -(3x^4 + 5x^3 + 6x + 1) \; 5 \bmod 7 = |3^{-1} \bmod 7 = 5|$

$= -15x^4 - 25x^3 - 30x - 5 \bmod 7 = -x^4 - 4x^3 - 2x - 5 =$

$= 6^{-1} \bmod m \qquad = 6x^4 + 3x^3 + 5x + 2$

Check:

$6^{-1} \cdot 6 \bmod m = 1$ ?

$(6x^4 + 3x^3 + 5x + 2)(x^3 - 4x + 1) =$

$= 6x^7 - 24x^5 + 6x^4 + 3x^6 - 12x^4 + 3x^3 + 5x^4 - 20x^2 +$

$+ 5x + 2x^3 - 8x + 2 = 6x^7 + 3x^6 - 24x^5 - x^4 + 5x^3 -$

$- 20x^2 - 3x + 2 = 6x^7 + 3x^6 - 3x^5 - x^4 + 5x^3 - 6x^2 - 3x + 2$

$\overline{6x^7} + 3x^6 - 3x^5 - x^4 + 5x^3 \overline{- 6x^2} - 3x + 2 \;\big|\; x^5 + 2x^3 - 1$

$- \overline{6x^7 + 12x^5 - 6x^2} \qquad\qquad 6x^2 + 3x - 1$

$\overline{3x^6} - 15x^5 - x^4 + 5x^3 \overline{- 3x} + 2$

$- \overline{3x^6 + 6x^4 - 3x}$

$-15x^5 - 7x^4 + 5x^3 + 2$

$= -\overline{15x^5} + 5x^3 + 2$

$- \overline{-15x^5 - 5x^3 + 1}$

$1$