EENG224 Homework I

1. Find $\boldsymbol{i_x(t)}$ if $\boldsymbol{v_s(t)}$ = 20 Cos(100t-40°) in the circuit of Fig.1

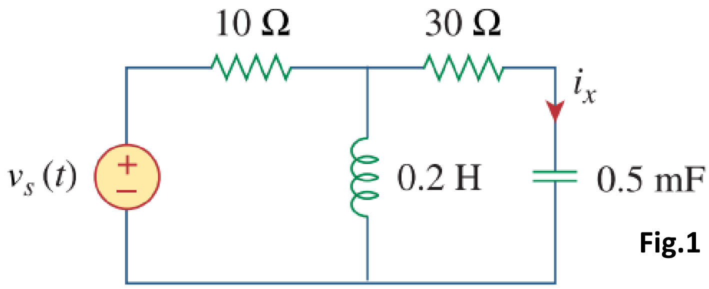
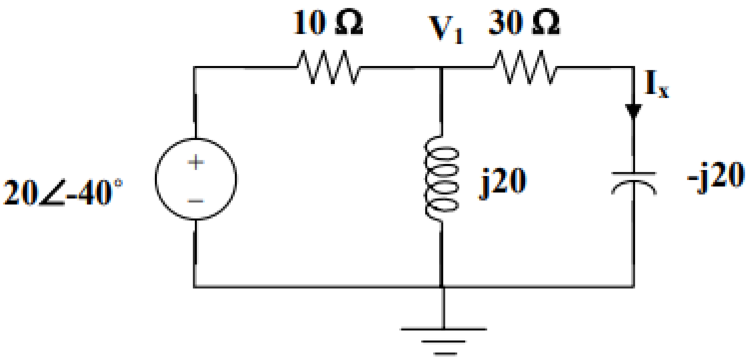


Fig.1

Soln.:

Converting the circuit to the frequency domain, we get:



We can solve this using nodal analysis.

$$\frac{V_1 - 20\angle -40^\circ}{10} + \frac{V_1 - 0}{j20} + \frac{V_1 - 0}{30 - j20} = 0$$

$$V_1(0.1 - j0.05 + 0.02307 + j0.01538) = 2\angle -40^\circ$$

$$V_1 = \frac{2\angle 40^\circ}{0.12307 - j0.03462} = 15.643\angle -24.29^\circ$$

$$I_X = \frac{15.643\angle -24.29^\circ}{30 - i20} = 0.4338\angle 9.4^\circ$$

$$i_X = 0.4338 \ \text{Cos}\ (100t + 9.4^\circ)\, A$$

2. By nodal analysis, find $i_o(t)$ in the circuit of Fig.2

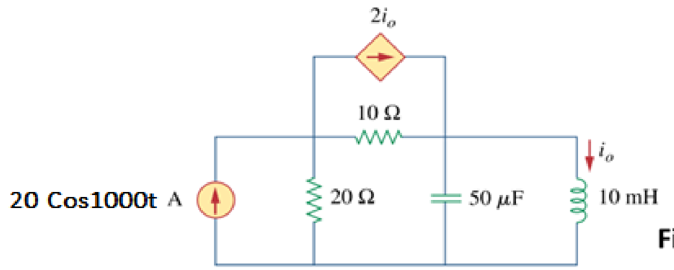


Fig.2

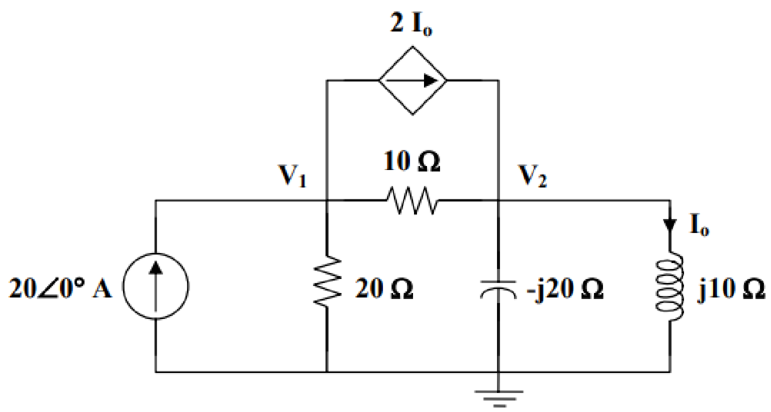
Soln.:

$$20\ \text{Cos}(1000t) \longrightarrow 20\angle 0°, \quad \omega = 1000$$

$$10\ \text{mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

Converting the circuit to the frequency domain, we get:



At node 1,

$$20 = 2\mathbf{I}_o + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}, \quad \text{where} \quad \mathbf{I}_o = \frac{\mathbf{V}_2}{j10}$$

$$20 = \frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}$$

$$400 = 3\mathbf{V}_1 - (2 + j4)\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{\mathbf{V}_2}{-j20} + \frac{\mathbf{V}_2}{j10}$$

$$j2\,\mathbf{V}_1 = (-3 + j2)\,\mathbf{V}_2$$

or

$$\mathbf{V}_1 = (1 + j1.5)\,\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$400 = (3 + j4.5)\mathbf{V}_2 - (2 + j4)\mathbf{V}_2 = (1 + j0.5)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{400}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10} = \frac{40}{j(1 + j0.5)} = 35.74\angle -116.6°$$

Therefore, $i_o(t) =$ **35.74 Cos (1000t – 116.6°) A**

3. By using mesh analysis, find I_1 and I_2 in the circuit in Fig. 3.

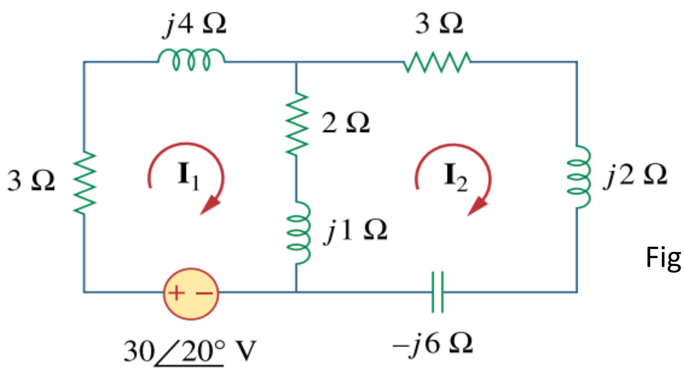


Fig.3

Soln.:

For mesh 1,

$$(5 + j5)\mathbf{I}_1 - (2 + j)\mathbf{I}_2 - 30\angle 20° = 0$$

$$30\angle 20° = (5 + j5)\mathbf{I}_1 - (2 + j)\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$(5 + j3 - j6)\mathbf{I}_2 - (2 + j)\mathbf{I}_1 = 0$$

$$0 = -(2 + j)\mathbf{I}_1 + (5 - j3)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 30\angle 20° \\ 0 \end{bmatrix} = \begin{bmatrix} 5 + j5 & -(2 + j) \\ -(2 + j) & 5 - j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 37 + j6 = 37.48\angle 9.21°$$

$$\Delta_1 = (30\angle 20°)(5.831\angle -30.96°) = 175\angle -10.96°$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \underline{\mathbf{4.67\angle -20.17°\ A}}$$

$$\Delta_2 = (30\angle 20°)(2.356\angle 26.56°) = 67.08\angle 46.56°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{1.79\angle 37.35°\ A}}$$

4. In the circuit shown in Fig.4, use **source transformation** to find the current $\boldsymbol{I_O}$

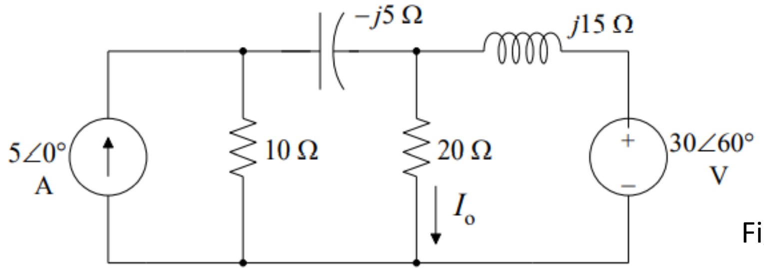
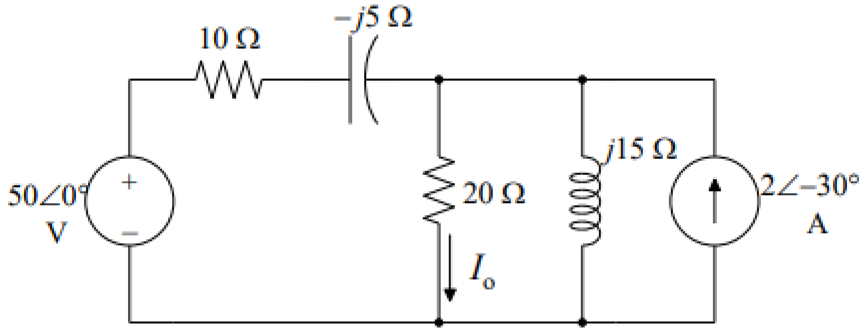


Fig.4

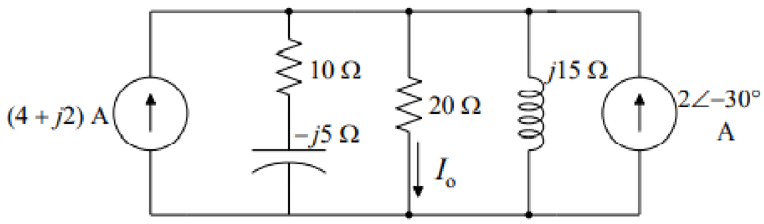
Soln.:

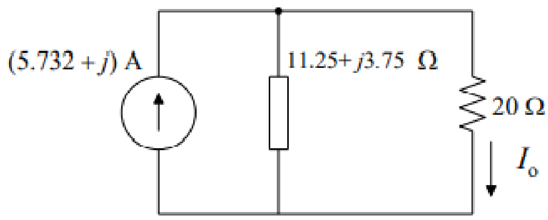
Transform the $5\angle 0°$ A current source into a voltage source, and the $30\angle 60°$ V source into a current source:



Transform the voltage source $50\angle 0°$ V source into a current source; then combine the current sources and the impedances in parallel:

$$\frac{50}{10 - j5} = 4 + j2$$





$$j15 \,||\, (10 - j5) = \frac{j15(10 - j5)}{10 + j10} = 11.25 + j3.75$$

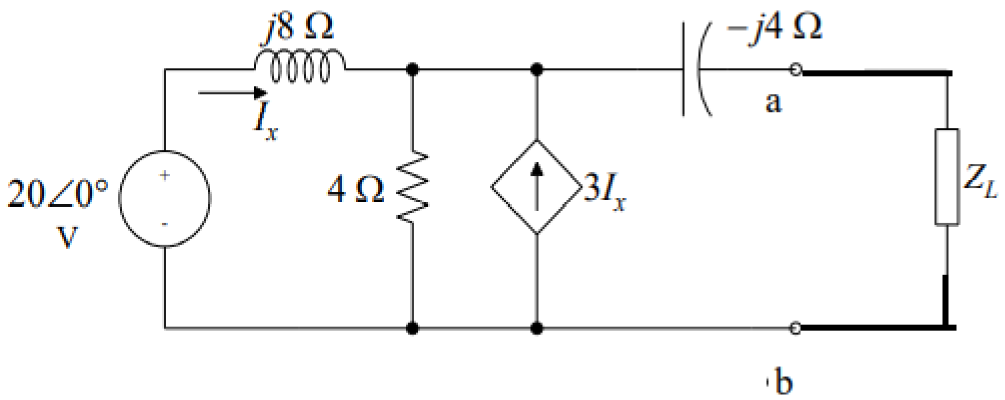
$$4 + j2 + 2\angle -30^{\circ} = 5.732 + j$$

By current division:

$$I_0 = \frac{11.25 + j3.75}{31.25 + j3.75} \times (5.732 + j) \text{ A}$$

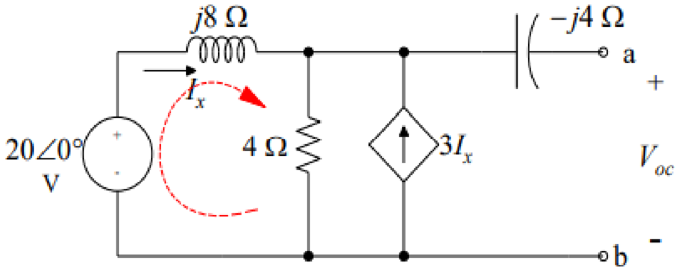
$$I_0 = 2.04 + j0.8031 = 2.1923\angle 21.49^{\circ} \text{ A}$$

5. Determine the load impedance Z_L that maximizes the average power drawn from the circuit shown in Fig.5. What is the maximum average power?



Soln.:

V_{Th}:



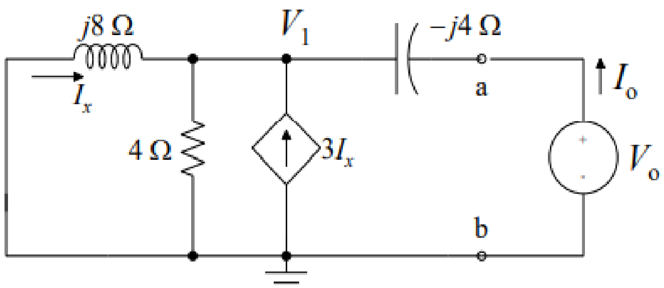
KVL for the loop:

$$-20\angle 0^\circ + j8I_x + 4(4I_x) = 0 \quad \Rightarrow \quad I_x = \frac{20\angle 0^\circ}{16 + j8}$$

$$V_{oc} = 4(4I_x) = 16I_x = \frac{16}{16 + j8} 20\angle 0^\circ$$

$$= 16 - j8 \text{ V} = 17.89\angle -26.57^\circ \text{ V}$$

Z_{Th}:



KCL eqn. at node V_1 :

$$\frac{V_1}{j8} + \frac{V_1}{4} + \frac{V_1 - V_o}{-j4} - 3I_x = 0$$

$$I_x = -\frac{V_1}{j8}$$

$$V_1 = -\frac{V_o}{1+j} \quad \Rightarrow \quad I_o = \frac{V_o - V_1}{-j4} = \frac{2+j}{(-j4)(1+j)} V_o \quad \Rightarrow \quad Z_{Th} = \frac{V_o}{I_o} = 0.8 - j2.4\,\Omega$$

The Load impedance draws the maximum power from the circuit when

$$Z_L = Z_{TH}^* = 0.8 + j\,2.4\ \Omega$$

The maximum average power is $P_{max} = \frac{|VTH|^2}{8R_{TH}} = \frac{17.89^2}{8x0.8} = 50$ W