Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG224 *Circuit Theory* II

FALL 2022-2023

<u>Instructor:</u> A. Doğanalp

MIDTERM EXAM
November 23, 2022

Duration :* 100 *minutes

Number of Problems: 4

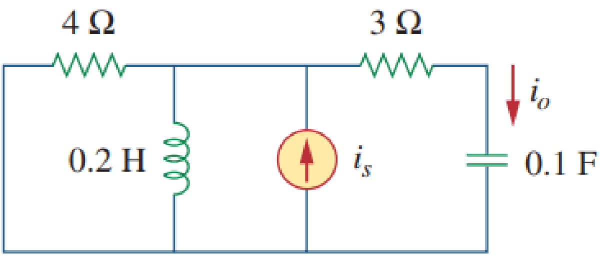
Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	

Problem		**Points**
1		15
2		25
3		30
4		30
TOTAL		100

PROBLEM 1

If $i_s = 5\,\text{Cos}(10\,t + 40^0)$ A in the circuit, find i_o .



SOLUTION:

Step 1: Transfer from time domain to phasor domain.

$$i_s = 5\cos(10t + 40°) \longrightarrow \mathbf{I}_s = 5\angle 40°$$

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.1)} = -j$$

$$0.2\text{ H} \longrightarrow j\omega L = j(10)(0.2) = j2$$

Step 2:

$$\text{Let} \quad \mathbf{Z}_1 = 4 \,||\, j2 = \frac{j8}{4 + j2} = 0.8 + j1.6 \quad \text{and} \quad \mathbf{Z}_2 = 3 - j$$

Step 3: By applying current division rule

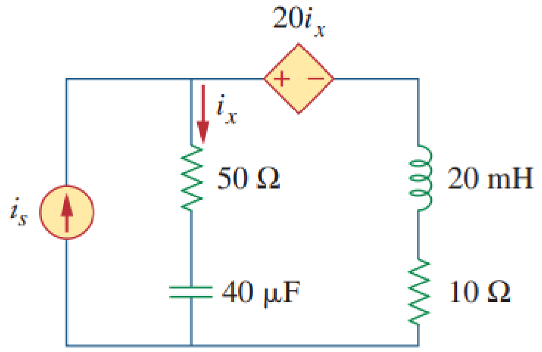
$$\mathbf{I}_o = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}_s = \frac{0.8 + j1.6}{3.8 + j0.6}(5\angle 40°)$$

$$\mathbf{I}_o = \frac{(1.789\angle 63.43°)(5\angle 40°)}{3.847\angle 8.97°} = 2.325\angle 94.46°$$

Step 4:

$$\text{Thus,} \quad i_o(t) = \mathbf{2.325\cos(10t + 94.46°)\ A}$$

PROBLEM 2



For the circuit shown i_S = 6 Cos (10^3 t) A.

Find the ***average power*** absorbed by 50 Ω resistor.

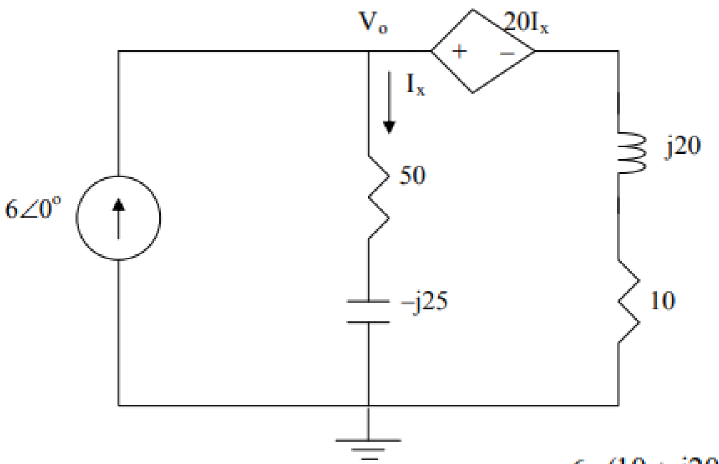
SOLUTION:

Step 1: Transfer from time domain to phasor domain.

$$20\text{ mH} \longrightarrow j\omega L = j10^3 x20x10^{-3} = j20$$

$$40\mu\text{F} \rightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 x40x10^{-6}} = -j25$$

Step 2: Apply Nodal analysis to the circuit below.



$$\frac{V_o - 20I_x}{10 + j20} + \frac{V_o}{50 - j25} = 6$$

$$\text{But} \quad I_x = \frac{V_o}{50 - j25}.$$

Substituting this and solving for V_o leads

$$6 \ (10 + j20)(50 - j25) = V_o \ [\ 10 + j20 + 50 - j25 \ - 20 \ ]$$

$$\left(\frac{1}{10 + j20} - \frac{20}{(10 + j20)} \frac{1}{(50 - j25)} + \frac{1}{50 - j25} \right) V_o = 6$$

$$\frac{6 \ (10 + j20)(50 - j25)}{40\text{-}j5} = V_o \quad \Longrightarrow \quad V_o = 186.05\angle 43.99^\circ \text{ volts.}$$

$$I_x = \frac{186.05\angle 43.99^\circ}{(50 - j25)} = \frac{186.05\angle 43.99^\circ}{55.9\angle -26.57^\circ} = 3.328 \ \angle 70.55^o \ \text{A}$$

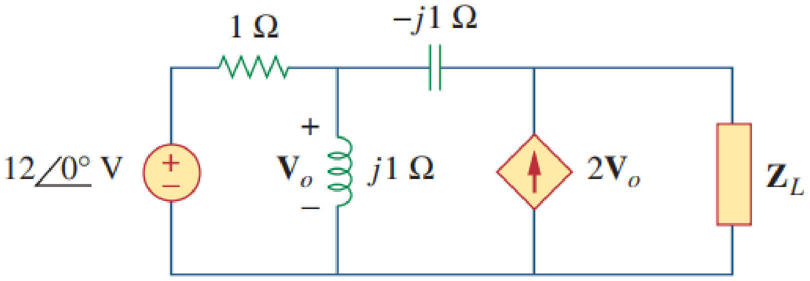
Step 3: We can now calculate the average power absorbed by the 50-Ω resistor.

$$P_{avg} = \frac{1}{2} |I_x|^2 R$$

$$P_{avg} = [(3.328)^2/2]x50 = \mathbf{276.8\ W}.$$

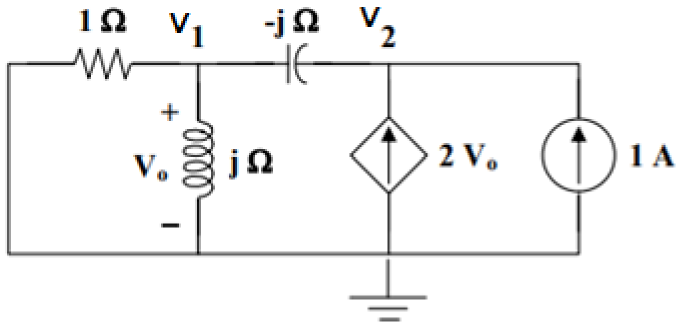
PROBLEM 3

In the circuit shown below, find ***the value of Z_L*** that will absorb the maximum power and the ***value of the maximum power***



SOLUTION:

Step 1: To find Z_{eq} insert a 1 A (arbirtrary number) current source at the load terminals as shown below.



At node 1,

$$\frac{V_o}{1}+\frac{V_o}{j}=\frac{V_2-V_o}{-j} \longrightarrow V_o = jV_2 \qquad (1)$$

At node 2,

$$1+2V_o=\frac{V_2-V_o}{-j} \longrightarrow 1 = jV_2-(2+j)V_o \qquad (2)$$

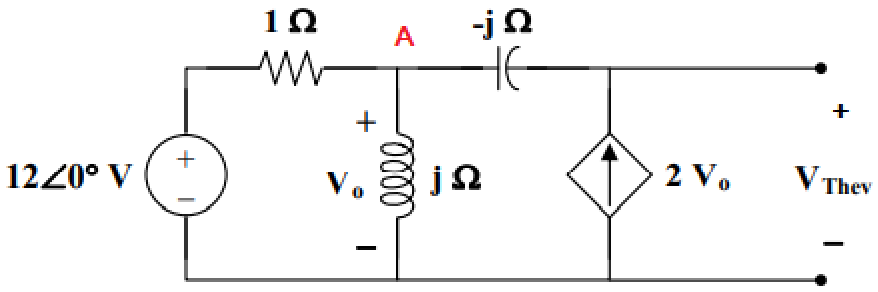
Substituting (1) into (2),

$$1 = jV_2-(2+j)(j)V_2=(1-j)V_2$$

$$V_2=\frac{1}{1-j}$$

$$Z_{eq}=\frac{V_2}{1}=\frac{1+j}{2}=0.5+j0.5 \Longrightarrow Z_L = Z_{eq}^* = [0.5-j0.5]\ \Omega$$

Step 2: To obtain V_{Thev} consider the circuit shown below.



Apply KCL to node A → $-2\mathbf{V}_o + \frac{\mathbf{V}_o - 12}{1} + \frac{\mathbf{V}_o}{j} = 0$ → $\mathbf{V}_o = \frac{-12}{1+j}$

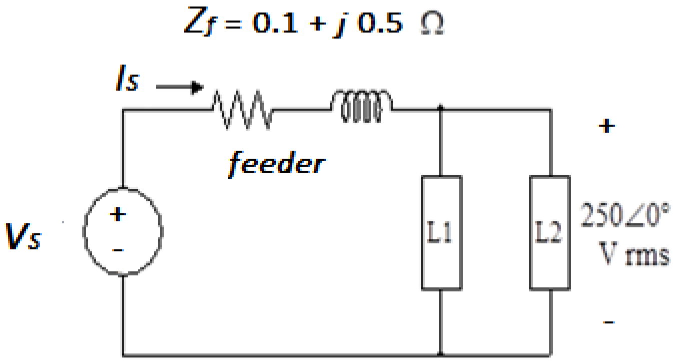
$$-\mathbf{V}_o - (-j \times 2\mathbf{V}_o) + \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Thev} = (1 - j2)\mathbf{V}_o = \frac{(-12)(1-j2)}{1+j} = \frac{-12 + j\,24}{1 + j} = \frac{26.83\ \angle 116.57^\circ}{1.41\ \angle 45^\circ} = 19.02\ \angle 71.57\ \text{volt}$$

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}} \;\Rightarrow\; P_{max} = \frac{|19.02|^2}{8 \times 0.5\ \text{Ohm}}$$

$$P_{max} = 90.44\ \text{Watt}$$

PROBLEM 4



In the circuit shown, a feeder with impedance $Z_f = 0.1 + j0.5\,\Omega$ supplies power to the loads **L1** and **L2**. The voltage across the loads $V_L = 250\angle 0^\circ$ V rms. The loads absorb the powers given below.

L1: 8 kW, power factor 0.8, leading.
L2: 20 kVA, power factor 0.6, lagging.

Find

(a) The total complex power absorbed by the loads. **(10 pts)**

(b) The current I_S and the voltage of the source V_S. **(6 pts)**

(c) The average power loss on the feeder. **(4 pts)**

(d) A capacitor is to be connected across the loads to raise the power factor of the loads to 1.0. Taking the frequency of the source to be 50 Hz, find the value C of the capacitance required. **(10 pts)**

SOLUTION:

(a) Reactive power absorbed by L1:

$\theta_1 = \cos^{-1}(0.8) = 36.87^\circ$, $Q_1 = -8 \times \tan(36.87^\circ) = -6$ kVAR

$\Rightarrow S_1 = (8 - j\,6)$ kVA

Average and reactive power absorbed by L2;

$\theta_2 = \cos^{-1}(0.6) = 53.13^\circ$ $\sin(\theta_2) = 0.8$

$\Rightarrow S_2 = 20(0.6 + j\,0.8) = (12 + j\,16)$ kVA

$$S_{\text{total}} = (20 + j\,10)\text{ kVA}$$

(b)

$$S_{\text{total}} = V_L I_s^* \quad \Rightarrow \quad I_s = \left(\frac{S_{\text{total}}}{V_L}\right)^* = \frac{20000 - j10000}{250} = 80 - j40\text{ A} = 89.44\angle -26.57^\circ\text{ A}$$

$$V_s = Z_f I_s + V_L = (0.1 + j0.5)(80 - j40) + 250 = 278 + j36\text{ V} = 280.32\angle 7.38^\circ\text{ V}$$

(c) $P_{loss} = 0.1 \times |I_s|^2 = 0.1 \times 89.44^2 = 799.95$ W

(d) The capacitor must generate the reactive power 10 kVAR $\Rightarrow$

$$\frac{|V_L|^2}{X_C} = 10^4 \quad \Rightarrow \quad X_C = \frac{250^2}{10^4} = 6.25\,\Omega = \frac{1}{2\pi fC} \quad \Rightarrow \quad C = \frac{1}{100\pi \times 6.25} = 509.3\,\mu\text{F}$$