CIVL471 DESIGN OF RC STRUCTURES

LECTURE NOTE #11

CHAPTER II

JOIST FLOORS

One-Way Joist Floors

- Solid slabs become thicker and heavier with increasing of span lengths. It is clear that tensile zones of slabs do not have any contribution to the bending resistance and rather increase the weight.

- The weight of the slab can be reduced by providing openings in the tensile zone.

- Joist floors (ribbed-slab) are this type of slabs (openings provided in the tensile zone).

- These ribs are in the form of T-Beams and they can also be described as closely spaced and monolithically cast reinforced concrete T-Beams.

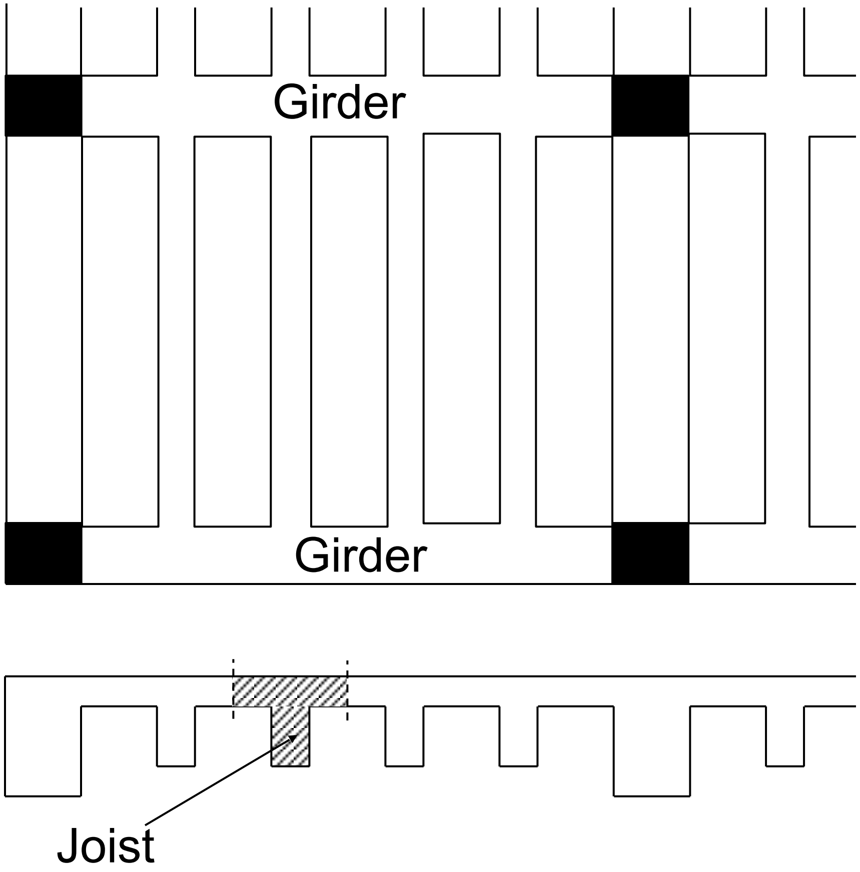


Figure 2.1

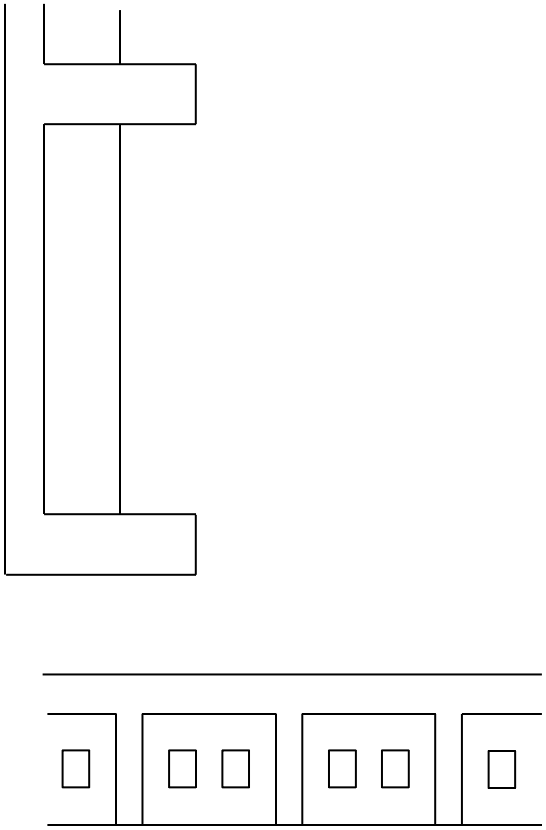


Figure 2.2

- According to the definition given by TS500 a slab system can be analyzed and designed as joist floor system if only the joists are not farther apart than 70 cm. face to face. If spacing between the joists is greater than 70 cm plates should be designed as one-way edge-supported slabs and joists should be designed as ordinary T beams.

Specifications for Dimensions and Reinforcement in TS500

The width of the ribs can not be less than 10 cm. The minimum thickness for top floor slab (plate) which forms the flanges of the T beams is either 1/10 of the clear spacing between the adjacent ribs or 5 cm, whichever is larger. According to earthquake regulations this thickness can not be less than 7 cm. Total thickness of the joist floor thickness should not be less than the following:

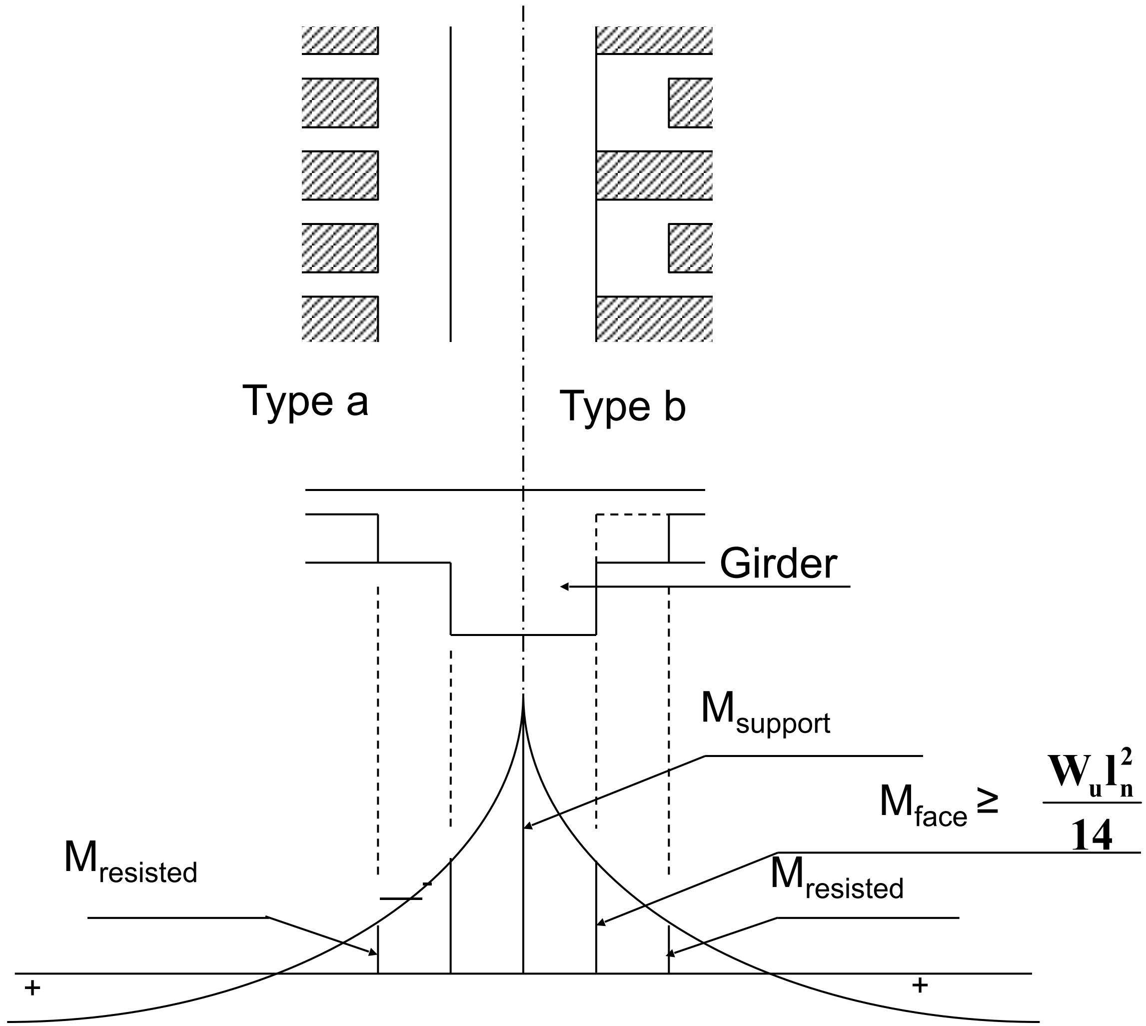
- 1/20 of the clear span of the joists in simply supported single span joists
- 1/25 of the clear span of the joists in continuous joists
- 1/10 of the clear span of the joists in cantilever joists

- If the span of the one-way joist exceeds 4 m joists perpendicular to the main joists should be provided. If the span length is between 4 m and 7 m one lateral joist is enough. For the spans greater than 7 m at least two joists should be provided. These lateral joists should be identical to the main joists. They stiffen the floor system and contribute to the distribution of local loads to the several joists. At the bottom of the plate distribution steel should be placed in two directions. The area of each layer should be equal to at least 0.0015 times the plate section.

Design of One-Way Joist Floors

- Bending moment and shear forces can be determined by any method developed for elastic beams. Also, approximate method discussed in Chapter I can also be used.
- Joists are designed as T-Beams in positive moment regions and as rectangular beams in negative moment regions.
- The width of the flange is equal to the distance between the centerlines of the two adjacent voids.
- At the supports bending moments which are at the faces of supports may be used. The moments can be computed as explained in one-way edge-supported slabs. However support width should not be taken greater than 2 times the slab thickness in these calculations.

- Dimensions of the joists are generally selected according to the positive moment. Therefore it is likely that they will not resist to the negative moment at the support. TS500 does not allow double reinforcement in slab designs. For this reason the only solution is enlargement of the ribs. They are enlarged starting from the section where the bending moment is equal to the resistance of the joist. In Fig. 2.3 two types of enlargement are shown though there are some other types.



Type a
Type b
Girder
$M_{support}$
$M_{face} \geq \frac{W_u l_n^2}{14}$
$M_{resisted}$
$M_{resisted}$
+
+
-

Figure 2.3

- At the supports joists are designed for $M_{resisted}$ which can be obtained by using a relevant ratio of steel less than ρ_{max}. This reinforcement should be checked at the face of the support. Because, both the width of the cross-section and the moment increase at the face of the support.

- Design shear force is also calculated at the face of the support. If design shear force (V_d) is greater than

  $V_{cr} = 0.65 f_{ctd} b_w d$, joists should be designed like ordinary T beams. Otherwise open stirrups with spacing not more than 25 cm may be used without any calculation. It is preferable to chose this option.

- Sometimes wide girders having the same total height as the joists are used as supports. This system is known as 'joist–band' system and has some advantages. Formwork is very simple, construction is fast and a completely flat ceiling provides freedom to the architect in the design. In Fig.2.4 a joist –band system is shown.

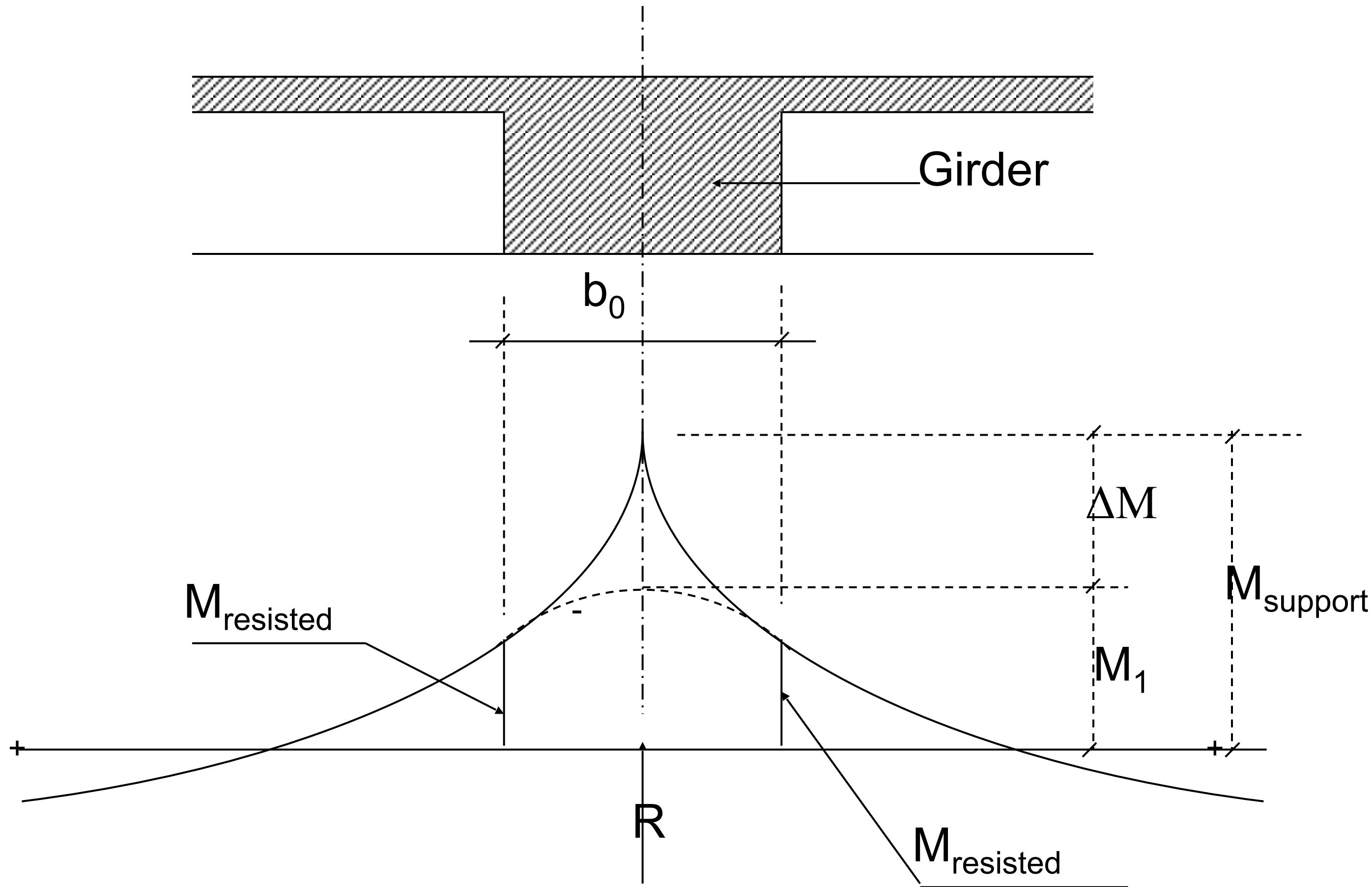


Figure 2.4

- In this system the width (b_0) of the girder must be selected such that negative moment at the support can be resisted. However reinforcement calculated at this section should be checked at the middle section where real moment is M1. This moment can be computed as follows:

$$M_1 = M_{support} - \Delta M \qquad (2.1)$$

$$\Delta M = \frac{b_0 R}{8} \qquad (2.2)$$

Where R is the reaction at this support.

- Generally it is assumed that, moments at the external supports are zero. However, a negative reinforcement equal to the half of the positive reinforcement provided at the span should be placed at this support.

- If the spans of supporting girders are not large, joists should be arranged in short direction. But especially in joist-band systems design of long girders may cause difficulty. For example abnormally wide beams may be required. Best solution may be obtained when the span of the girder is about 2/3 of the joist span.

- Example 2.1
  Design the one-way joist floor illustrated in Fig.2.5. Live load: 2 kN/m^2
  Materials: C20 and S220

  See page 34 for the solution

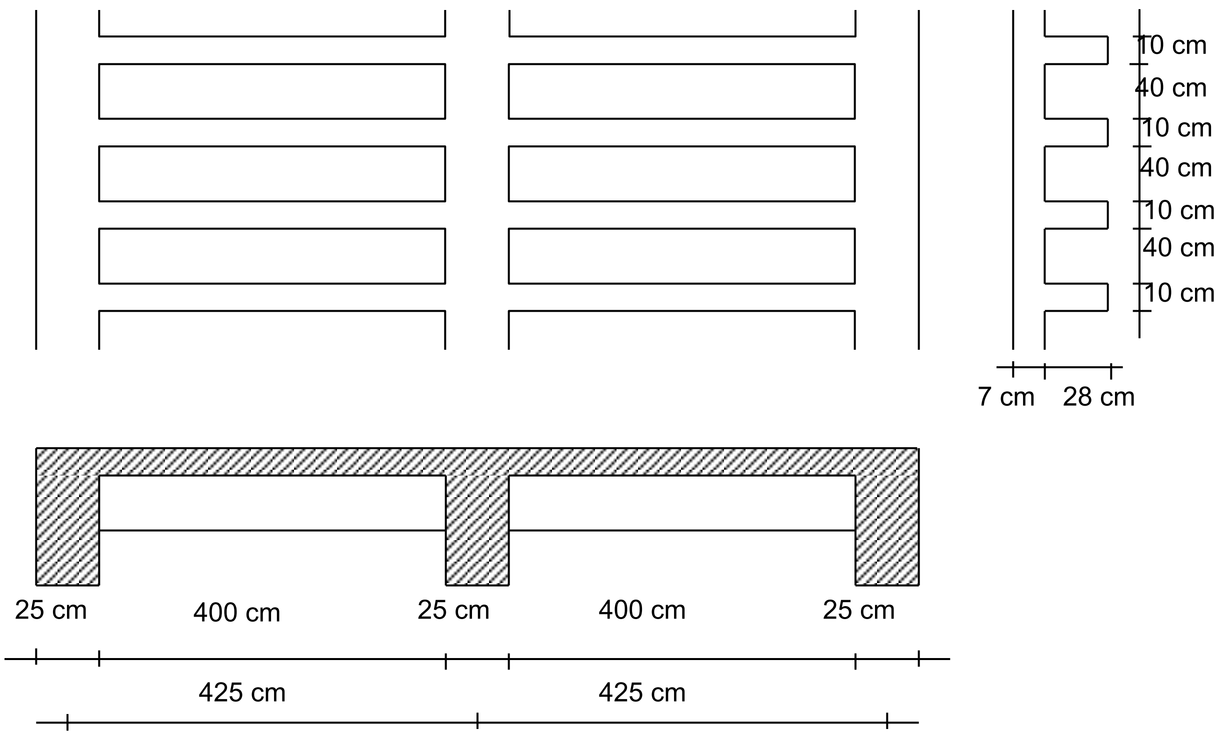


Figure 2.5

Solution:

Slab loads (for 1 m wide strip):

Self-weight of the plate: 0.07*25 = 1.75 kN/m

Weights of two ribs: 2*0.10*0.28*25 = 1.40 kN/m

Weights of plaster, floor finish etc. (assumed): 1.25 kN/m

Dead load: W_d = 4.40 kN/m

This strip can be analyzed as a continuous beam having two spans. Bending moment diagrams showing maximum moments is shown in Fig.2.6. These moments can be computed by any method developed for the beams.

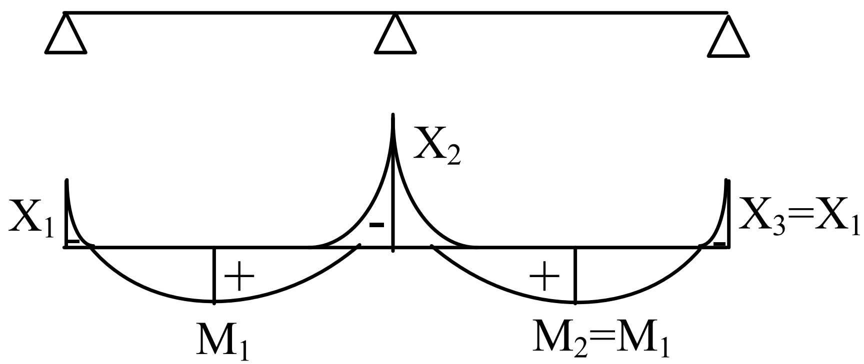


Figure 2.6

At the freely rotating external supports bending moment should be zero, but in real structures a completely free rotation is not possible. Therefore it is advisable to assume the presence of a negative moment at the external supports. At the external supports of slabs existance of negative moment is much more likely.

In this problem moment coefficients can be used for the calculation of moments, because:

- Spans are equal
- Loads are uniformly distributed
- $W_l = 2$ kN/m $< 2W_d = 2*4.4 = 8.8$ kN/m

Bending moments:

Design Load: $W_u = 1.4W_d + 1.6W_l = 1.4*4.4 + 1.6*2 = 9.36$ kN/m

$$X_1 = -\frac{1}{24} 9.36*4.25^2 = -7.04 \text{ kN-m} = X_3$$

$$M_1 = \frac{1}{11} 9.36*4.25^2 = 15.37 \text{ kN-m} = M_2$$

$$X_2 = -\frac{1}{8} 9.36*\left(\frac{4.25+4.25}{2}\right)^2 = -21.13 \text{ kN-m}$$

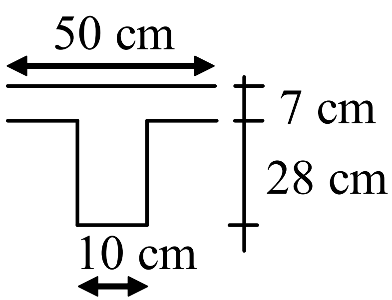
In this strip there are two ribs, which means that each rib is subjected to half of these moments. Due to approximate nature of the analysis support moments will not be adjusted.

$$\overline{X}_1 = -7.04 / 2 = -3.52 \text{ kN-m}$$

$$\overline{M}_1 = 15.37 / 2 = 7.69 \text{ kN-m}$$

$$\overline{X}_2 = -21.13 / 2 = -10.57 \text{ kN-m}$$

Design of a rib for bending moment:



Assume d′ = 3 cm $d = h - d' = 35 - 3 = 32$ cm

$$\overline{X}_1 = -3.52 \text{ kN-m} = -35200 \text{ Kg-cm}$$

$$R = \frac{35200}{10 * 32^2} = 3.44 \text{ Kg/cm}^2 \qquad \rho = 0.0018 < \rho_{min} = 0.0046$$

$$A_s = \rho_{min}\, b_w d = 0.0046 * 10 * 32 = 1.47 \text{ cm}^2$$

Selected: 2Ø10 (1.57 cm^2)

$\overline{M}_1$ = 7.69 kN-m = 76900 Kg-cm, let "a" be the height of the equivalent stress block. Flange thickness h_f = 7 cm and assume $a \leq h_f$. Rectangular beam equations or tables can be used.

$$R = \frac{76900}{50*32^2} = 1.50 \text{ Kg/cm}^2 \qquad \rho = 0.0008 < \rho_{max} \text{ (underreinforced)}$$

Check for the validity of the assumption:

$$a = \frac{\rho f_{yd} d}{0.85 f_{cd}} = \frac{0.0008*1910*32}{0.85*130} = 0.44\text{cm} < h_f = 7\text{cm} \qquad \text{OK.}$$

$$A_s = \rho bd = 0.0008*50*32 = 1.28 \text{ cm}^2$$

Check against the minimum steel value:

$$\rho_w = \frac{A_s}{b_w d} = \frac{1.28}{10*32} = 0.004 < \rho_{min} = 0.0046 \qquad \text{Adjust } A_s\text{:}$$

$$A_s = 0.0046*10*32 = 1.47 \text{ cm}^2 \qquad \text{Selected: } 2\text{Ø}10 \ (1.57 \text{ cm}^2)$$

$\overline{X}_2 = -10.57$ kN-m $= -105700$ Kg-cm

$$R = \frac{105700}{10*32^2} = 10.32\,Kg/cm^2 \qquad \rho = 0.006 > \rho_{min}$$

$A_s = 0.006*10*32 = 1.92$ cm^2 Selected: 2Ø12 (2.26 cm^2)

Design for shear forces:

Continuous beams can be analyzed as statically determinate systems after the determination of unknown support moments. In Fig.2.7 shear forces in a rib and statically determinate identical system are shown.

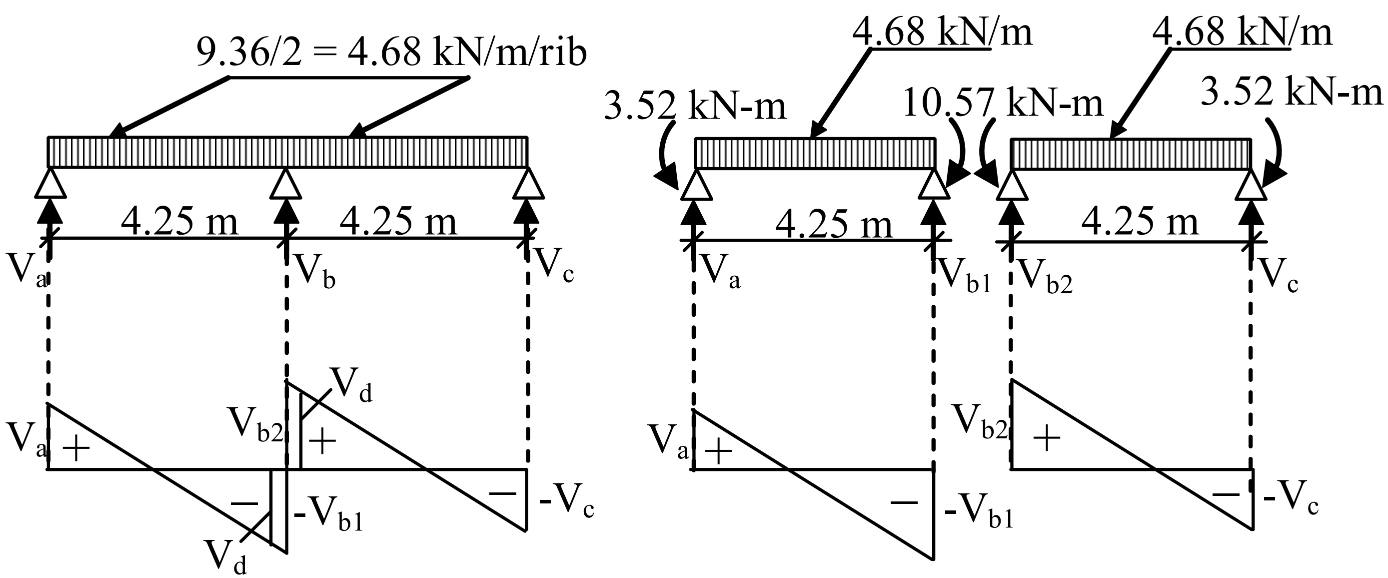


Figure 2.7

Reaction forces can easyly be computed by using superposition rule:

$$V_a = \frac{4.68*4.25}{2} + \frac{3.52-10.57}{4.25} = 9.95 - 1.66 = 8.29 \text{ kN}$$

$$V_{b1} = \frac{4.68*4.25}{2} - \frac{3.52-10.57}{4.25} = 9.95 + 1.66 = 11.61 \text{ kN}$$

Because of symmetry $V_{b2} = V_{b1}$ and $V_c = V_a$. Therefore only one span will be considered in design. For the design positive and negative shears can be treated separetly but in most cases stirrups corresponding to maximum shear force are placed in the whole span. In joist floors design shear force is calculated at the face of supporting beam. Therefore,

$$V_d = 11.61 - 4.68*(0.25/2) = 11.02 \text{ kN}$$

$$V_{cr} = 0.65 f_{ctd} b_w d = 0.65*1.1*100*320 = 22880 \text{ N} = 22.88 \text{ kN} > V_d$$

Ø6/25cm stirrups will be used.

Distribution steel:

$$0.0015*100*7 = 1.05\ cm^2 \quad \text{Selected} \quad Ø6/25cm\ (1.13\ cm^2)$$

There should be one lateral joist in each span since 4 m < l = 4.25 m < 7 m. Details are shown in Fig. 2.8.

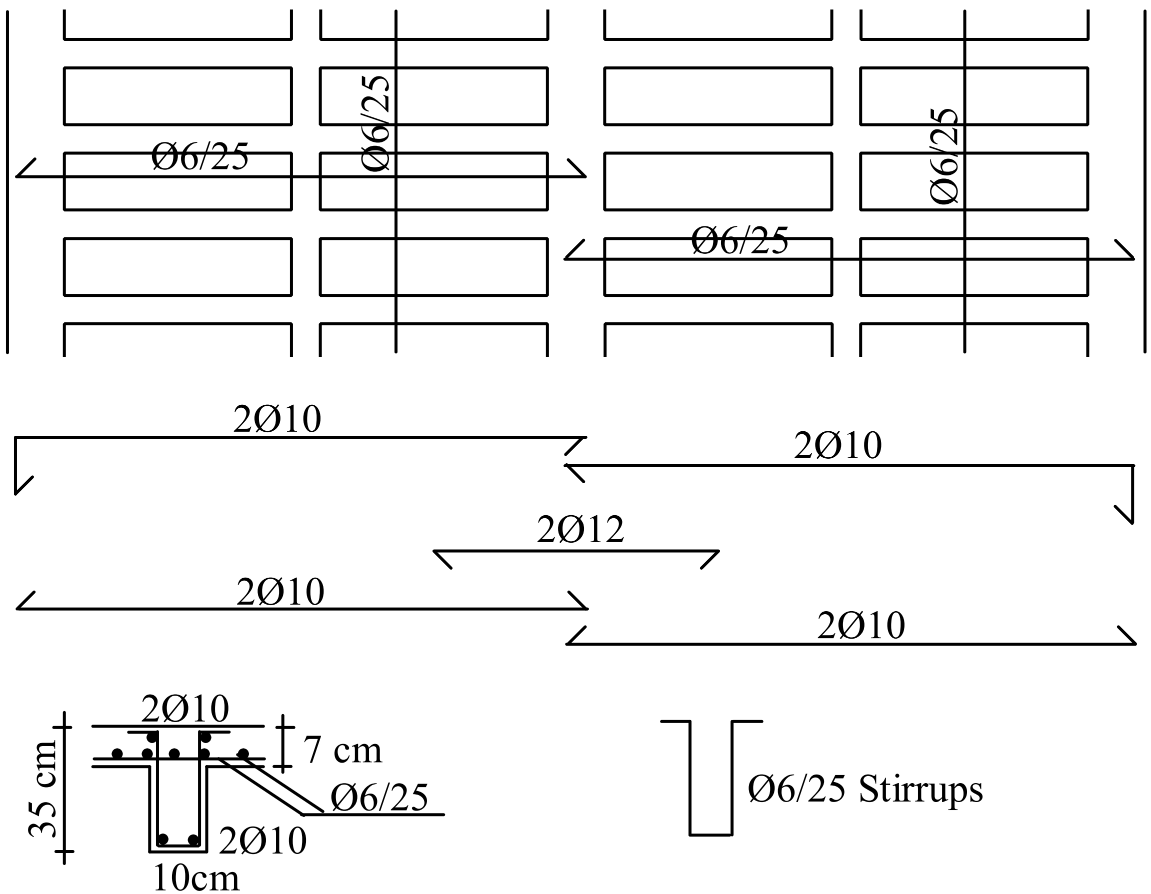


Figure 2.8

TWO-WAY JOIST FLOORS

- One-way joist floor system is not economical and suitable for the slabs if the ratio of the sides is less than two. Slabs of this kind should be two-way by arranging the joists in two directions. Two-way joist slab systems are highly suitable if spans are large in both directions and loads are heavy. They are also called grid slabs.

- For the analysis of two-way joist slabs some methods developed for two-way solid plates may be used. Because grid slabs are not too much different than solid plates. Only difference is the voids at the bottom of the plate. These voids may be filled with permanent light-weight blocks or reusable fillers. Approximate methods can also be used. For example the moment coefficients given in Table 1.2 may be used if the slab is supported by beams at the edges. Prof. Dr. Ersoy advises to increase the positive moments 30% if this table is used. Another method is analyzing the joists in two directions independently. For this purpose slab load should be distributed in these two directions. Using equalities of the deflections in two directions distribution coefficients can be obtained. Coefficient tables are available in some reinforced concrete design books and hand-books (see “Betonarme Yapı Elemanları”, Aka, Keskinel and Arda).

- Similar to one-way joist slabs the weights of the joists are assumed uniformly distributed.

Example 2.2

Design of two-way joist floor

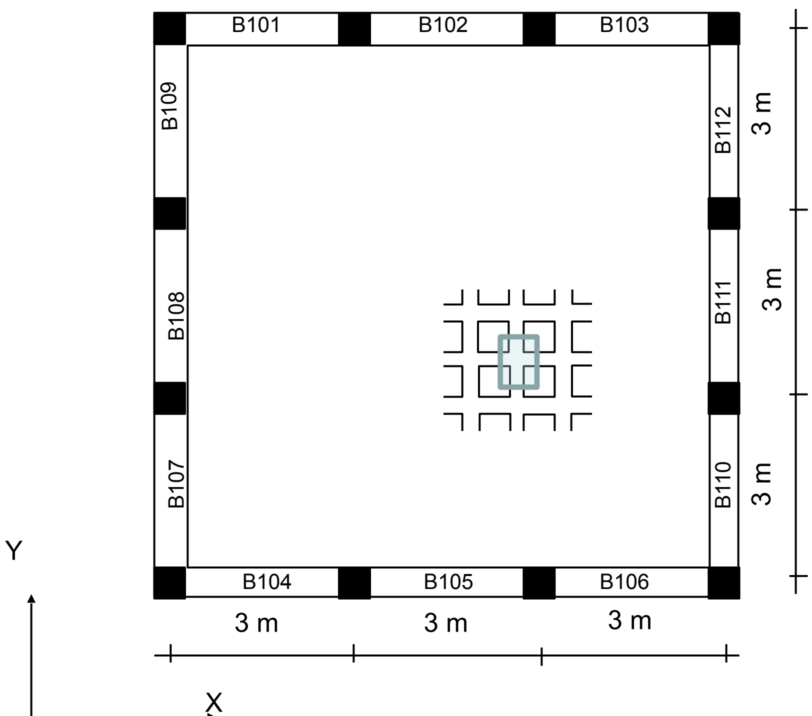
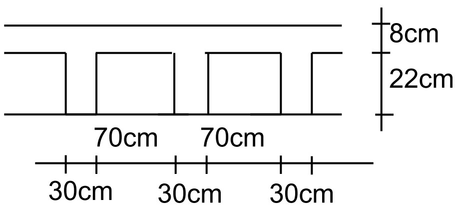


Figure 2.9

Joists:



Live Load: 5 kN/m^2
Additional dead load: 1 kN/m^2

Materials: C20, S420

For Solution See page 38 from your text book

Solution:

Loads: Self-weight of the plate: $0.08*25 = 2.00 \text{ kN/m}^2$

Weights of the ribs: $0.3*0.22*25*1.7 = 2.81 \text{ kN/m}^2$

Additional dead load: 1.00 kN/m^2

$W_d = 5.81 \text{ kN/m}^2$

Design load:

$$W_u = 1.4*5.81 + 1.6*5 = 16.13 \text{ kN/m}^2$$

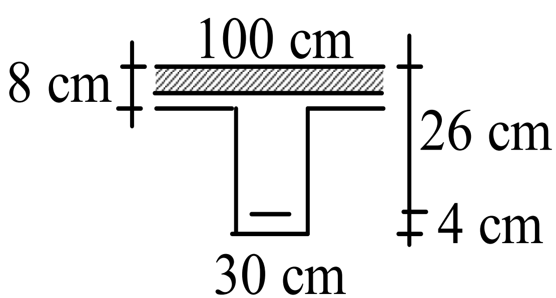
This slab may be assumed simply supported at four edges. Since it is a square slab loads are equally distributed in two directions.

$$W_{ux} = W_{uy} = 16.13 / 2 = 8.07 \text{ kN/m}^2$$

Maximum bending moment for 1 m wide strip in each direction:

$$M_{max} = \frac{8.07 * 9^2}{8} = 81.71 \text{ kN-m} = 817100 \text{ Kg-cm}$$

In the strip there is only one joist which is a T shaped beam. If "a" is the height of the equivalent rectangular stress block and assumed equal or less than the flange thickness rectangular beam tables can be used for the design.



$h = 30$ cm $\quad d = 30 - 4 = 26$ cm

$$R = \frac{817100}{100 * 26^2} = 12.09 \text{ Kg/cm}^2$$

$\rho = 0.0035 < \rho_{max} = 0.0136$ Underreinforced

Check the validity of the assumption:

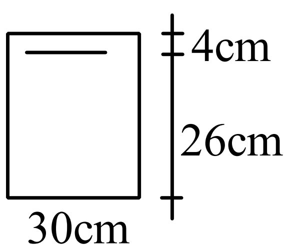
$$a = \frac{\rho f_{yd} d}{0.85 f_{cd}} = \frac{0.0035*3650*26}{0.85*130} = 3.01 \text{ cm} < h_f = 8 \text{ cm} \quad \text{OK}$$

$$A_s = \rho bd = 0.0035*100*26 = 9.10 \text{ cm}^2$$

$$\text{Check: } \rho_w = \frac{A_s}{b_w d} = \frac{9.10}{30*26} = 0.0117 > \rho_{min} = 0.0024 \quad \text{OK}$$

Selected: 2Ø16 (straight) + 2Ø18 (bent-up)   ($4.02 + 5.09 = 9.11 \text{ cm}^2$)

Theoretically bending moments at the supports are zero. However it is advised to provide top steel for a negative moment of $W_u l^2/16$.



$$X = -\frac{8.07*9^2}{16} = -40.85 \text{ kN-m} = -408500 \text{ Kg-cm}$$

$$R = \frac{408500}{30*26^2} = 20.14 \text{ Kg/cm}^2 \quad \rho = 0.0061 \begin{matrix} < \rho_{max} \\ > \rho_{min} \end{matrix}$$

$A_s = 0.0061*30*26 = 4.76 \text{ cm}^2$ Available 2Ø18 bent bars (5.09 cm^2)

Shear design:

Reaction forces: $V_A = V_B = \dfrac{W_u l}{2} = \dfrac{8.07*9}{2} = 36.32$ kN

If the width of the supporting beam is 30 cm,
Design shear force: $V_d = 36.32 - 0.15*8.07 = 36.32 - 1.21 = 35.11$ kN

$V_{cr} = 0.65*1.1*300*260 = 55770$ N $= 55.77$ kN $> V_d$

Ø8/25 stirrups are selected.

Distribution steel:

$$0.0015*100*8 = 128\ cm^2 \qquad \text{Selected: Ø6/23 } (1.23\ cm^2)$$

Cross-section of a joist and provided reinforcement are shown in Fig.2.10.

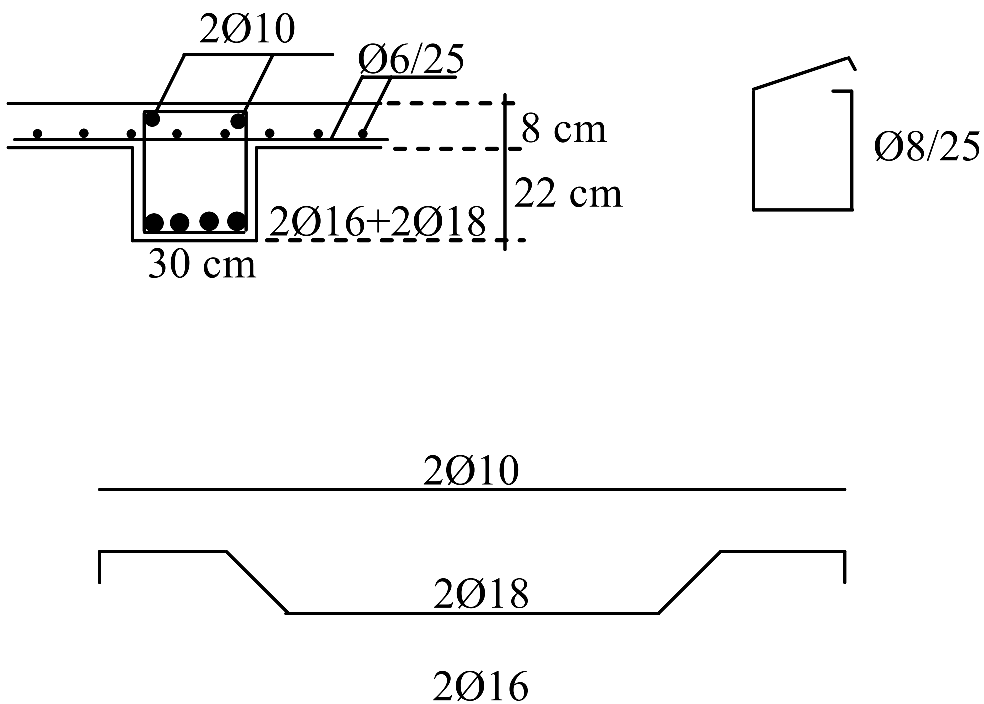


Figure 2.10