

CIVL471 DESIGN OF REINFORCED STRUCTURES

LECTURE NOTE #2

CHAPTER III

SHORT COLUMNS- REVIEW -
Continued

COMPRESSION PLUS BENDING OF COLUMNS

- Columns are rarely subjected to concentric loads. Columns receive bending moments at the rigid joints since they are the members of the frame.
- In addition, there may be eccentricity (eg. Inevitable imperfection in the construction may cause unexpected bending of a column).
- Therefore, TS 500 defines the minimum eccentricity as follows:

$$e_{min} = 15 \text{ mm} + 0.03h$$

where h is dimension of the column in bending direction. If bending moment is zero or eccentricity is less than e_{min} design moment should be adjusted by using e_{min} .

- A column subjected to axial compression plus bending moment is shown in Fig.3.3a. In Fig.3.3b equivalent eccentric loading for the same column is shown in which $e=M/N$.

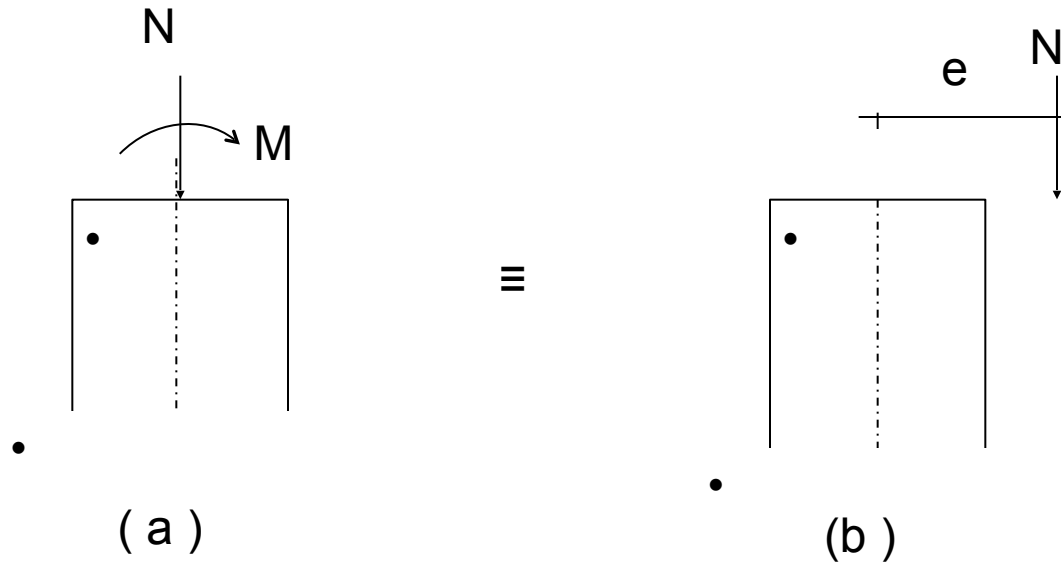


Figure 3.3

STRENGTH ANALYSES OF RECTANGULAR COLUMNS SUBJECTED TO COMBINED FLEXURE AND AXIAL LOAD

- The results of numerous tests reveal that columns fail by crushing of concrete when eccentrically compressed. At this moment reinforcement either in the compression side or in the tension side may be yielding or not. Section of the column may be completely under compression or partly in compression partly in tension depending on the value of eccentricity.

- In design the strain corresponding to the crushing of concrete is assumed as **0.003** like in pure bending. It is also assumed that the real stress distribution in concrete can be replaced by an equivalent rectangular stress block. The equivalent stress is **$0.85f_{cd}$** and the height is "**a**" in this block. As in the case of pure bending **$a = k_1c$** , where **k_1** varies with the strength of concrete. These assumptions are in good agreement with the test results. In Fig.3.4 a rectangular column section, assumed strain distribution at the moment of failure and internal forces balancing external force are shown.

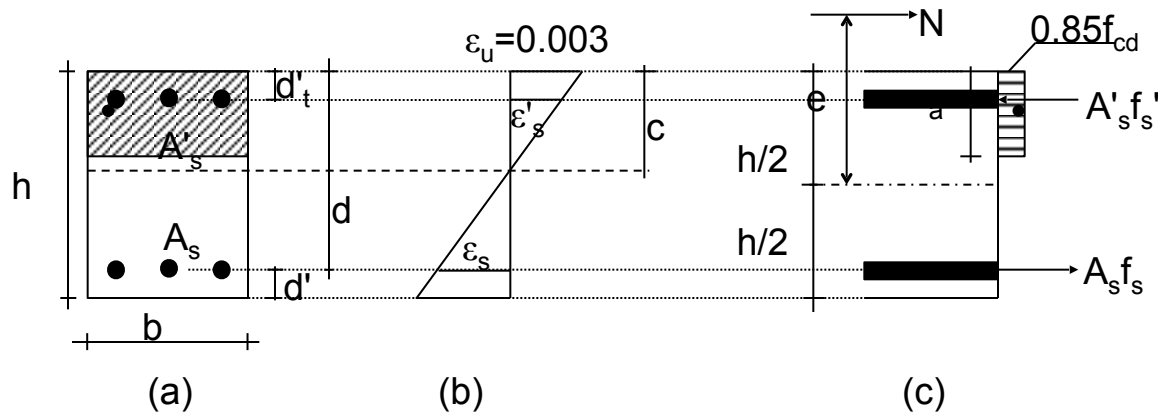


Figure 3.4

- Between the external and internal forces the following equilibrium equations can be written:

$$N = 0.85f_{cd} ab + A'_s f'_s - A_s f_s \quad (3.12)$$

$$M = Ne = 0.85f_{cd} ab \left(\frac{h}{2} - \frac{a}{2} \right) + A'_s f'_s \left(\frac{h}{2} - d'_t \right) + A_s f_s \left(d - \frac{h}{2} \right) \quad (3.13)$$

- In the equations above it is assumed that f_s is tension as shown in Fig.3.4. Otherwise opposite signs should be used in the last terms of the equations. Strains in tension and compression bars can be calculated from the similar triangles in the strain diagram as follows:

$$\varepsilon_s = 0.003 \frac{d - c}{c} \quad (3.14)$$

$$\varepsilon'_s = 0.003 \frac{c - d'_t}{c} \quad (3.15)$$

Steel stresses are determined by Hook's Law:

$$f_s = \varepsilon_s E_s = 0.003 \frac{d - c}{c} E_s \leq f_{yd} \quad (3.16)$$

$$f'_s = \varepsilon'_s E_s = 0.003 \frac{c - d'_t}{c} E_s \leq f_{yd} \quad (3.17)$$

- Replacing “**c**” by (\mathbf{a} / k_1) and substituting Eqs. (3.16) and (3.17), Eqs. (3.12) and (3.13) can be written in terms of “**a**”. Thus **N** and “**a**” will be two unknowns in the equations if eccentricity is known. It is possible to assume that these two unknowns can be calculated by the simultaneous solution of Eqs. (3.12) and (3.13). However this process is very complicated, because “**a**” is not a free variable. It can not be greater than “**h**” and it can not take the values corresponding to the steel stresses higher than f_{yd} .

- For this reason in practice strength interaction diagrams (briefly interaction diagrams) are used for hand calculations. For the construction of this diagram first position of neutral axis “ c ” is assumed. Then “ $a (\leq h)$ ”, ϵ_s , ϵ'_s , f_s and f'_s are computed. Finally N and M are determined from Eqs. (3.12) and (3.13). A series of such calculations makes it possible to plot a diagram which shows the strength couples N_r and M_r for a particular column. In Fig.3.5 an interaction diagram is shown.

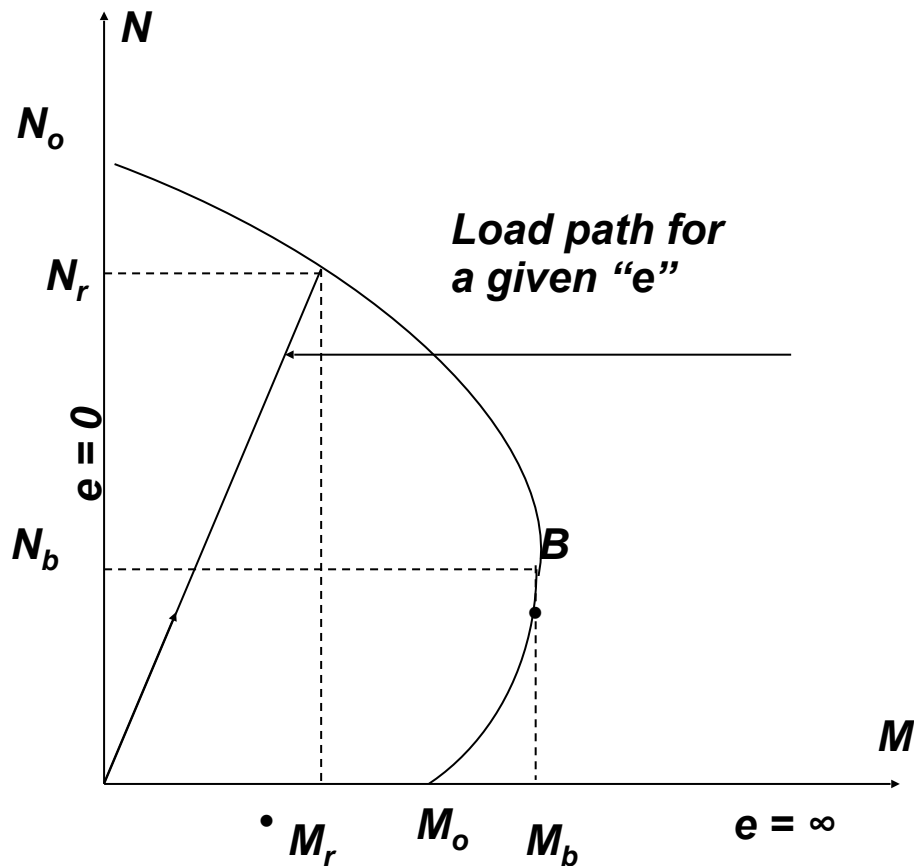


Figure 3.5

Radial lines in this diagram represent the constant eccentricity $e = M / N$, that is, if external load is increased by keeping the eccentricity constant, points corresponding to the pairs M and N will be on this line. The point where the line reaches the curve represents the load pair N and $M = Ne$ causing the failure of the column with the eccentricity "e". Vertical axis corresponds to zero eccentricity, that is, N_o is the strength of the column if axially loaded. Horizontal axis corresponds to infinite eccentricity, that is, M_o is the pure bending strength of the column. It is clear that the diagram covers all possible eccentricities between zero and infinity.

BALANCED FAILURE

- Two distinct type of failures can be observed in short columns. If compression area is too small, that is, “ c ” is very small, ϵ_s' and as a result f_s' will be small whereas ϵ_s will be large (see Fig.3.4). Therefore, tension steel may yield and initiate the failure. This is the same type of failure observed in under-reinforced beams and known as tension failure.

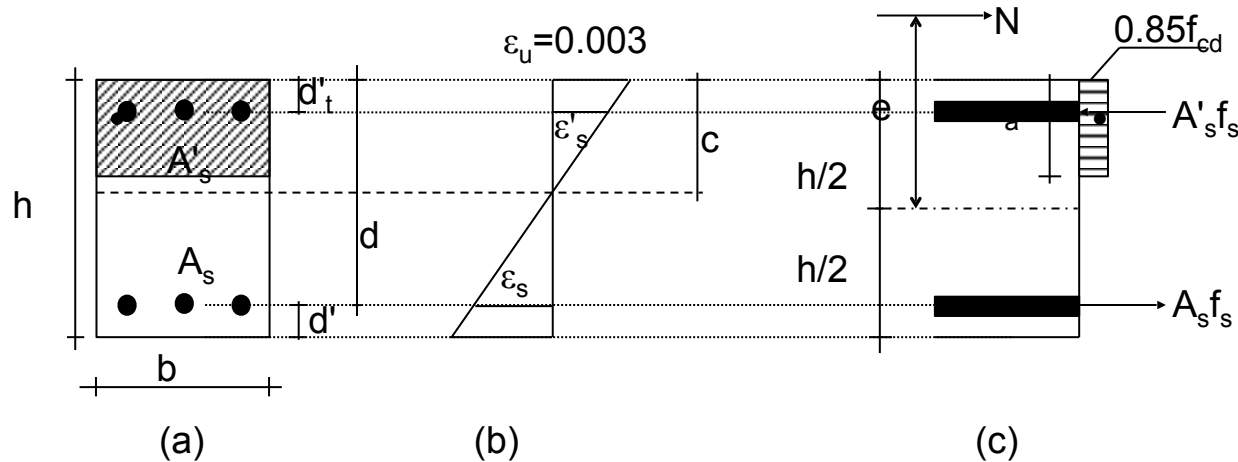
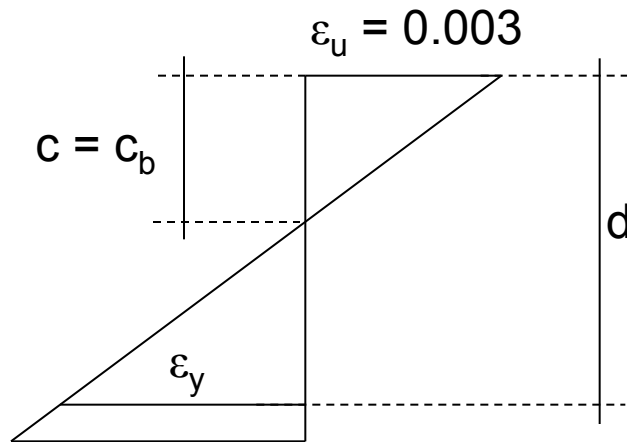


Figure 3.4

Tension failures correspond to large eccentricities. Infinite eccentricity (pure bending) is the limit case for them as shown in Fig.3.5. On the other hand as “ c ” increases compression strains also increase but tension strains decrease. If strain in tension steel is less than the yielding strain, this steel will not yield. In such cases failure will be initiated by crushing of the concrete as observed in over-reinforced beams. Failures of this type are known as compression failures and correspond to small eccentricities. In fact for the values of “ c ” equal or greater than “ h ” cross-section of the column becomes completely under compression and f_s also becomes compression. **Zero eccentricity is the limit case for compression failures.**

However, there is a very special case in which tension steel and concrete reach the ultimate strength values simultaneously. That is concrete strain reaches to ϵ_u and steel strain to ϵ_y at the same time. Such a failure is known as “Balanced Failure”. Strain diagram corresponding to balanced failure is shown in Fig.3.6.



From similar triangles,

$$c_b = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d \quad (3.18)$$

Fig.3.6

It is clear that if “ c ” is less than c_b it will be tension failure, if greater than c_b it will be compression failure. Balanced failure is the border between these two failure types. By the substitution of $a_b = k_1 c_b$ into Eqs.(3.12) and (3.13) N_b and M_b which is the pair corresponding to balanced failure can be calculated (See Fig.3.5). Although there is a similarity between the failure modes, a definite type of failure can not be prescribed for the columns as it can be done for the beams. Failure type in the columns depends on the degree of eccentricity which in turn depends on the properties of the structure. Once established, structural system is not changed too much by the designer. Nevertheless the point B on the interaction diagram is significant since the points over B correspond to brittle failure whereas the points below B correspond to ductile failures.

INTERACTION DIAGRAMS USED IN DESIGN (DESIGN CHARTS)

An interaction diagram shown in Fig. 3.5 can be used only for a particular column. But in design there must be diagrams which can be used as many times as possible. Such diagrams can be obtained by converting equilibrium equations into non-dimensional forms. In the following, construction of design charts for symmetrically reinforced rectangular columns which do not have middle bars will be explained. Nearly all columns in practice are symmetrically reinforced. Such a column is shown in Fig. 3.9. If total steel area in the column is A_{st} , there will be half of this steel ($A_{st}/2$) at each side. For the rectangular columns without middle bars Eqs. (3.12) and (3.13) were obtained as two equilibrium equations. If two sides of Eq. (3.12) is divided by bhf_{cd} and two sides of Eq. (3.13) by $bh2fcd$ these equations become non-dimensional as follows:

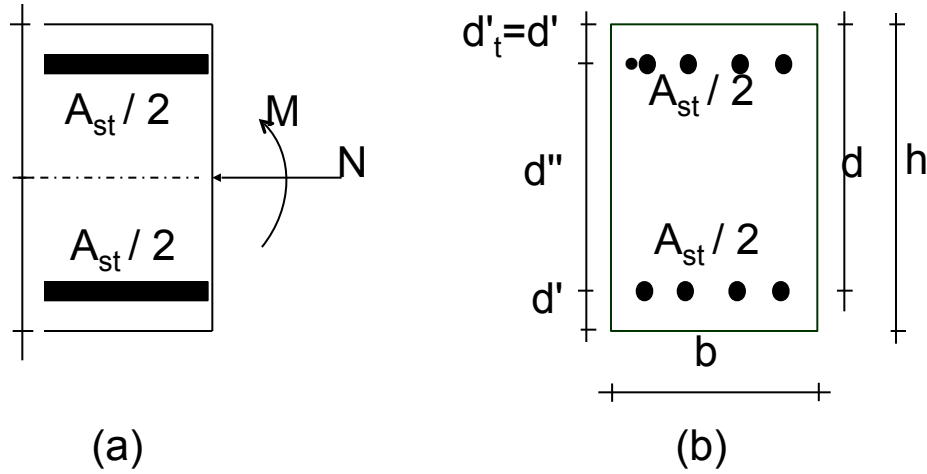


Figure 3.9

$$\frac{N}{bhf_{cd}} = 0.85 \frac{a}{h} + \frac{A_{st}}{2bh} \frac{1}{f_{cd}} (f'_s - f_s) \quad (3.19)$$

$$\frac{M}{bh^2 f_{cd}} = \frac{N}{bhf_{cd}} \frac{e}{h} = 0.85 \frac{a}{h} \left(\frac{1}{2} - \frac{a}{2h} \right) + \frac{A_{st}}{2bh} \frac{f'_s}{f_{cd}} \left(\frac{1}{2} - \frac{d'_t}{h} \right) + \frac{A_{st}}{2bh} \frac{f_s}{f_{cd}} \left(\frac{d}{h} - \frac{1}{2} \right) \quad (3.20)$$

From the geometry of the section

$$\frac{d'_t}{h} = \frac{d'}{h} = \frac{h-d''}{2h} = \frac{1}{2} \left(1 - \frac{d''}{h}\right) \quad \text{and} \quad \frac{d}{h} = \frac{h-d'}{h} = 1 - \frac{d'}{h} = \frac{1}{2} \left(1 + \frac{d''}{h}\right)$$

Let $\rho_t = \frac{A_{st}}{bh}$ (percentage of the total steel area in the column) and

$m = \frac{f_{yd}}{f_{cd}}$ Eqs. (3.19) and (3.20) can now be written as follows:

$$\frac{N}{bhf_{cd}} = 0.85 \frac{a}{h} + \frac{\rho_t m}{2 f_{yd}} (f'_s - f_s) \quad (3.21)$$

$$\frac{M}{bh^2 f_{cd}} = \frac{N}{bhf_{cd}} \frac{e}{h} = 0.425 \frac{a}{h} \left(1 - \frac{a}{h}\right) + 0.25 \frac{\rho_t m d''}{f_{yd} h} (f'_s + f_s) \quad (3.22)$$

- For a certain steel grade and for a particular d'' / h value a family of interaction diagrams corresponding to various $\rho_t m$ values can be plotted. Fig.3.10 illustrates such a design chart. The horizontal line corresponding to $N / bhf_{cd} = 0.9$ is TS500 limit that should not be exceeded. It is obvious that radial lines in design chart correspond to e/h ratios. Similar design charts may be produced for the sections with the reinforcement distributed around the perimeter of the section and for the circular sections. Some design charts are given in the Appendix.

S220 $d''/h = 0.8$ $\lambda = 0$

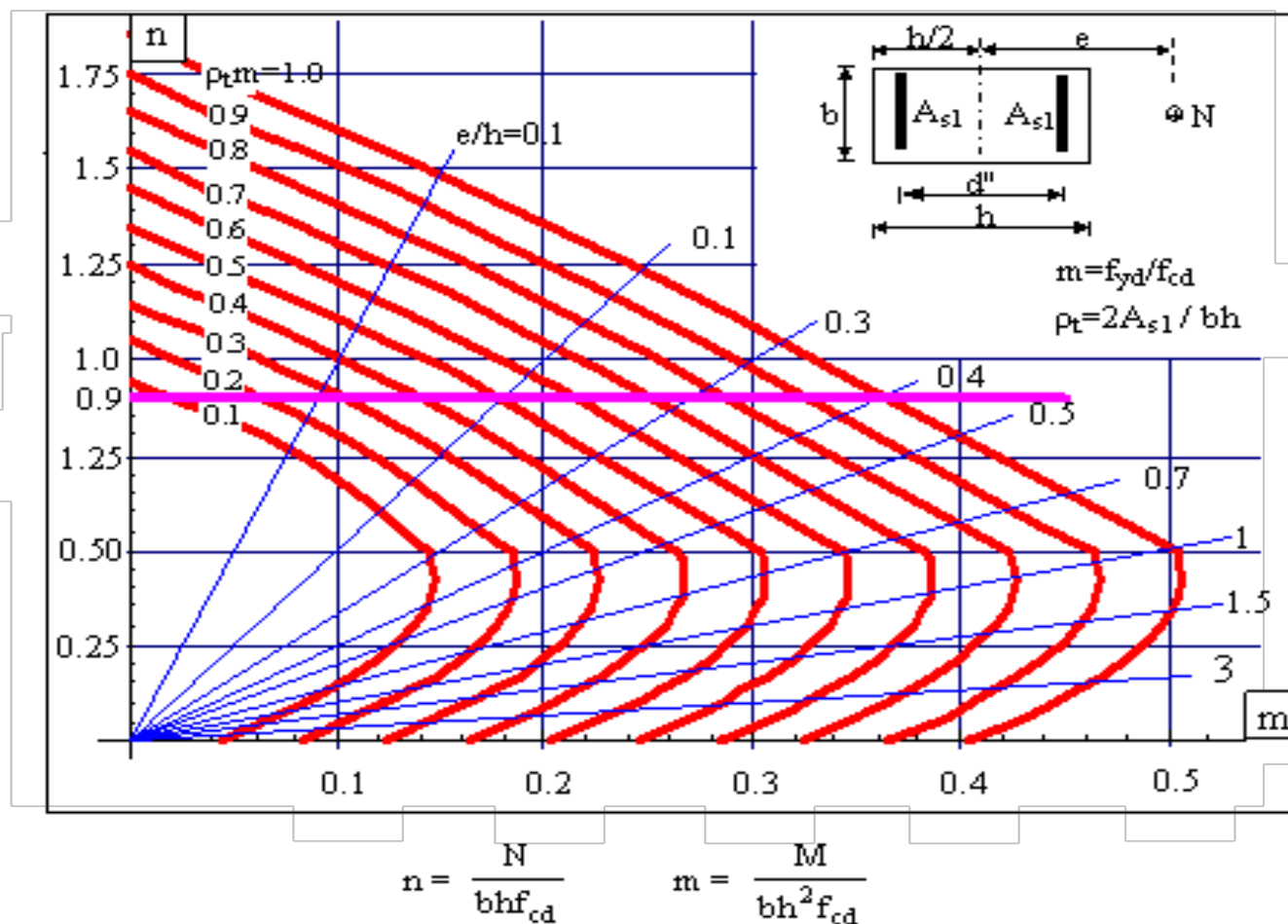


Figure 3.10

These charts may be used essentially in two ways for a given design (factored) load pair N and M .

- 1- (a) Select the dimensions b and h .
(b) Establish d' , calculate d''/h and select the corresponding design chart.
(c) Calculate N/bhf_{cd} and M/bh^2f_{cd} .
(d) Find the corresponding point on the graph and read the value of $\rho_t m$.
(e) Calculate $m = f_{yd} / f_{cd}$ and then ρ_t . Steel ratio should be less than $\max \rho_t$. Otherwise change the dimensions and repeat the calculations. If $\rho_t < \min \rho_t$, $\min \rho_t$ should be used in the column. However if calculated steel ratio is too small dimensions may be reduced and calculations may be repeated.
(d) Calculate $A_{st} = \rho_t bh$ and select the bars.
- 2- (a) Choose the desired steel ratio and calculate $\rho_t m$.
(b) Select h , establish d' and calculate d''/h . Select the corresponding graph.
(c) Calculate $e = M / N$ and then e/h .
(d) Find the intersection point of the e/h line and the $\rho_t m$ curve. Read the value of N/bhf_{cd} on the vertical axis
(e) Calculate b . Repeat the calculations if a better proportioned section is desired.
(f) Calculate $A_{st} = \rho_t bh$ and select the bars

Some other problems can also be solved by using these graphs. For example, if N is known for a given column M can easily be determined. Similarly if M is known N can be determined. If eccentricity is known for a given column both N and M can be determined.

Example 3.1

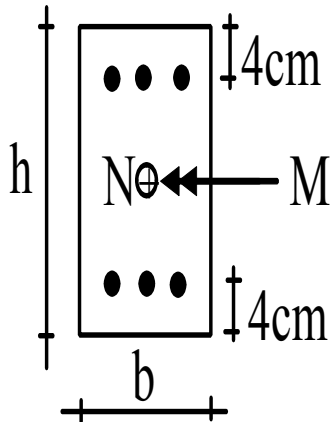


Figure 3.11

The column shown in Fig.3.11 is subjected to a design load pair of $N = 1200$ kN and $M = 200$ kN-m. Determine the dimensions and the reinforcement of the column if the reinforcement is placed at the sides as shown in the figure.

Materials: C25 and S420

Solution:

Minimum section area allowed by TS500:

$$\text{Min } A_c = \frac{N}{0.6f_{ck}} = \frac{1200000}{0.6 * 25} = 80000 \text{ mm}^2$$

Selected dimensions: $b = 30 \text{ cm}$, $h = 40 \text{ cm}$ ($300 * 400 = 120000 \text{ mm}^2 > 80000 \text{ mm}^2$).

$$d'' = 40 - 2 * 4 = 32 \text{ cm.} \quad d'' / h = 32 / 40 = 0.8 \quad \lambda = 0 \text{ (no middle bars)}$$

Chart A-3 corresponds to this column.

$$\frac{N}{bh f_{cd}} = \frac{1200 * 10^3}{300 * 400 * 17} = 0.59 \quad \frac{M}{bh^2 f_{cd}} = \frac{200 * 10^6}{300 * 400^2 * 17} = 0.25$$

$$\rho_t m = 0.46 \text{ (read from the chart)} \quad m = 365 / 17 = 21.5$$

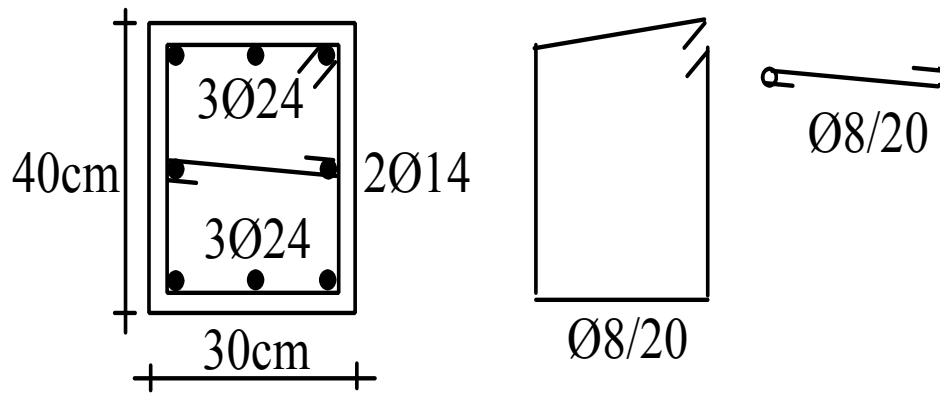
$$\rho_t = \frac{0.46}{21.5} = 0.021 \quad \min \rho_t = 0.01 < \rho_t = 0.021 < \max \rho_t = 0.04 \quad \text{OK.}$$

$$A_{st} = 0.021 * 30 * 40 = 25.7 \text{ cm}^2 \quad \text{Select: } 6\text{Ø}24 \text{ (} 27.14 \text{ cm}^2 \text{)}$$

$$\text{Ties: selected } \text{Ø}8 \quad \text{Check: } 8 = 24 / 3 = 8 \text{ mm} \quad \text{OK.}$$

$$\text{Spacing of ties: } s_{\max} = 12 * 24 = 288 \text{ mm} = 28.8 \text{ cm} > 20 \text{ cm.} \quad s = 20 \text{ cm.}$$

Details are shown in Fig.3.12.



Because the distance between the corner bars in the long direction is more than 30 cm, 2 $\text{Ø}14$ bars are placed between them and they are tied with $\text{Ø}8/20$ cross-ties.

Figure 3.12