Slenderness Effect in Columns

In the structures, all columns carry bending moment and these moments form curvature in the column. The curvature causes displacement between the two ends of the column. Sometimes these displacements may occur (relative) due to lateral displacement. These displacements result in increased eccentricity and increased moment.

General Method

The dimensioning and reinforcement design of the elements which are under axial load and also bending, is made according to the axial force and moment values obtained from the second order structural analyzes considering non-linear material behavior, cracking, creep and shrinkage of concrete under unfavorable load components. However, the following approximate method can be used in the calculation of the elements where the slenderness limit does not exceed the following limit;

$$(l_k/i) \leq 100$$

Approximate Method (Moment Amplification Method):

In this approximate method that is applied to columns whose cross-section and axial force does not change along the height, the design moment to be used in the design is determined by multiplying the maximum end moment obtained from the solution based on linear elastic assumptions, which has to meet the minimum eccentricity condition.

a) Lateral Shift Criteria

- It can be assumed that lateral displacement is prevented if there are shear walls, stiff walls or similar elements in the structural system which have sufficient rigidity against horizontal forces.

- It can be assumed that lateral displacement is avoided in cases where the column end moments obtained from the second order structural analysis under horizontal and vertical loads with the assumption of linear material behavior are not differ more than 5%, compared with the column end moments obtained from the first order analysis under the same assumptions and loads.

- If the second order analysis is not performed, it can be assumed that the stability index (ϕ) calculated for any floor of the structure considering the whole of the structural system does not exceed the limit stated below and that there is sufficient rigidity in the floor and lateral displacement is prevented.

$$\varphi=1.5\Delta_i \frac{\sum N_d / l_i}{V_{fi}} \leq 0.05$$

In these calculations, the assumption of uncracked section and

$$F_d=1.0G+1.0Q+1.0E$$

$$F_d=1.0G+1.3Q+1.3W$$

V_{fi}: sum of base shear force on the ith floor

Δ_i: the drift displacement of the ith floor

unfavorable value of the load compositions is taken into account

b) Effective length of Column

- The free height of the column is the distance between slabs, beams or other elements that provide lateral support to the column. In cases where the column cap or drop panel is present, the column free length is measured from the face of the cap or drop panel. In cases where a more reliable analysis method is not used, the effective length of the column can be obtained by multiplying the free length of the column by the coefficient “k”, which is associated with inhibition of rotation at the column ends.

$l_k = k\, l_n$ (l_n:column free length)

Column effective length coefficient “k” is defined below for both braced and un braced story columns.

For braced columns:

$$k = 0.7 + 0.05\,(\alpha_1 + \alpha_2)$$

But, $k \leq (0.85 + 0.05\,\alpha_1)$

$$k \leq 1.0$$

If the calculation is not made, k = 1.0 is taken for the braced columns.

For unbraced columns

if $\alpha_m < 2$: $$k = \frac{20 - \alpha_m}{20} \sqrt{1 + \alpha_m}$$

if $\alpha_m \geq 2$ $$k = 0.9 \sqrt{1 + \alpha_m}$$

For unbraced columns with hinged one end; $$k = 2 + 0.3\, \alpha_2$$

$$\alpha_{1,2} = \frac{\sum (I/l)_{column}}{\sum (I/l)_{beam}}, \quad \alpha_m = 0.5\, (\alpha_1 + \alpha_2)$$

For columns with fixed end: $\alpha = 0$

For columns with hinged end: $\alpha = \infty$

$(I/l)_{beam}$ is considered for only in the direction of bending

The column moment of inertia is calculated as the gross section moment of inertia. In practice, the cracked section moment of inertia of the beam is taken into account because of the fact that, capillary cracking of the beams can always occur.

The cracked section moment of inertia is considered to be approximately half of the section moment of inertia.

- According to TS500, second order moments may be omitted if the following conditions are met;

(c) Conditions where the slenderness effect may be neglected:

Lateral displacement-prevented (braced) floor columns:

M_2: Large moment in the column

M_1: Small moment in the column

i=0.3 h (rectangular)

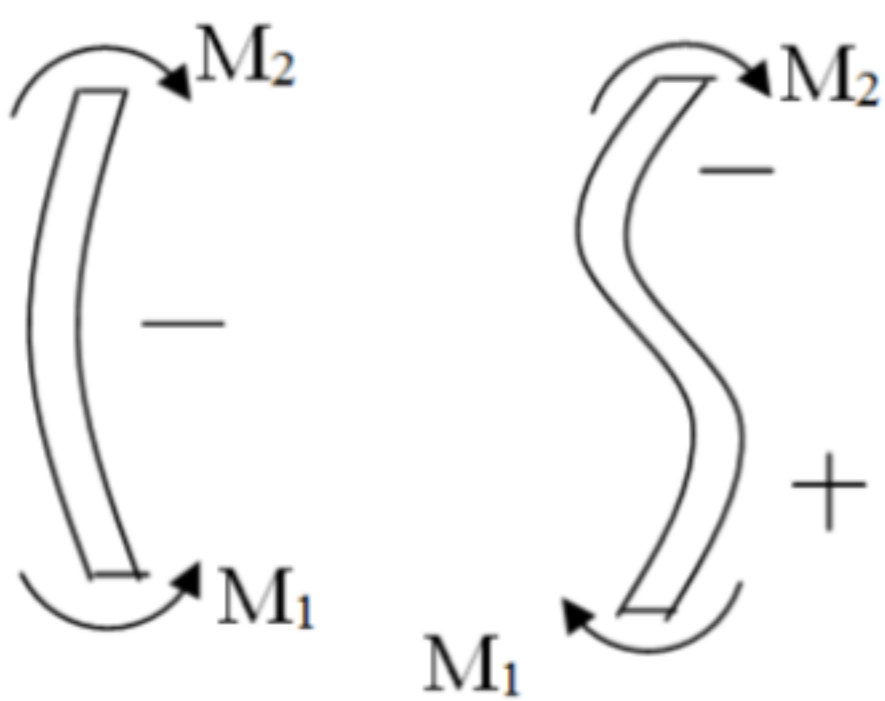
i=0.25 d (circular)

$$l_k/i \le 34 - 12\frac{M_1}{M_2} \le 40$$

If the condition is met, the slenderness effect can be neglected.

If M_1 and M_2 generate pressure on the same side of the column (single curved column), the ratio (M1 / M2) is positive (a), But for the vice versa case (double curved column) this ratio is negative (b).

$M_2 > M_1$



(a) (b)

For unbraced story columns:

$l_k/i \le 22$ If the condition is met, the slenderness effect can be neglected.

(d) Buckling Load Calculation:

Buckling Critical Load according to Euler Formula $N_k = \frac{\pi^2 EI}{(l_k)^2}$

Column effective bending stiffness (EI) can be obtained by the following equation in cases where a more reliable calculation is not performed.

$$EI = \frac{0.2\, E_c I_c + E_s I_s}{1 + R_m} \quad \text{or} \quad EI = \frac{0.4 E_c I_c}{1 + R_m}$$

Here, $E_c I_c$ is the bending stiffness of the entire concrete section and $E_s I_s$ is the bending stiffness of the longitudinal reinforcement section according to the center of gravity of the element section.

In the braced system: The creep ratio R_m is the ratio of the permanent load contribution to the column axial force obtained from the vertical loads to the total axial load value.

$$R_m = \frac{N_{gd}}{N_d}$$

In the unbraced system: The creep ratio R_m is the ratio of the sum of the permanent load contribution to the total calculated shear forces in all floor columns to the total shear forces (Vd).

$$R_m = \frac{\sum V_{gd}}{\sum V_d}$$ **for the whole floor**

Calculation Method for Slender Column

(e) Moment Magnifier Method

For braced columns

$$\beta = \frac{C_m}{1 - 1.3\frac{N_d}{N_k}} \geq 1.0$$

Where,

$$C_m = (0.6 + 0.4\frac{M_1}{M_2}) \geq 0.4\ ;\ \ M_1 \leq M_2$$

The ratio (M_1/M_2) is taken as positive in single curvature columns and negative in double curvature columns.

If there is any horizontal load acting between the column ends, Cm = 1.0 is taken.

Design moment to be used in design; $M_d' = \beta\ M_2$

For unbraced columns

For all columns of story $$\beta_s = \frac{1}{1 - 1.3\frac{\Sigma N_d}{\Sigma N_k}} \geq 1.0$$

Here, ΣN_d and ΣN_k are the sum of the axial design loads carried by the compression elements on that floor and the sum of the column critical loads. These values must meet the following condition, if not, the column sizes should be increased.

$$\Sigma N_d \leq 0.45 \Sigma N_k$$

for unbraced columns, individual β values should also be calculated for each columns of the story.

In these calculations C_m=1.0.

The greater of β and β_s values are used to find the calculation moment

$$M_d' = \beta M_2$$

$$M_d' = \beta_s M_2$$

Take the big one

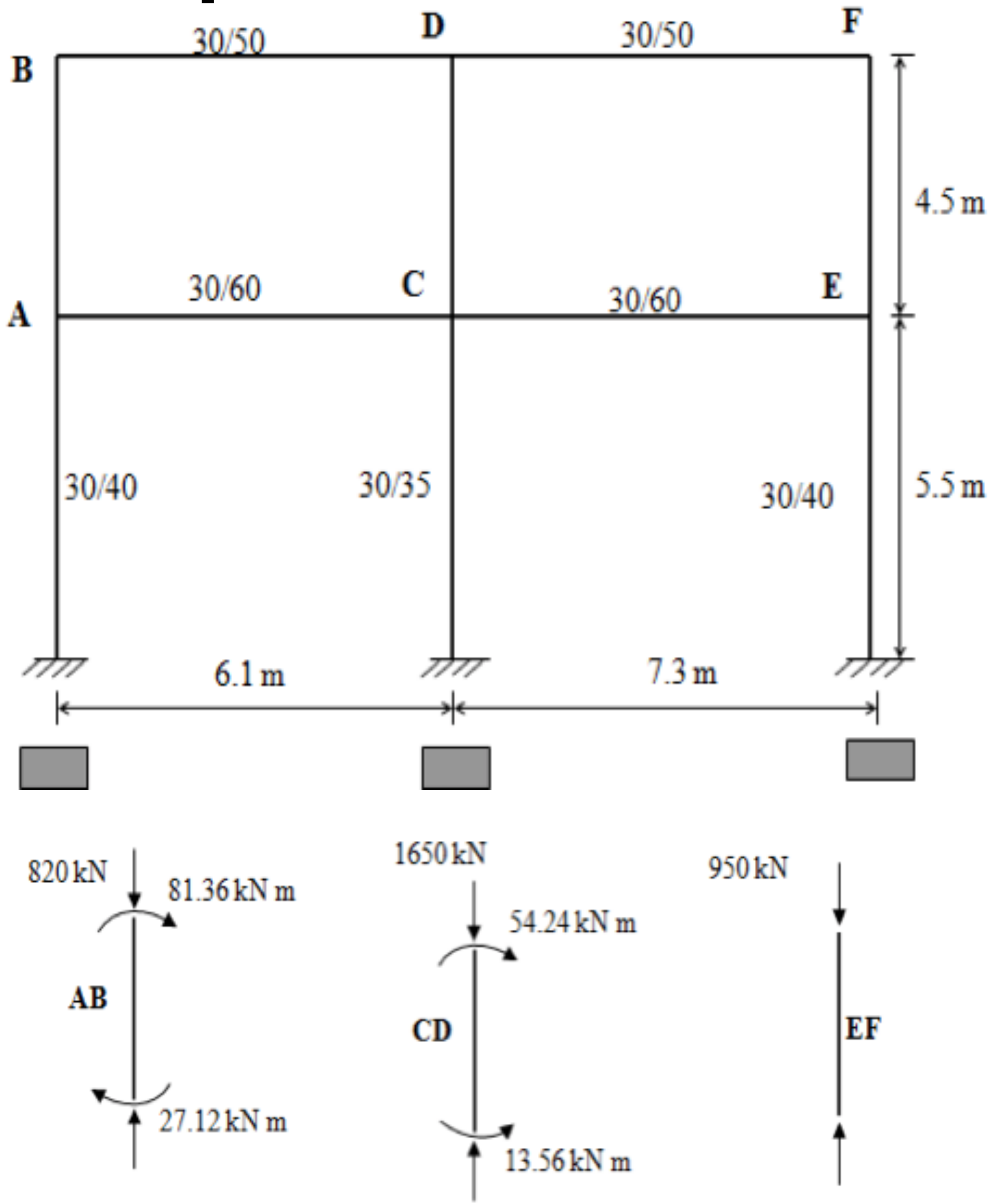
However, if the slenderness ratio found with the free height,

$$\left(\frac{L}{i}\right) > \frac{35}{\sqrt{\frac{N_d}{f_{ck} A_c}}}$$

The magnified moment

$$M_d' = \beta \beta_s M_2$$

Example



The frame in the figure is in its plane and unbraced.

Define that, columns A-B, C-D and E-F are slender or short columns.

Design columns A-B, and C-D.

R_m=0.4, E_c=30000 Mpa, f_{cd}=17 Mpa, f_{yd}=365 Mpa,

concrete cover thickness: 25 mm

Column A-B

$I_c=0.3*0.4^3/12=0.0016\ m^4$

$I_{B\ (A\text{-}C)}=0.3*0.6^3/12=0.0054\ m^4$, $I_{B,cr}=0.5*0.0054=0.0027\ m^4$

$I_{B\ (B\text{-}D)}=0.3*0.5^3/12=0.0031\ m^4$, $I_{B,cr}=0.00155\ m^4$

$$\alpha=\frac{\sum(I/L)_{kolon}}{\sum(I/L)_{kiriş}} ,$$

$$\alpha_A=\frac{\frac{0.0016}{4.5}+\frac{0.0016}{5.5}}{\frac{0.0027}{6.1}}=1.46,\ \alpha_B=\frac{\frac{0.0016}{4.5}}{\frac{0.00155}{6.1}}=1.4$$

$\alpha_m=0.5(\alpha_1+\alpha_2)=0.5(1.46+1.4)=1.43<2$

$$k=\frac{20-\alpha_m}{20}\sqrt{1+\alpha_m}=1.45$$

$L_{AB}=1.45*4.5=6.5$ m

$$\frac{kL}{i}=\frac{6.5}{0.3*0.4}=54.16>22$$ **(Slender Column)** **(i=0.3h)**

$$EI=\frac{E_c I_c}{2.5*(1+R_m)}=\frac{30*10^6*0.0016}{2.5*(1+0.4)}=13714.3 \text{ kN m}^2$$

Critical Load, $N_k=\frac{\pi^2 EI}{(kL)^2}=\frac{\pi^2*13714.3}{6.5^2}=3203.7$ kN

$$\frac{l}{i}=\frac{4.5}{0.3\ 0.4}=37.5, \qquad \frac{35}{\sqrt{\frac{N_d}{f_{ck}\ A_c}}}=\frac{35}{\sqrt{\frac{820x10^3}{25\ 300\ 400}}}=66.94$$

$$\frac{l}{i} < \frac{35}{\sqrt{\dfrac{N_d}{f_{ck}\ A_c}}}$$

There is no need to multiply β and β_s

Column C-D

$I_c = 0.3 * 0.35^3 / 12 = 0.0011\ m^4$

$$\alpha_C = \frac{\dfrac{0.0011}{4.5} + \dfrac{0.0011}{5.5}}{\dfrac{0.0027}{6.1} + \dfrac{0.0027}{7.3}} = 0.547\ \text{(alt)}$$

$$\alpha_D = \frac{\dfrac{0.0011}{4.5}}{\dfrac{0.00155}{6.1} + \dfrac{0.00155}{7.3}} = 0.524\ \text{(üst)},$$

$\alpha_m = 0.5(\alpha_1 + \alpha_2) = 0.5(0.5 + 0.547) = 0.535 < 2$

$$k=\frac{20-\alpha_m}{20}\sqrt{1+\alpha_m}=1.2$$

$$\frac{kL}{i}=\frac{1.2*4.5}{0.3*0.35}=51.43>22$$ **Slender Column**

$$EI=\frac{30*10^6*0.0011}{2.5*(1+0.4)}=9428.6\ kN\,m^2$$

$$N_k=\frac{\pi^2*9428.6}{(5.4)^2}=3191.2\ kN$$

$$\frac{l}{i}=42.86<\frac{35}{\sqrt{\frac{N_d}{f_{ck}\ A_c}}}=44.14$$ There is no need to multiply β and β_s

Column E-F

$$\alpha_E = \frac{\frac{0.0016}{4.5} + \frac{0.0016}{5.5}}{\frac{0.0027}{7.3}} = 1.75\,,\quad \alpha_F = \frac{\frac{0.0016}{4.5}}{\frac{0.00155}{7.3}} = 1.67,$$

$$\alpha_m = 0.5(\alpha_1 + \alpha_2) = 0.5(1.75 + 1.67) = 1.71 < 2$$

$$k = \frac{20 - \alpha_m}{20}\sqrt{1 + \alpha_m} = 1.505$$

$$\frac{kL}{i} = \frac{1.505 * 4.5}{0.3 * 0.4} = 56.43 > 22$$ **(Slender Column)**

$$EI = \frac{30 * 10^6 * 0.0016}{2.5 * (1 + 0.4)} = 13714.3\ \text{kN m}^2$$

$$N_k = \frac{\pi^2 * 13714.3}{(6.77)^2} = 2953.2\ \text{kN}$$

$$\frac{l}{i} = 37.5 < \frac{35}{\sqrt{\frac{N_d}{f_{ck}\ A_c}}} = 62.2$$ There is no need to multiply β and β_s

$\Sigma N_d = 3420$ kN, $\Sigma N_k = 9348.1$ kN $\Sigma N_d < 0.45 \Sigma N_k = 4206.6$ **appropriate**

$$\beta_s = \frac{C_m}{1 - 1.3\frac{\sum N_d}{\sum N_k}} = \frac{1}{1 - 1.3\frac{3420}{9348.1}} = 1.91$$

Cross Section Design

$\beta=1.50<\beta_s=1.91$

Column A-B

$M_d'=\beta M_d=1.91*81.36=155.4$ kN m , $N_d=820$ kN,

$d'=2.5$ cm, $d''=35$ cm $\frac{d''}{h}=\frac{35}{40}=0.9$, $\lambda=0$, S420, **S420**

$$\frac{N_d}{b\,h\,f_{cd}}=\frac{820*10^3}{300*400*17}=0.4\ ,\quad \frac{M_d'}{b\,h^2\,f_{cd}}=\frac{155.4*10^6}{300*400^2*17}=0.19$$

$\rho_t m=0.18$, $m=\frac{f_{yd}}{f_{cd}}=\frac{365}{17}=21.47$,

$\rho_t=0.18/21.47=0.0084<\rho_{t\,(min)}=0.01$

$A_{st}=0.01*300*400=1200\ mm^2$ ($8\phi14=1232\ mm^2$)

$$\boxed{\beta > \beta_s = 1.91}$$

Column C-D

$M_d' = \beta\, M_d = 3.03 * 54.24 = 164.35$ kN m , $N_d = 1650$ kN

$d' = 2.5$ cm, $d'' = 30$ cm , $\dfrac{d''}{h} = \dfrac{30}{35} \cong 0.9$, $\lambda = 0$, S420.

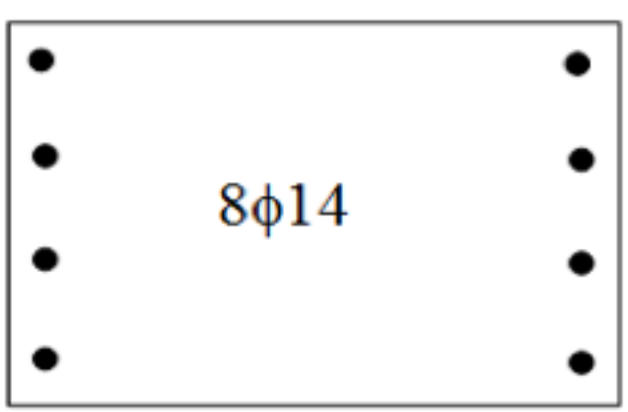
$$\frac{N_d}{b\,h\,f_{cd}} = \frac{1650 * 10^3}{300 * 350 * 17} = 0.92 > 0.9$$

can be accepted approx.=0.9

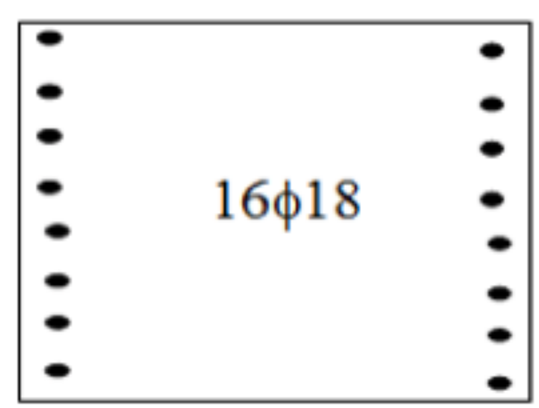
$$\frac{M_d'}{b\,h^2\,f_{cd}} = \frac{164.35x10^6}{300 * 350^2 * 17} = 0.263 , \quad \rho_t m = 0.82$$

$m = 21.47$, $\rho_t = 0.82/21.47 = 0.0382$

$A_{st} = 0.0382 * 300 * 350 = 4010\ mm^2$ ($16\phi18 = 4071\ mm^2$).

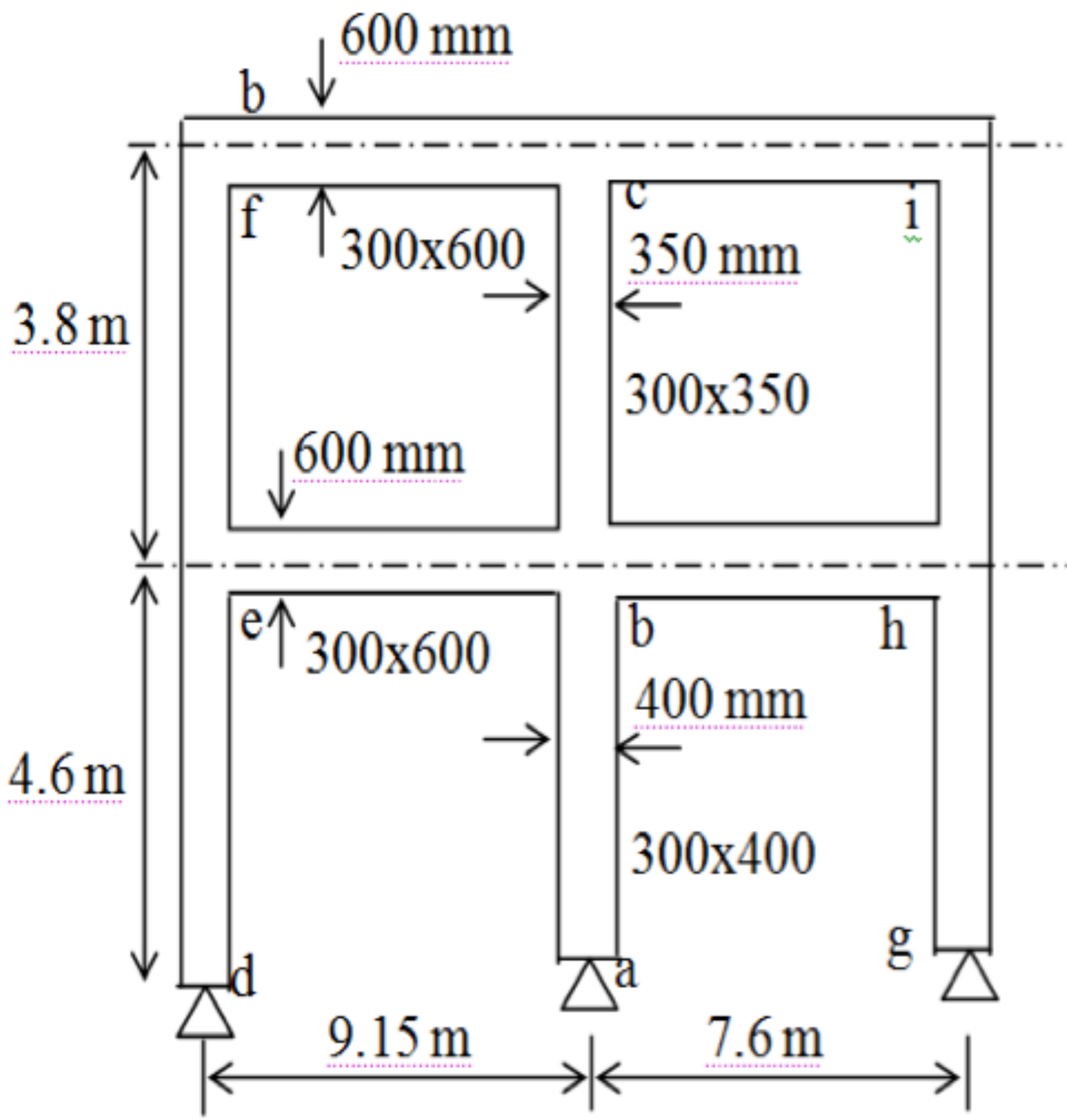


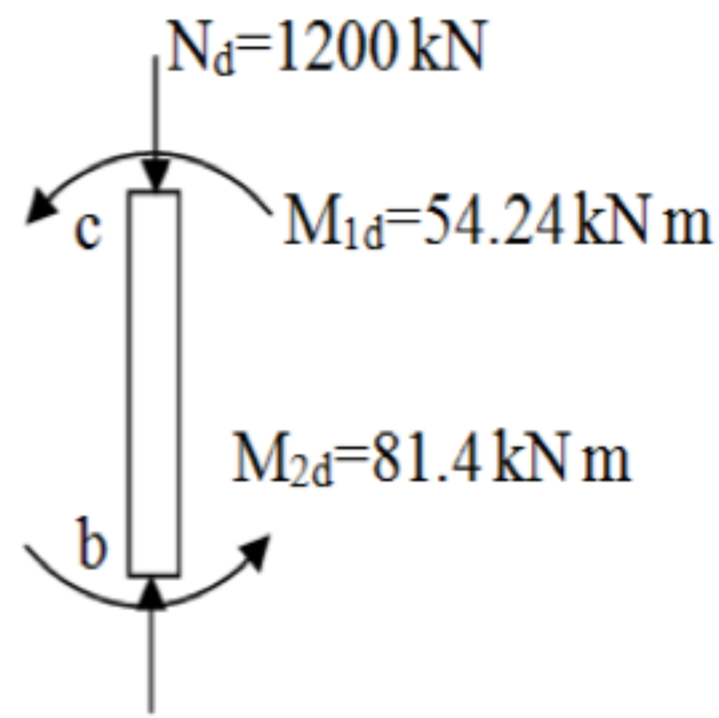
Column A-B



Column C-D

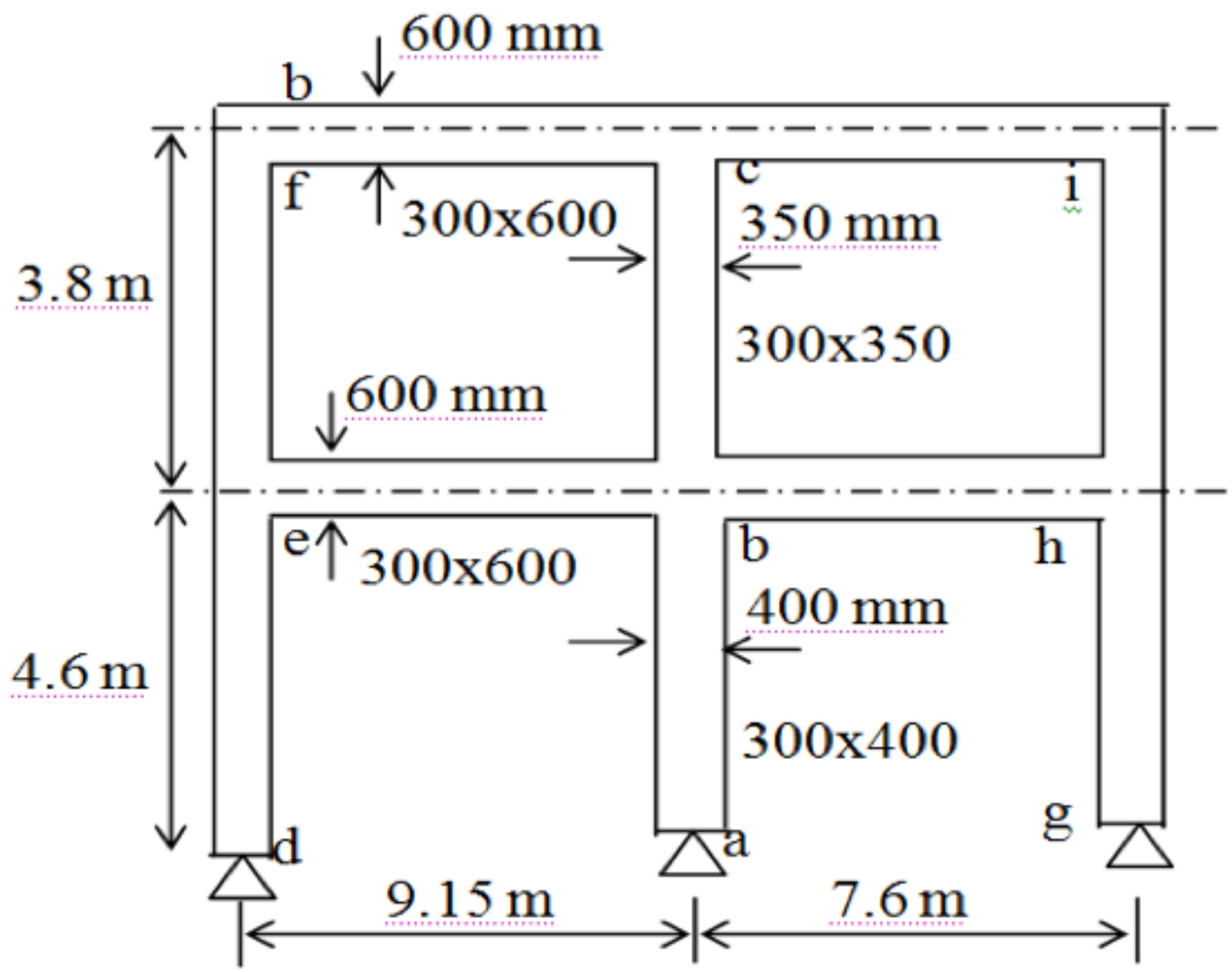
Example





$N_{(e\text{-}f)d}$=700 kN, $N_{(h\text{-}i)d}$=600 kN,

R_m=0.5, E_c=30250 N/mm^2



The members of the planar frame have rectangular sections and the frame is loaded in its plane.

a) Define a-b and b-c columns are slender or short column for braced and unbraced frame assumptions.

b) Design column b-c by considering that the frame is unbraced. The materials: C25 and S420, d’=35 mm. The given loads are design loads.

Solution

a)

columns (e-f), (b-c), (h-i):

$$I_C = 0.3 \times 0.35^3/12 = 0.00107 \ m^4,$$

column (a-b):

$$I_C = 0.3 \times 0.4^3/12 = 0.0016$$

Beams:

$$I_B = 0.3 \times 0.6^3/12 = 0.0054 \ m^4, \quad I_{B,cr} = 0.5 \times I_B = 0.0027 \, m^4$$

column (b-c):

$$\alpha_b = \frac{\frac{0.00107}{3.8} + \frac{0.0016}{4.6}}{\frac{0.0027}{9.15} + \frac{0.0027}{7.6}} = 0.97, \quad \alpha_c = \frac{\frac{0.00107}{3.8}}{\frac{0.0027}{9.15} + \frac{0.0027}{7.6}} = 0.43$$

Braced frame:

$$k=0.7+0.05(\alpha_1+\alpha_2)\leq(0.85+0.05\alpha_1)\leq1$$

$$k=0.7+0.05\times(0.97+0.43)=0.77$$

At braced frame column:

$$\frac{k\,L}{i}\leq 34-12\frac{M_1}{M_2}\leq 40$$ **if satisfied, slenderness can be neglected**

$$\frac{0.77\times3.8}{0.3\times0.35}=27.87\leq 34-12\times\frac{(-54.24)}{81.4}=47.3$$ **so, the column is not slender**

column (a-b):

$\alpha_b=0.97,\ \alpha_a=\infty$ **(Hinge support)**

$k=0.85+0.05\times\alpha_1=0.9$ **(can be calculated)**

$$\frac{k\,L}{i}=\frac{0.9\times4.6}{0.3\times0.4}=34.5,\quad 34-12\times\frac{M_1}{M_2}=34+12\times\frac{0}{81.4}=34\approx34.5$$ **so, the column is not slender**

Unbraced frame:

column (b-c):

$\alpha_b=0.97,\ \alpha_c=0.43,\quad \alpha_m=0.5(\alpha_1+\alpha_2)=0.5\times(0.97+0.43)=0.7<2$

$$k=\frac{20-\alpha_m}{20}\sqrt{1+\alpha_m}=1.26$$

$$\frac{k\ L}{i}=\frac{1.26\times(3.8)}{0.3\times(0.35)}=45.6>22$$ **so, the column is slender**

column (a-b):

$\alpha_b=0.97,\ \alpha_a=\infty,\quad k=2+0.3\alpha_2=2.29$

$$\frac{k\ L}{i}=\frac{2.29\times4.6}{0.3\times0.4}=87.8>22$$ **so, the column is slender**

b)

for unbraced column (b-c):

$$\frac{k\,L}{i} > 22$$ **Slender Column R_m=0.5**

$$EI = \frac{0.4 E_c\, I_c}{1+R_m} = \frac{0.4 \times 30250 \times 10^3 \times 0.00107}{1+0.5} = 8631.3\ kNm^2$$

$$N_k = \frac{\pi^2 EI}{(k\,L)^2} = \frac{\pi^2 \times 8631.3}{(1.26 \times 3.8)^2} = 3715.93$$

column (e-f):

$$\alpha_e = \frac{\frac{0.00107}{3.8} + \frac{0.0016}{4.6}}{\frac{0.0027}{9.15}} = 2.13, \quad \alpha_f = \frac{\frac{0.00107}{3.8}}{\frac{0.0027}{9.15}} = 0.954$$

$$\alpha_m = 0.5(\alpha_1 + \alpha_2) = 0.5 \times (2.13 + 0.954) = 1.542 < 2$$

$$k = \frac{20 - 1.542}{20} \times \sqrt{1 + 1.542} = 1.47, \quad \frac{k\,L}{i} > 22$$ **Slender Column Rm=0.5**

$$EI = 8631.3\,kNm^2, \quad N_k = \frac{\pi^2 \times 8631.3}{(1.47 \times 3.8)^2} = 2730\,kN$$

column (h-i):

$$\alpha_h = \frac{\frac{0.00107}{3.8} + \frac{0.0016}{4.6}}{\frac{0.0027}{7.6}} = 1.77, \quad \alpha_i = \frac{\frac{0.00107}{3.8}}{\frac{0.0027}{7.6}} = 0.79$$

$$\alpha_m = 0.5(\alpha_1 + \alpha_2) = 0.5 \times (1.77 + 0.79) = 1.28 < 2$$

$$k = \frac{20 - 1.28}{20} \times \sqrt{1 + 1.28} = 1.41, \quad \frac{k\,L}{i} > 22$$ **Slender Column Rm=0.5**

$$EI = 8631.3\,kNm^2, \quad N_k = \frac{\pi^2 \times 8631.3}{(1.41 \times 3.8)^2} = 2967.4\,kN$$

individual column (b-c): β; (C_m=1)

$$\beta=\frac{C_m}{1-1.3\frac{N_d}{N_k}}=\frac{1}{1-1.3\times\frac{1200}{3715.93}}=1.723$$

if, $\frac{k\,L}{i}>\frac{35}{\sqrt{\frac{N_d}{f_{ck}\,A_c}}}$ $\beta\;\beta_s$ **must be used**

$$\frac{k\,L}{i}=45.6,\quad \frac{35}{\sqrt{\frac{N_d}{f_{ck}\,A_c}}}=\frac{35}{\sqrt{\frac{1200\times10^3}{25\times300\times350}}}=51.76>45.6$$

So, no need for $\beta\;\beta_s$

Calculation of β_s for the story (C_m=1):

$$\beta_s = \frac{C_m}{1-1.3\frac{\sum N_d}{\sum N_k}} = \frac{1}{1-1.3\times\frac{1200+700+600}{3715.93+2730+2967.4}} = 1.527$$

According to TS500 $\sum N_d \le 0.45 \sum N_k$

$2500\,kN < 0.45\times 9413.3 = 4236\,kN$ **the rule of the code is satisfied.**

$\beta > \beta_s$ **so** $\beta = 1.723$ **should be used**

Designing of column (b-c):

$M_d' = 1.723\times 81.4 = 140.25\,kNm,\ N_d = 1200\,kN$

$$\frac{N_d}{b\,h\,f_{cd}} = \frac{1200\times 10^3}{300\times 350\times 17} = 0.67,\quad \frac{M_d}{b\,h^2\,f_{cd}} = \frac{140.25\times 10^6}{300\times 350^2\times 17} = 0.224,\quad \frac{d''}{h} = 0.8\quad \lambda = 0$$

From the chart

$\rho_t m = 0.45$ $\quad m = f_{yd}/f_{cd} = 21.47,\quad \rho_t = \frac{0.45}{21.47} = 0.02096 > 0.01$

$A_{st} = \rho_t\, b\, h = 0.02096\times 300\times 350 = 2200\,mm^2$ **Chosen** $(10\phi 18 = 2545\,mm^2)$