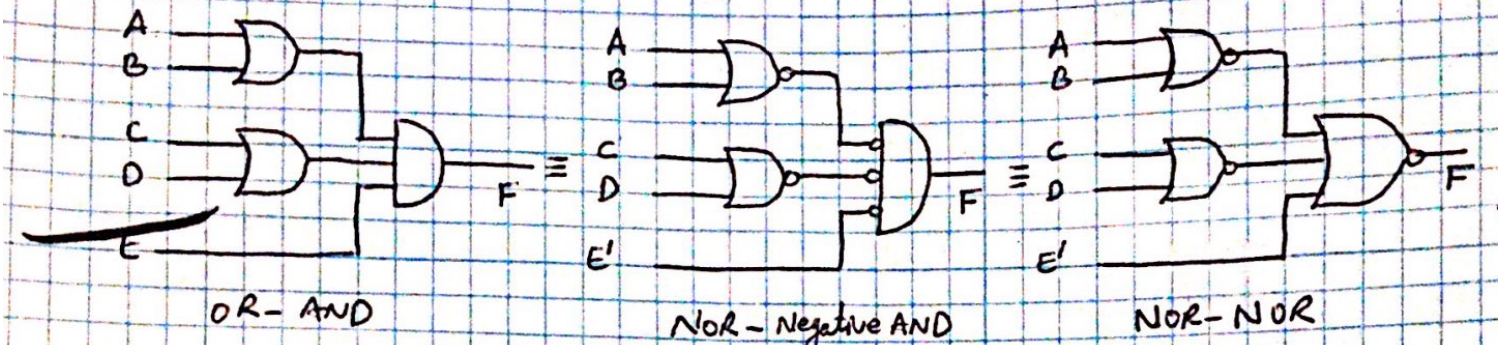
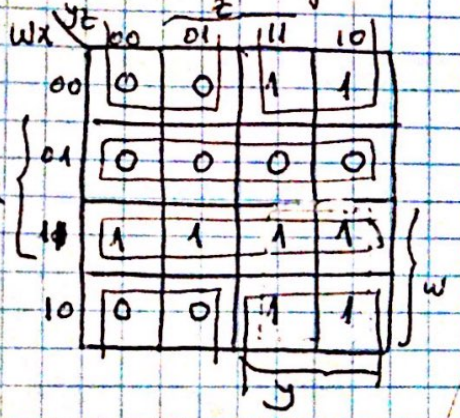
Example: Implement the function $F = (A+B)(C+D)E$ with NOR gates.



OR-AND NOR-Negative AND NOR-NOR

Example: Implement the function $F(w,x,y,z) = \sum(2,3,10,11,12,13,14,15)$ with NOR gates

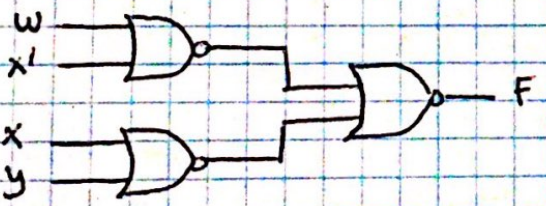


$\Rightarrow F = wx + x'y$

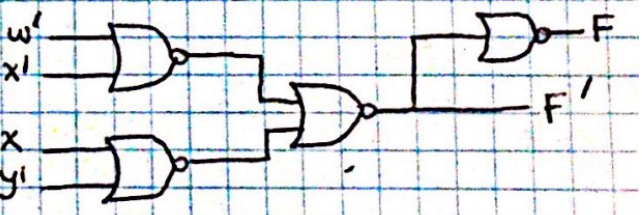
$F' = w'x + x'y'$ group of 0's

$F = (F')' = (w+x')(x+y)$ or

$F' = (w'+x')(x+y')$



Two-level



Three-level

Summary: A function in the standard form can be implemented in different ways as follows:

(Combine 1's in the K-map)

Sum of Products

AND - OR

NAND - NAND

NOR - OR

OR - NAND

(Combine 0's in the K-map to obtain F' then take the complement)

Product of Sums

OR - AND

NOR - NOR

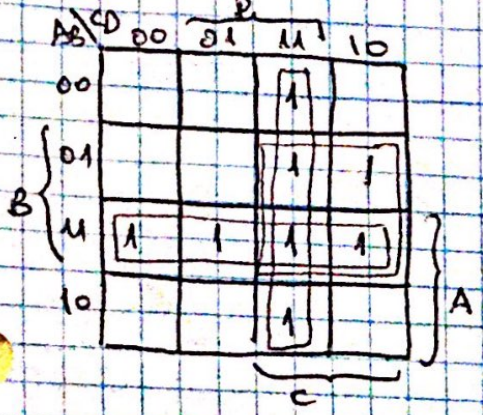
NAND - AND

AND - NOR

Example: (Sum of products implementation) ←

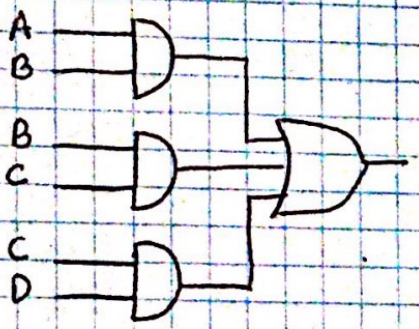
Implement the function $F(A,B,C,D) = \sum (3,6,7,11,12,13,14,15)$ by i) AND-OR ii) NAND-NAND iii) NOR-OR iv) OR-NAND

Sum of products: Combine 1's in the k-map:

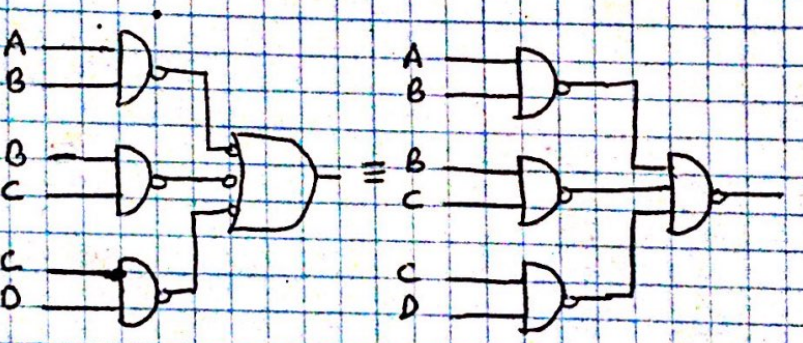


$$F = AB + BC + CD$$

i) AND-OR

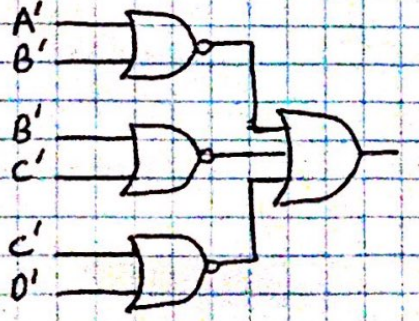


ii) NAND-NAND.

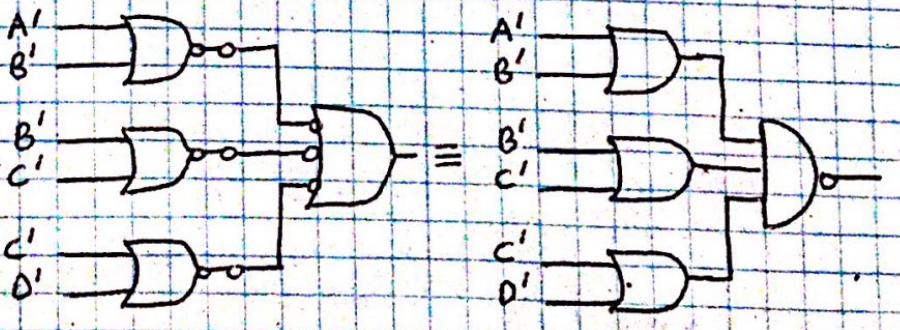


$$F' = (AB + BC + CD)' = (A'+B')(B'+C')(C'+D')$$

$$F = (F')' = [(A'+B')(B'+C')(C'+D')]' = (A'+B')' + (B'+C')' + (C'+D')'$$



iii) NOR-OR



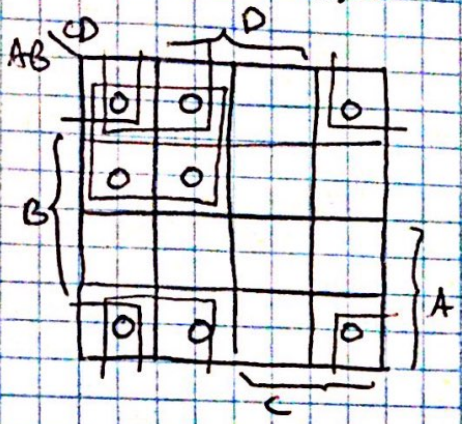
iv) OR-NAND

Product of Sums Implementation: (Combine 0's in K-map ⇒ F' then take the complement to obtain F)

Example:

Implement the function $F(A,B,C,D) = \sum(3,6,7,11,12,13,14,15)$ by

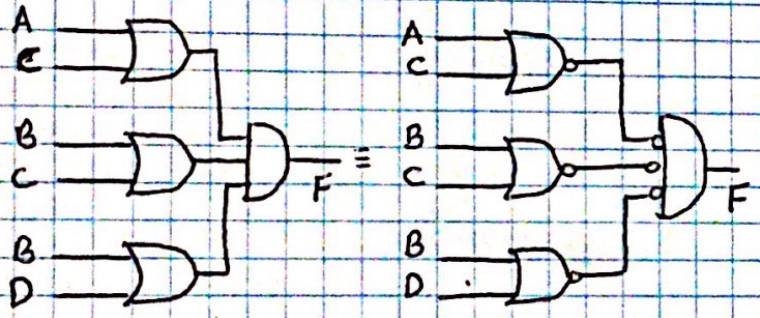
i) OR-AND ii) NOR-NOR iii) NAND-AND iv) AND-NOR



$$F' = A'C' + B'C' + B'D'$$

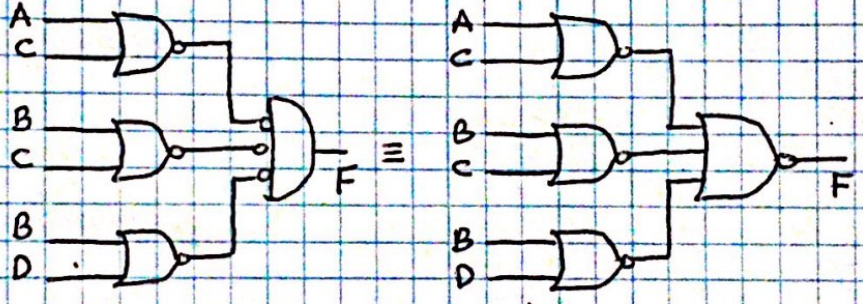
$$F = (F')' = (A+C)(B+C)(B+D)$$

i) OR-AND



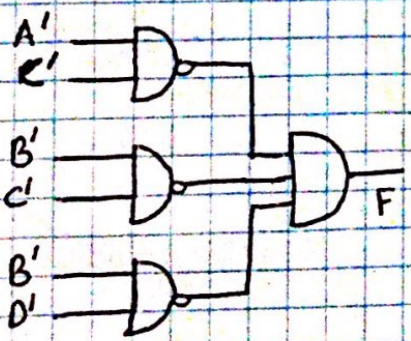
≡

ii) NOR-NOR



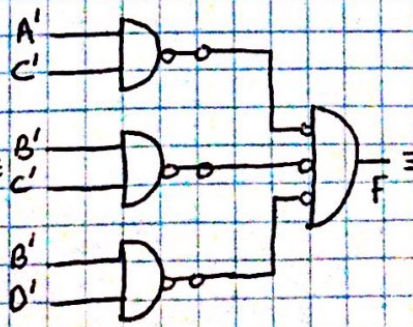
Now $F' = A'C' + B'C' + B'D'$

$$F = (F')' = (A'C')' \cdot (B'C')' \cdot (B'D')'$$



iii) NAND-AND

≡



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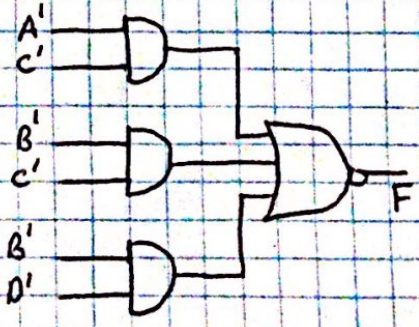


iv) AND-NOR

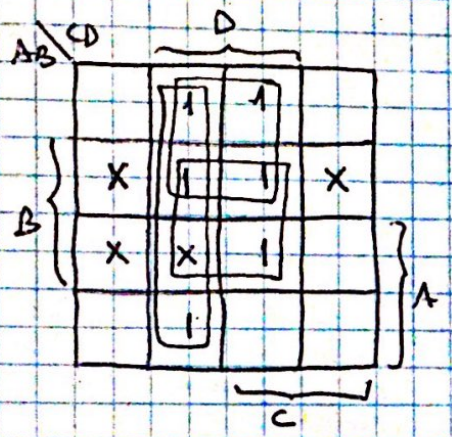
DON'T CARE CONDITIONS:

In some practical applications, the function is not specified for some combinations of the variables. for example, 4-bit BCD for decimal digits has six combinations that are not used. i.e the value of the function is unspecified for some minterms ⇒ These minterms are called don't care conditions (X). {i.e include X to have the power of 2 ← max. no. of combined 1's or 0's}

Example:

Simplify the following function: $F(A,B,C,D) = \sum(1,3,5,7,9,15)$

$d(A,B,C,D) = \sum(4,6,12,13)$



$F = A'D + BD + C'D$

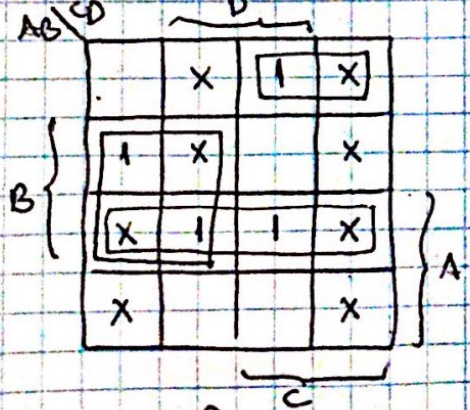
$= \sum(1,3,5,7,9,13,15)$

(DO not combine those squares that contain only x's)

Example:

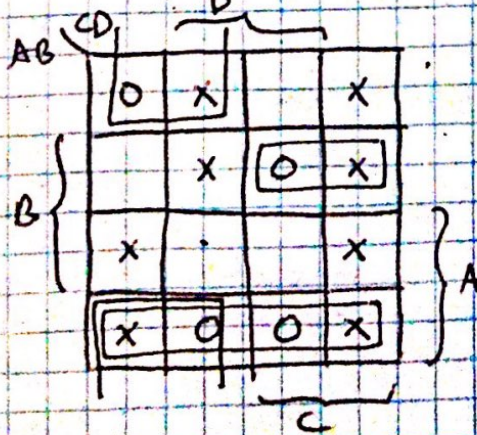
Simplify $F(A,B,C,D) = \sum(3,4,13,15)$; $d(A,B,C,D) = \sum(1,2,5,6,8,10,12,14)$

in i) SOP ii) POS



$F = AB + BC' + A'B'C$

$= \sum(2,3,4,5,12,13,14,15)$



$F' = AB' + B'C' + A'BC$

$F = (F')' = (A'+B)(B+C)(A+B'+C')$

← Simplification by using the tabulation (prime implicants) method:

Example: Determine the prime implicants of the function $F_1 = \Sigma(0,1,2,5,6,7,8,9,10,14)$ by using the tabulation (prime implicants) method.

		Column I		Column II			Column III	
		w x y z			w x y z			w x y z
group 1	0	0 0 0 0	✓	0,1	0 0 0 −	✓	0,1,8,9	− 0 0 −
				0,2	0 0 − 0	✓	0,2,8,10	− 0 − 0
group 2	1	0 0 0 1	✓	0,8	− 0 0 0	✓	0,8,1,9	− 0 0 −
	2	0 0 1 0	✓				0,8,2,10	− 0 − 0
	8	1 0 0 0	✓	1,5	0 − 0 1			
				1,9	− 0 0 1	✓	2,6,10,14	− − 1 0
group 3	5	0 1 0 1	✓	2,6	0 0 1 0	✓	2,10,6,14	− − 1 0
	6	0 1 1 0	✓	2,10	− 0 1 0	✓		
	9	1 0 0 1	✓	8,9	1 0 0 −	✓		
	10	1 0 1 0	✓	8,10	1 0 − 0	✓		
group 4	7	0 1 1 1	✓	5,7	0 1 − 1			
	14	1 1 1 0	✓	6,7	0 1 1 −			
				6,14	− 1 1 0	✓		
				10,14	1 − 1 0	✓		

1,5 5,7 6,7 0,1,8,9 0,2,8,10 2,6,10,14

$\Rightarrow F_1 = w'y'z + w'xz + w'xy + x'y' + x'z' + yz'$

$(0-01)\quad (01-1)\quad (011-)\quad (-00-)\quad (-0-0)\quad (--10)$

Very important note: Not that F_1 is not in the minimal sum of products. A minimum set of these prime implicants must be chosen to have the function in the minimal form. Goto (45)

(45) Goto *

* Note that the tabulation method can be used also by replacing binary numbers with decimals.

Example: Repeat the previous example

16 8 4 2 1

Column I			Column II			Column III	
0 0 0 0	0	✓	0,1	(1)	✓	0,1,8,9	(1,8)
			0,2	(2)	✓	0,2,8,10	(2,8)
0 0 0 1	1	✓	0,8	(8)	✓	0,8,1,9	(8,1)
0 0 1 0	2	✓				0,8,2,10	(8,2)
1 0 0 0	8	✓	1,5	(4)			
			1,9	(8)	✓	2,6,10,14	(4,8)
0 1 0 1	5	✓	2,6	(4)	✓	2,10,6,14	(8,4)
0 1 1 0	6	✓	2,10	(8)	✓		
1 0 0 1	9	✓	8,9	(1)	✓		
1 0 1 0	10	✓	8,10	(2)	✓		
			5,7	(2)			
0 1 1 1	7	✓	6,7	(1)			
1 1 1 0	14	✓	6,14	(8)	✓		
			10,14	(4)	✓		

Goto (45)

Simplification of Boolean functions using the Tabulation (Prime implicants) meth

A step-by-step procedure that guarantees a minimal expression. It can be used with any number of variables.

Step1: Minterms are sorted into groups according to the number of 1's they contain.

Step2: Any two minterms can be combined if they differ in exactly one variable, and the unmatched variable is removed (-).
(i.e each minterm in a group is compared with each minterm in the next group) and a check sign ✓ is placed.
The results of these comparisons are listed in the first group of column II. {# of variables will be reduced by 1}

Step3: Repeat the comparing process for the groups in column II and place the results in column III. {# of variables will be reduced by 1}.
Delete any duplication and repeat for groups in column III if possible.

Step4: The unchecked terms which cannot be combined with other terms are called the prime implicants.

Step5: Minimize the function by selecting some of the prime implicants (to be explained later):

back to (44) **

$F = \Sigma$

- Numbers in the parenthesis indicates the position of the dash in the binary number.
- For the comparison, pairs of numbers in the below group must be greater than the pairs of numbers in the above group, and the difference should be always a power of 2.
- only those terms with the same index in parenthesis in adjacent groups are compared.
- All remaining (unchecked) prime implicants are converted to binary by writing the binary equivalent and placing a dash in the position(s) indicated in the parenthesis.

Decimal	Prime implicants Binary w x y z	Term
1,5 (4)	0 - 0 1	$w'y'z$
5,7 (2)	0 1 - 1	$w'xz$
6,7 (1)	0 1 1 -	$w'xy$
0,1,8,9 (1,8)	- 0 0 -	$x'y'$
0,2,8,10 (2,8)	- 0 - 0	$x'z'$
2,6,10,14 (4,8)	- - 1 0	yz'

Example: Determine the prime implicants of the function

$$F_2(w,x,y,z) = \sum(1,4,6,7,8,9,10,11,15)$$

Column I		Column II		Column III	
0 0 0 1	1 ✓	1,9 (8)		8,9,10,11 (1,2)	
0 1 0 0	4 ✓	4,6 (2)		8,10,9,11 (2,1)	duplications are deleted
1 0 0 0	8 ✓	8,9 (1)	✓		
0 1 1 0	6 ✓	8,10 (2)	✓		
1 0 0 1	9 ✓	6,7 (1)			
1 0 1 0	10 ✓	9,11 (2)	✓		
0 1 1 1	7 ✓	10,11 (1)	✓		
1 0 1 1	11 ✓	7,15 (8)			
1 1 1 1	15 ✓	11,15 (4)			

Decimal	PI (binary) w x y z	Term
1,9 (8)	- 0 0 1	$x'y'z$
4,6 (2)	0 1 - 0	$w'xz'$
6,7 (1)	0 1 1 -	$w'xy$
7,15 (8)	- 1 1 1	xyz
11,15 (4)	1 - 1 1	wyz
8,9,10,11 (1,2)	1 0 - -	wx'

Minimization stage (i.e selection of prime implicants)

The second part of the tabulation method employs a prime implicant table to select a minimum set of prime implicants.

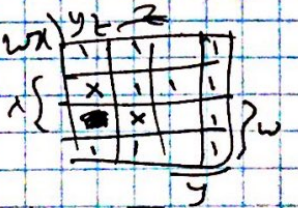
Example: Minimize the function $F_1 = \sum(0,1,2,5,6,7,8,9,10,14)$

We obtained the prime implicants of this function in a previous example.

The prime implicant table of this function will be as follows:

	Prime implicants	Minterms	0	1	2	5	6	7	8	9	10	14
	$w'y'z$	1,5		X		X						
	$w'xz$	5,7				X		X				
	$w'xy$	6,7					X	X				
✓	$x'y'$	0,1,8,9	X	X					X	X		
	$x'z'$	0,2,8,10	X		X				X		X	
✓	yz'	2,6,10,14			X		X				X	X
			✓	✓	✓		✓		✓	✓	✓	✓

- Cross out rows then columns of essential prime implicants (i.e where we have a single ⊗)
- Choose a minimum set of prime implicants to cover the remaining minterms.



$\Rightarrow F_1 = x'y' + yz' + w'xz$

Example: Minimize the function $F_2 = \sum(1,4,6,7,8,9,10,11,15)$ page 66

We've obtained the prime implicants of this function in a previous example. ⇒ the prime implicant table:

	Prime implicants		1	4	6	7	8	9	10	11	15
✓	$x'y'z$	1,9	⊗					X			
✓	$w'xz'$	4,6		⊗	X						
	$w'xy$	6,7			X	X					
	xyz	7,15				X					X
	wyz	15,15									X
✓	wx'	8,9,10,11					⊗	X	⊗	X	
			✓	✓	✓		✓	✓	✓	✓	

The essential prime implicants: $x'y'z$, $w'xz'$ and wx' cover all the minterms except 7 and 15 ⇒ include xyz which covers both

∴ $F = x'y'z + w'xz' + wx' + xyz$

Example: Select the prime implicants from the table given below to obtain the minimum SOP expression

Prime implicants		Minterms: 0	4	5	6	7	8	9	10	13	15
	$A'C'D'$	0,4	X	X							
	$B'C'D'$	0,8	X				X				
	$AB'C'$	8,9					X	X			
✓	$AB'D'$	8,10					X		⊗		
	$AC'D$	9,13						X		X	
✓	$A'B$	4,5,6,7	X	X	⊗	X					
✓	BD	5,7,13,15		X		X				X	⊗
			✓	✓	✓	✓	✓		✓	✓	✓

⇒ $F = AB'D' + A'B + BD + A'C'D' + AB'C'$

Note: The tabulation method can also be used to give a simplified expression in Product of sums.

⇒ we should start with F' by combining 0's, and continue to obtain F' simplified in SOP. Then take complement of F' to obtain F minimized in POS.