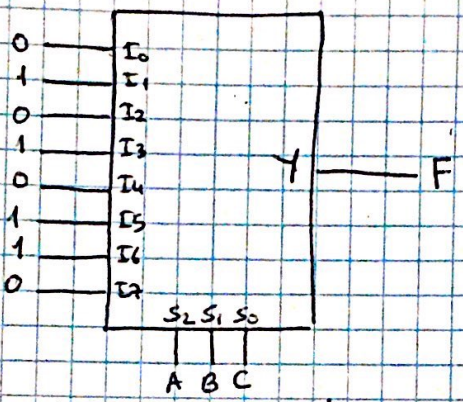
## Implementing Boolean functions with MUXs:

Any Boolean function can be implemented with MUXs. If a function has n input variables, it can be implemented by using $2^n$-to-1 MUX. The function should be expressed in sum of minterms form.

Example: Implement the function $F(A,B,C) = \sum(1,3,5,6)$ using MUX.

| A | B | C | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

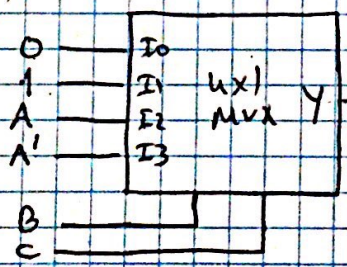



However, it is possible to generate any function of n+1 variables with a $2^n$-to-1 multiplexer. We connect n of the variables to the selection lines and the remaining single variable is used for the inputs of the MUX.

Example: Implement the previous function with 4-to-1 MUX.

implementation table

| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| $A'$ | 0 | (1) | 2 | (3) |
| $A$ | 4 | (5) | (6) | 7 |
| MUX inputs | 0 | 1 | A | A' |




1- Express F in sum of minterms

2- Use the leftmost variable (Assume it is A) for the inputs of the MUX and the remaining n variables are connected to the selection lines $S_{n-1}, S_{n-2}, \dots, S_0$

3- List the inputs of the MUX and under them list all the minterms in two rows (1st row corresponds to the complement of the leftmost variable (A') and 2nd row corresponds to the leftmost variable (A)

4- Circle the minterms of the function. Apply 0 of the two minterms.

5. • If the two minterms in a column are not circled ⇒ apply 0 to MUX input
   • If " " " " " circled ⇒ apply 1 to MUX input
   • If there is a single circle in the bottom minterm ⇒ apply A.
   • " " " " " " top " ⇒ apply A'

Note: It is not necessary to choose the leftmost variable for the data inputs of the MUX. Any of the variables can be chosen provided that the minterms are listed accordingly.

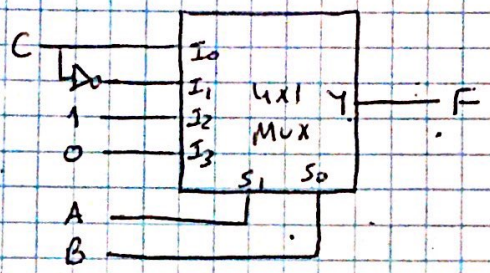
Example: Implement the function $F(A,B,C) = \sum(1,2,4,5)$ with MUX.

| A | B | C | F | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | F=C |
| 0 | 0 | 1 | 1 | |
| 0 | 1 | 0 | 1 | F=C' |
| 0 | 1 | 1 | 0 | |
| 1 | 0 | 0 | 1 | F=1 |
| 1 | 0 | 1 | 1 | |
| 1 | 1 | 0 | 0 | F=0 |
| 1 | 1 | 1 | 0 | |

lets choose C

| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| C' | 0 | (2) | (4) | 6 |
| C | (1) | 3 | (5) | 7 |
| | C | C' | 1 | 0 |

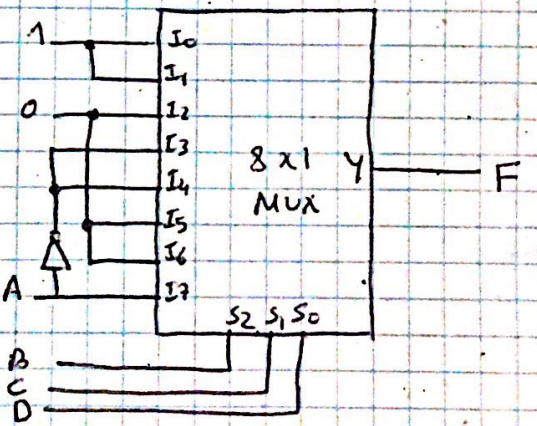
Implementation table




Example: Implement the function $F(A,B,C,D) = \sum(0,1,3,4,8,9,15)$ with 8x1 MUX. (lets choose A) for MUX inputs data

| | $I_0$ | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ |
|---|---|---|---|---|---|---|---|---|
| A' | (0) | (1) | 2 | (3) | (4) | 5 | 6 | 7 |
| A | (8) | (9) | 10 | 11 | 12 | 13 | 14 | (15) |
| | 1 | 1 | 0 | A' | A' | 0 | 0 | A |

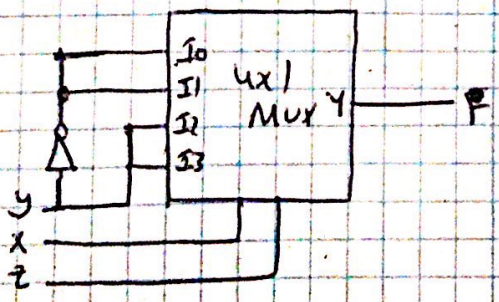
Implementation table




Example: Implement the function $F(x,y,z) = \sum(0,1,6,7)$ with a MUX.

Choose y for the MUX data inputs.

| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| y' | (0) | (1) | 4 | 5 |
| y | 2 | 3 | (6) | (7) |
| | y' | y' | y | y |




| x | y | z | |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | → |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | → |
| 1 | 0 | 1 | → |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

Example: Implement the Boolean function $F(A,B,C,D) = \sum(0,1,2,3,5,7,11,13)$ with a MUX. Exercise 11 ← choose $I_0$ HW: 7, 10, 15, 18, 23, 24