13–13 A gas mixture consists of 5 kg of O_2, 8 kg of N_2, and 10 kg of CO_2. Determine (a) the mass fraction of each component, (b) the mole fraction of each component, and (c) the average molar mass and gas constant of the mixture.

13-13 The masses of the constituents of a gas mixture are given. The mass fractions, the mole fractions, the average molar mass, and gas constant are to be determined.

Properties The molar masses of O_2, N_2, and CO_2 are 32.0, 28.0 and 44.0 kg/kmol, respectively (Table A-1)

Analysis (a) The total mass of the mixture is

$$m_m = m_{O_2} + m_{N_2} + m_{CO_2} = 5\,\text{kg} + 8\,\text{kg} + 10\,\text{kg} = 23\,\text{kg}$$

Then the mass fraction of each component becomes

$$mf_{O_2} = \frac{m_{O_2}}{m_m} = \frac{5\,\text{kg}}{23\,\text{kg}} = 0.217$$

$$mf_{N_2} = \frac{m_{N_2}}{m_m} = \frac{8\,\text{kg}}{23\,\text{kg}} = 0.348$$

$$mf_{CO_2} = \frac{m_{CO_2}}{m_m} = \frac{10\,\text{kg}}{23\,\text{kg}} = 0.435$$

| 5 kg O_2 |
| 8 kg N_2 |
| 10 kg CO_2 |

(b) To find the mole fractions, we need to determine the mole numbers of each component first,

$$N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{5\,\text{kg}}{32\,\text{kg/kmol}} = 0.156\,\text{kmol}$$

$$N_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{8\,\text{kg}}{28\,\text{kg/kmol}} = 0.286\,\text{kmol}$$

$$N_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{10\,\text{kg}}{44\,\text{kg/kmol}} = 0.227\,\text{kmol}$$

Thus,

$$N_m = N_{O_2} + N_{N_2} + N_{CO_2} = 0.156\,\text{kmol} + 0.286\,\text{kmol} + 0.227\,\text{kmol} = 0.669\,\text{kmol}$$

and

$$y_{O_2} = \frac{N_{O_2}}{N_m} = \frac{0.156\,\text{kmol}}{0.699\,\text{kmol}} = 0.233$$

$$y_{N_2} = \frac{N_{N_2}}{N_m} = \frac{0.286\,\text{kmol}}{0.669\,\text{kmol}} = 0.428$$

$$y_{CO_2} = \frac{N_{CO_2}}{N_m} = \frac{0.227\,\text{kmol}}{0.669\,\text{kmol}} = 0.339$$

(c) The average molar mass and gas constant of the mixture are determined from their definitions:

$$M_m = \frac{m_m}{N_m} = \frac{23\,\text{kg}}{0.669\,\text{kmol}} = 34.4\,\text{kg/kmol}$$

and

$$R_m = \frac{R_u}{M_m} = \frac{8.314\,\text{kJ/kmol·K}}{34.4\,\text{kg/kmol}} = 0.242\,\text{kJ/kg·K}$$

EXAMPLE 13–3 Mixing Two Ideal Gases in a Tank

An insulated rigid tank is divided into two compartments by a partition, as shown in Fig. 13–14. One compartment contains 7 kg of oxygen gas at 40°C and 100 kPa, and the other compartment contains 4 kg of nitrogen gas at 20°C and 150 kPa. Now the partition is removed, and the two gases are allowed to mix. Determine (a) the mixture temperature and (b) the mixture pressure after equilibrium has been established.

Solution A rigid tank contains two gases separated by a partition. The pressure and temperature of the mixture are to be determined after the partition is removed.

Assumptions 1 We assume both gases to be ideal gases, and their mixture to be an ideal-gas mixture. This assumption is reasonable since both the oxygen and nitrogen are well above their critical temperatures and well below their critical pressures. 2 The tank is insulated and thus there is no heat transfer. 3 There are no other forms of work involved.

Properties The constant-volume specific heats of N_2 and O_2 at room temperature are $c_{v,N_2} = 0.743$ kJ/kg · K and $c_{v,O_2} = 0.658$ kJ/kg · K (Table A–2a).

Analysis We take the entire contents of the tank (both compartments) as the system. This is a *closed system* since no mass crosses the boundary during the process. We note that the volume of a rigid tank is constant and thus there is no boundary work done.

(a) Noting that there is no energy transfer to or from the tank, the energy balance for the system can be expressed as

$$E_{in} - E_{out} = \Delta E_{system}$$

$$0 = \Delta U = \Delta U_{N_2} + \Delta U_{O_2}$$

$$[mc_v(T_m - T_1)]_{N_2} + [mc_v(T_m - T_1)]_{O_2} = 0$$

By using c_v values at room temperature, the final temperature of the mixture is determined to be

$$(4\text{ kg})(0.743\text{ kJ/kg} \cdot \text{K})(T_m - 20°C) + (7\text{ kg})(0.658\text{ kJ/kg} \cdot \text{K})(T_m - 40°C) = 0$$

$$T_m = 32.2°C$$

(b) The final pressure of the mixture is determined from the ideal-gas relation

$$P_m V_m = N_m R_u T_m$$

where

$$N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{7\text{ kg}}{32\text{ kg/kmol}} = 0.219\text{ kmol}$$

$$N_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{4\text{ kg}}{28\text{ kg/kmol}} = 0.143\text{ kmol}$$

$$N_m = N_{O_2} + N_{N_2} = 0.219 + 0.143 = 0.362\text{ kmol}$$

O_2	N_2
7 kg	4 kg
40°C	20°C
100 kPa	150 kPa
	←Partition

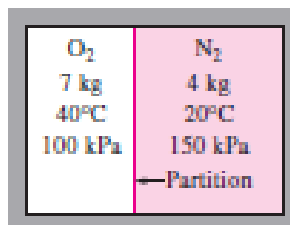**FIGURE 13–14**
Schematic for Example 13–3.

and

$$V_{O_2} = \left(\frac{NR_uT_1}{P_1}\right)_{O_2} = \frac{(0.219 \text{ kmol})(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(313 \text{ K})}{100 \text{ kPa}} = 5.70 \text{ m}^3$$

$$V_{N_2} = \left(\frac{NR_uT_1}{P_1}\right)_{N_2} = \frac{(0.143 \text{ kmol})(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(293 \text{ K})}{150 \text{ kPa}} = 2.32 \text{ m}^3$$

$$V_m = V_{O_2} + V_{N_2} = 5.70 + 2.32 = 8.02 \text{ m}^3$$

Thus,

$$P_m = \frac{N_m R_u T_m}{V_m} = \frac{(0.362 \text{ kmol})(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(305.2 \text{ K})}{8.02 \text{ m}^3} = 114.5 \text{ kPa}$$

Discussion We could also determine the mixture pressure by using $P_m V_m = m_m R_m T_m$, where R_m is the apparent gas constant of the mixture. This would require a knowledge of mixture composition in terms of mass or mole fractions.

FIGURE 13–15
Schematic for Example 13–4.

EXAMPLE 13–4   Exergy Destruction during Mixing of Ideal Gases

An insulated rigid tank is divided into two compartments by a partition, as shown in Fig. 13–15. One compartment contains 3 kmol of O_2, and the other compartment contains 5 kmol of CO_2. Both gases are initially at 25°C and 200 kPa. Now the partition is removed, and the two gases are allowed to mix. Assuming the surroundings are at 25°C and both gases behave as ideal gases, determine the entropy change and exergy destruction associated with this process.

Solution A rigid tank contains two gases separated by a partition. The entropy change and exergy destroyed after the partition is removed are to be determined.

Assumptions Both gases and their mixture are ideal gases.

Analysis We take the entire contents of the tank (both compartments) as the system. This is a *closed system* since no mass crosses the boundary during the process. We note that the volume of a rigid tank is constant, and there is no energy transfer as heat or work. Also, both gases are initially at the same temperature and pressure.

When two ideal gases initially at the same temperature and pressure are mixed by removing a partition between them, the mixture will also be at the same temperature and pressure. (Can you prove it? Will this be true for non-ideal gases?) Therefore, the temperature and pressure in the tank will still be 25°C and 200 kPa, respectively, after the mixing. The entropy change of each component gas can be determined from Eqs. 13–18 and 13–25:

$$\Delta S_m = \sum \Delta S_i = \sum N_i \, \Delta \bar{s}_i = \sum N_i \left(\bar{c}_{p,i} \ln \frac{T_{i,2}^{\,\nearrow 0}}{T_{i,1}} - R_u \ln \frac{P_{i,2}}{P_{i,1}} \right)$$

$$= -R_u \sum N_i \ln \frac{y_{i,2} P_{m,2}}{P_{i,1}} = -R_u \sum N_i \ln y_{i,2}$$

since $P_{m,2} = P_{i,1} = 200$ kPa. It is obvious that the entropy change is independent of the composition of the mixture in this case and depends on only

the mole fraction of the gases in the mixture. What is not so obvious is that if the same gas in two different chambers is mixed at constant temperature and pressure, the entropy change is zero.

Substituting the known values, the entropy change becomes

$$N_m = N_{O_2} + N_{CO_2} = (3 + 5) \text{ kmol} = 8 \text{ kmol}$$

$$y_{O_2} = \frac{N_{O_2}}{N_m} = \frac{3 \text{ kmol}}{8 \text{ kmol}} = 0.375$$

$$y_{CO_2} = \frac{N_{CO_2}}{N_m} = \frac{5 \text{ kmol}}{8 \text{ kmol}} = 0.625$$

$$\Delta S_m = -R_u (N_{O_2} \ln y_{O_2} + N_{CO_2} \ln y_{CO_2})$$

$$= -(8.314 \text{ kJ/kmol} \cdot \text{K})[(3 \text{ kmol})(\ln 0.375) + (5 \text{ kmol})(\ln 0.625)]$$

$$= 44.0 \text{ kJ/K}$$

The exergy destruction associated with this mixing process is determined from

$$X_{destroyed} = T_0 S_{gen} = T_0 \, \Delta S_{sys}$$

$$= (298 \text{ K})(44.0 \text{ kJ/K})$$

$$= 13.1 \text{ MJ}$$

Discussion This large value of exergy destruction shows that mixing processes are highly irreversible.

EXAMPLE 13–2   P-v-T Behavior of Nonideal Gas Mixtures

A rigid tank contains 2 kmol of N_2 and 6 kmol of CO_2 gases at 300 K and 15 MPa (Fig. 13–10). Estimate the volume of the tank on the basis of (a) the ideal-gas equation of state, (b) Kay's rule, (c) compressibility factors and Amagat's law, and (d) compressibility factors and Dalton's law.

Solution The composition of a mixture in a rigid tank is given. The volume of the tank is to be determined using four different approaches.

Assumptions Stated in each section.

Analysis (a) When the mixture is assumed to behave as an ideal gas, the volume of the mixture is easily determined from the ideal-gas relation for the mixture:

$$V_m = \frac{N_m R_u T_m}{P_m} = \frac{(8 \text{ kmol})(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(300 \text{ K})}{15{,}000 \text{ kPa}} = 1.330 \text{ m}^3$$

since

$$N_m = N_{N_2} + N_{CO_2} = 2 + 6 = 8 \text{ kmol}$$

(b) To use Kay's rule, we need to determine the pseudocritical temperature and pseudocritical pressure of the mixture by using the critical-point properties of N_2 and CO_2 from Table A–1. However, first we need to determine the mole fraction of each component:

$$y_{N_2} = \frac{N_{N_2}}{N_m} = \frac{2 \text{ kmol}}{8 \text{ kmol}} = 0.25 \quad \text{and} \quad y_{CO_2} = \frac{N_{CO_2}}{N_m} = \frac{6 \text{ kmol}}{8 \text{ kmol}} = 0.75$$

$$T'_{cr,m} = \sum y_i T_{cr,i} = y_{N_2} T_{cr,N_2} + y_{CO_2} T_{cr,CO_2}$$
$$= (0.25)(126.2 \text{ K}) + (0.75)(304.2 \text{ K}) = 259.7 \text{ K}$$

$$P'_{cr,m} = \sum y_i P_{cr,i} = y_{N_2} P_{cr,N_2} + y_{CO_2} P_{cr,CO_2}$$
$$= (0.25)(3.39 \text{ MPa}) + (0.75)(7.39 \text{ MPa}) = 6.39 \text{ MPa}$$

Then,

$$\left. \begin{array}{l} T_R = \dfrac{T_m}{T'_{cr,m}} = \dfrac{300 \text{ K}}{259.7 \text{ K}} = 1.16 \\[2mm] P_R = \dfrac{P_m}{P'_{cr,m}} = \dfrac{15 \text{ MPa}}{6.39 \text{ MPa}} = 2.35 \end{array} \right\} Z_m = 0.49 \quad \text{(Fig. A–15b)}$$

Thus,

$$V_m = \frac{Z_m N_m R_u T_m}{P_m} = Z_m V_{ideal} = (0.49)(1.330 \text{ m}^3) = 0.652 \text{ m}^3$$

2 kmol N_2
6 kmol CO_2
300 K
15 MPa
$V_m = ?$

FIGURE 13–10

Schematic for Example 13–2.

(c) When Amagat's law is used in conjunction with compressibility factors, Z_m is determined from Eq. 13–10. But first we need to determine the Z of each component on the basis of Amagat's law:

N_2: $\left.\begin{array}{l} T_{R,N_2} = \dfrac{T_m}{T_{cr,N_2}} = \dfrac{300\text{ K}}{126.2\text{ K}} = 2.38 \\[2mm] P_{R,N_2} = \dfrac{P_m}{P_{cr,N_2}} = \dfrac{15\text{ MPa}}{3.39\text{ MPa}} = 4.42 \end{array}\right\}$ $Z_{N_2} = 1.02$ (Fig. A–15b)

CO_2: $\left.\begin{array}{l} T_{R,CO_2} = \dfrac{T_m}{T_{cr,CO_2}} = \dfrac{300\text{ K}}{304.2\text{ K}} = 0.99 \\[2mm] P_{R,CO_2} = \dfrac{P_m}{P_{cr,CO_2}} = \dfrac{15\text{ MPa}}{7.39\text{ MPa}} = 2.03 \end{array}\right\}$ $Z_{CO_2} = 0.30$ (Fig. A–15b)

Mixture: $\quad Z_m = \sum y_i Z_i = y_{N_2} Z_{N_2} + y_{CO_2} Z_{CO_2}$
$$= (0.25)(1.02) + (0.75)(0.30) = 0.48$$

Thus,

$$V_m = \frac{Z_m N_m R_u T_m}{P_m} = Z_m V_{ideal} = (0.48)(1.330\text{ m}^3) = \mathbf{0.638\text{ m}^3}$$

The compressibility factor in this case turned out to be almost the same as the one determined by using Kay's rule.

(d) When Dalton's law is used in conjunction with compressibility factors, Z_m is again determined from Eq. 13–10. However, this time the Z of each component is to be determined at the mixture temperature and volume, which is not known. Therefore, an iterative solution is required. We start the calculations by assuming that the volume of the gas mixture is 1.330 m³, the value determined by assuming ideal-gas behavior.

The T_R values in this case are identical to those obtained in part (c) and remain constant. The pseudoreduced volume is determined from its definition in Chap. 3:

$$v_{R,N_2} = \frac{\bar{v}_{N_2}}{R_u T_{cr,N_2}/P_{cr,N_2}} = \frac{V_m/N_{N_2}}{R_u T_{cr,N_2}/P_{cr,N_2}}$$

$$= \frac{(1.33\text{ m}^3)/(2\text{ kmol})}{(8.314\text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(126.2\text{ K})/(3390\text{ kPa})} = 2.15$$

Similarly,

$$v_{R,CO_2} = \frac{(1.33\text{ m}^3)/(6\text{ kmol})}{(8.314\text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(304.2\text{ K})/(7390\text{ kPa})} = 0.648$$

From Fig. A–15, we read $Z_{N_2} = 0.99$ and $Z_{CO_2} = 0.56$. Thus,

$$Z_m = y_{N_2} Z_{N_2} + y_{CO_2} Z_{CO_2} = (0.25)(0.99) + (0.75)(0.56) = 0.67$$

and

$$V_m = \frac{Z_m N_m R T_m}{P_m} = Z_m V_{ideal} = (0.67)(1.330\text{ m}^3) = 0.891\text{ m}^3$$

This is 33 percent lower than the assumed value. Therefore, we should repeat the calculations, using the new value of V_m. When the calculations are repeated we obtain 0.738 m³ after the second iteration, 0.678 m³ after the third iteration, and 0.648 m³ after the fourth iteration. This value does not change with more iterations. Therefore,

$$V_m = \mathbf{0.648\text{ m}^3}$$

Discussion Notice that the results obtained in parts (b), (c), and (d) are very close. But they are very different from the ideal-gas values. Therefore, treating a mixture of gases as an ideal gas may yield unacceptable errors at high pressures.