Final Exam Topics

I. INTRODUCTION

- Differences between (mathematical) modeling and (computer) simulation
- Trade-offs between tractable and realistic models
- Importance of analyzing stochastic systems either by modeling or by simulation
- Different kinds of commercially/freely available simulation programs

II. ELEMENTS OF PROBABILITY

- Sample spaces, events, probabilities
- Axioms of probability
- Conditional probability
- Independence
- Random variables (discrete and continuous)
- Probability mass function (pmf), probability density function (pdf), cumulative distribution function (cdf)
- Joint distributions
- Expectation, variance, standard deviation, covariance, correlation
- Sample vs. population
- Histograms
- The Strong Law of Large Numbers
- Examples of discrete random variables (Bernoulli, binomial, Poisson, geometric)
- Examples of continuous random variables (uniform, normal (Gaussian), exponential)
- The Central Limit Theorem
- The Poisson process
- Conditional expectation

III. RANDOM NUMBER GENERATION

- C library support for generating random numbers
- Generation of uniform random numbers in C: The rand() function
- Specifying the "seed" for rand(): The srand() function
- Monte Carlo Integration
- Estimation of PI

IV. DISCRETE RV GENERATION

- Discrete inverse transform method
- Generation of geometric RVs
- Algorithm for generating Poisson RVs
- Algorithm for generating binomial RVs

V. CONTINUOUS RV GENERATION

- Inverse transform method
- Generation of exponential RVs
- Algorithms for generating normal RVs

  - Box-Muller transformations
  - Polar method
- Generating a Poisson process
- The Rejection method (or the Accept/Reject method) of generating RVs

VI. MONTE CARLO SIMULATION OF COMMUNICATION SYSTEMS
- A communication system: transmitter, channel, and receiver
- The binary symmetric channel
- The entropy as a measure of "disorder"
- Channel capacity
- Additive White Gaussian Noise
- Signal-to-noise ratio
- Simple binary transmission over the Gaussian channel
- Monte Carlo simulation (history and applications)

VII. DISCRETE EVENT SIMULATION
- Key elements of discrete event simulation approach: Events and variables
- Time variable, counter variables, system state variables
- Event lists
- Simulation of a single-server FIFO queueing system
- Trace-driven simulation

VIII. STATISTICAL ANALYSIS OF SIMULATED DATA
- Population versus sample
- Estimators for population mean, population variance, and population standard deviation: sample mean, sample variance, and sample standard deviation
- Unbiased estimators
- "Worth" of an estimator: mean square error (MSE)
- Standard error
- Determining how many times a simulation program must be run to report an estimate of a quantity of interest with a certain degree of confidence
- Interval estimates of population mean, confidence intervals
- Transient removal: Long runs, proper initialization, truncation, initial data deletion, moving average of independent replications, batch means
- Estimation with correlated observations

IX. STATISTICAL VALIDATION TECHNIQUES
- Comparing simulation results with real-system measurements: Mann-Whitney U Test (or Two-Sample Rank-Sum Test)
- Comparing two alternatives (testing for zero mean in difference for paired observations)