Supply Chain Management

Part 7

Aggregate Planning in a Supply Chain

The role of aggregate planning in supply chain

Imagine a limitless & free world

- In which manufacturing, transportation, warehousing, information sharing ***are all*** without capacity constraint, without time delays, without costs!
- In this world, we would not need to do any planning to anticipate the demand.

BUT in the real world, this being not the case, we need to plan in order to be able to satisfy the demand. ***Because***, building and having the capacity needed has a cost and lead times are often long.

So we need to answer questions that relate to the required capacity, production levels, outsource or not, inventory levels, etc.

The role of aggregate planning in supply chain

Aggregate planning:

- Process by which a company determines planned levels of capacity, production, subcontracting, inventory, stockouts, and pricing over a specified time horizon.
- Goal is to build a plan that will satisfy demand & maximize profit.
- Decisions made at a product family (not SKU) level.
- Time frame of 3 to 18 months: too early to plan production at SKU level, but too late to arrange for additional capacity.
- How can a firm best use the facilities it has?

The role of aggregate planning in supply chain

Main objective is to identify the following operational parameters over the specified time horizon:

- ***Production rate:*** The number of units to be completed per unit time (such as per week or per month).
- ***Workforce:*** The number of workers or units of labor capacity required.
- ***Overtime:*** The amount of overtime production planned.
- ***Machine capacity level:*** The number of units of machine capacity needed for production.
- ***Subcontracting:*** The subcontracted capacity required over the planning horizon.
- ***Backlog:*** Demand not satisfied in the period in which it arises, but is carried over to future periods.
- ***Inventory on hand:*** The planned inventory carried over the various periods in the planning horizon.

Information needed for an aggregate plan

- Demand forecast in each period.
- Production costs:
  - labor costs, regular time ($/hr) and overtime ($/hr).
  - subcontracting costs ($/hr or $/unit).
  - cost of changing capacity: hiring or layoff ($/worker) and cost of adding or reducing machine capacity ($/machine).
- Labor/machine hours required per unit.
- Inventory holding cost ($/unit/period).
- Stockout or backlog cost ($/unit/period).
- Constraints: limits on overtime, layoffs, capital available, stockouts and backlogs.

Outputs of aggregate plan

- Production quantity from regular time, overtime, and subcontracted time, # of workers needed per category, suppliers purchase levels.
- Inventory held: warehousing space needed to store it, and working capital required.
- Backlog/stockout quantity: used to determine customer service levels.
- Workforce hired / laid off: watch out early for labor issues
- Machine capacity increase/decrease: determine new equipment for purchase, or available idle to sell

Aggregate planning strategies

- **Level Strategy**: level strategy looks to maintain a steady production rate and workforce level. Advantage of level strategy is steady workforce. Disadvantage of level strategy is high inventory and increase back logs.
- ***Chase Strategy:*** chase strategy looks to dynamically match demand with production. Advantage of chase strategy is lower inventory levels and back logs. Disadvantage is lower productivity, quality and depressed work force.
- **Hybrid Strategy:** hybrid strategy looks to balance between level strategy and chase strategy.

Aggregate planning using linear programming

Month	Demand Forecast
January	1,600
February	3,000
March	3,200
April	3,800
May	2,200
June	2,200

Item	Cost
Material cost	$10/unit
Inventory holding cost	$2/unit/month
Marginal cost of stockout/backlog	$5/unit/month
Hiring and training costs	$300/worker
Layoff cost	$500/worker
Labor hours required	4/unit
Regular time cost	$4/hour (no of units produced or regular time = 40/mo)
Overtime cost	$6/hour
Cost of subcontracting	$30/unit

Aggregate planning using linear programming

- Each product is sold through retailers for 40$.
- Starting inventory of 1,000 tools.
- Workforce of 80 employees; 20 working days in each month.
- 8 hour per day for each employee + 2 overtime.
- Capacity is determined by total labor hours worked.
- No limits on subcontracting, inventory and stockouts/backlogs.
- All stockouts are backlogged and supplied from next months production.
- Inventory costs appear at the end of each month.
- Pay $4/hr for regular time; overtime has to be $\leq$ 10 hrs/mo.
- What is the optimal aggregated plan that allows at the end of June to have at least 500 units of inventory.

Aggregate planning using linear programming

- ***Decision Variables:***

For month t = 1, ..., 6

W_t = Workforce size for month t

H_t = Number of employees hired at the beginning of month t

L_t = Number of employees laid off at the beginning of month t

P_t = Production in month t

I_t = Inventory at the end of month t

S_t = Number of units stocked out or backlogged at the end of month t

C_t = Number of units subcontracted for month t

O_t = Number of overtime hours worked in month t

Aggregate planning using linear programming Objective function

- ***Objective function:***

The objective function is to minimize the total cost incurred during the planning horizon. The cost incurred has the following components:

- Regular-time labor cost
- Overtime labor cost
- Cost of hiring and layoffs
- Cost of holding inventory
- Cost of stocking out
- Cost of subcontracting
- Material cost

Aggregate planning using linear programming Objective function

- These costs are evaluated as follows:

1- Regular-time labor cost:

A worker is paid a regular-time wage of \$640 (\$4/hour * 8 hours/day * 20 days/month)

Regular-time labor cost $= \sum_{t=1}^{6} 640W_t$

2- Overtime labor cost: As overtime labor cost is \$6 per hour and O_t represents the number of overtime hours worked in Period t, the overtime cost over the planning horizon is

Overtime labor cost $= \sum_{t=1}^{6} 6O_t$

Aggregate planning using linear programming Objective function

3- Cost of hiring and layoffs: the cost of hiring a worker is \$300 and the cost of laying off a worker is \$500. H_t and L_t represent the number hired and the number laid off, respectively, in Period t.

Cost of hiring and firing $= \sum_{t=1}^{6} 300H_t + \sum_{t=1}^{6} 500L_t$

4- Cost of inventory and stockout: the cost of carrying inventory is \$2 per unit per month, and the cost of stocking out is \$5 per unit per month. I_t and S_t represent the units in inventory and the units stocked out, respectively, in Period t.

Cost of inventory and stockout $= \sum_{t=1}^{6} 2I_t + \sum_{t=1}^{6} 5S_t$

Aggregate planning using linear programming Objective function

5- *Cost of materials and subcontracting:* The material cost is \$10 per unit and the subcontracting cost is \$30/unit. P_t represents the quantity produced and C_t represents the quantity subcontracted in Period t.

Cost of materials and subcontracting = $\sum_{t=1}^{6} 10P_t + \sum_{t=1}^{6} 30C_t$

The objective function is to minimize the sum of all the aforementioned cost:

$$Min \sum_{t=1}^{6} 640W_t + \sum_{t=1}^{6} 6O_t + \sum_{t=1}^{6} 300H_t + \sum_{t=1}^{6} 500L_t + \sum_{t=1}^{6} 2I_t + \sum_{t=1}^{6} 5S_t + \sum_{t=1}^{6} 10P_t + \sum_{t=1}^{6} 30C_t$$

Aggregate planning using linear programming Constraints

1- *Workforce, hiring, and layoff constraints:* the workforce size W_t in Period t is obtained by adding the number hired H_t in Period t to the workforce size in Period , and subtracting the number laid off L_t in Period t as follows:

$$W_t = W_{t-1} + H_t - L_t \quad \text{for } t = 1, \ldots, 6$$

The starting workforce size is given by $W_0 = 80$.

2- *Capacity constraint:* in each period, the amount produced cannot exceed the available capacity. As each worker can produce 40 units per month on regular time (four hours per unit) and one unit for every four hours of overtime, we have the following:

$$P_t \leq 40W_t + \frac{O_t}{4} \qquad \text{for } t = 1, \ldots, 6$$

Aggregate planning using linear programming Constraints

3- ***Inventory balance constraints:*** net demand for Period *t* is obtained as the sum of the current demand D_t and the previous backlog S_{t-1}. This demand is either filled from current production (in-house production P_t or subcontracted production C_t) and previous inventory I_{t-1}.

This relationship is captured by the following equation:

$$I_t = I_{t-1} + P_t + C_t - D_t - S_{t-1} + S_t \qquad \text{for } t = 1, \ldots, 6$$

The starting inventory is given by $I_0 = 1000$, the ending inventory must be at least 500 units (i.e., $I_6 = 500$), and initially there are no backlogs (i.e., $S_0 = 0$).

Aggregate planning using linear programming Constraints

3- Overtime limit constraints: no employee work more than 10 hours of overtime each month. This requirement limits the total amount of overtime hours available as follows:

$O_t \leq 10W_t$ for t = 1, …, 6

In addition, each variable must be nonnegative and there must be no backlog at the end of Period 6 (i.e., $S_6 = 0$).

Aggregate planning using linear programming Constraints

For this aggregate plan, we have the following:

Total Cost over planning horizon = $ 422,660

Period, t	No. Hired, H_t	No. Laid Off, L_t	Workforce Size, W_t	Overtime, O_t	Inventory, I_t	Stockout, S_t	Subcontract, C_t	Total Production, P_t	Demand, D_t
0	0	0	80	0	1,000	0	0		
1	0	16	64	0	1,960	0	0	2,560	1,600
2	0	0	64	0	1,520	0	0	2,560	3,000
3	0	0	64	0	880	0	0	2,560	3,200
4	0	0	64	0	0	220	140	2,560	3,800
5	0	0	64	0	140	0	0	2,560	2,200
6	0	0	64	0	500	0	0	2,560	2,200