Multiplication rule:

$$P(A \cap B) = P(A).P({}^{B}/_{A}) = P(B).P({}^{A}/_{B})$$

Example: suppose that 2 people are randomly chosen from a group of 4 women and 6 men.

- What is the probability that both are women?
- What is the probability that one is woman and one is man?

$W_i = ith\ member\ is\ woman$

$M_i = ith\ member\ is\ man$

$$P(W_1 \cap W_2) = P(W_1)P\left({}^{W_2}/_{W_1}\right) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$$

$$P(W_1 \cap M_2) + P(M_1 \cap W_2) = P(W_1)P\left({}^{M_2}/_{W_1}\right) + P(M_1)P\left({}^{W_2}/_{M_1}\right) = \frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} = \frac{8}{15}$$

Independency:

- Events A and B are independent, $P(A \cap B) = P(A).P(B)$
- Then $P({}^{A}/_{B}) = P(A)$ & $P({}^{B}/_{A}) = P(B)$
- If $A_1, A_2, \ldots, A_n$ are independent, then
- $P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1).P(A_2), \ldots P(A_n)$

Example: A couple is planning to have children, assume that each child is equally likely to be of either gender and the gender of the children is independent. Find the probability that:

- All three children will be girl?
- At least one child will be girl?

$G_i = ith\ child\ is\ girl,\ B_i = ith\ child\ is\ boy$

$$P(\text{All three children will be girl}) = P(G_1 \cap G_2 \cap G_3) = P(G_1) \times P(G_2) \times P(G_3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(\text{At least one child will be girl}) = 1 - P(No\ girl) = 1 - \left(P(B_1) \times P(B_2) \times P(B_3)\right)$$

$$= 1 - (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) = \frac{7}{8}$$

Example: A game club has 120 members, 40 members play chess, 56 members Play Bridge, 26 members play both chess & bridge.

If a member of the club is randomly chosen, find the conditional probability that he or she

- Plays chess given that he or she plays bridge?
- Plays bridge given that he or she plays chess?

$n(S) = 120$

$C = Chess\ player \rightarrow n(C) = 40$

$B = Bridg\ player \rightarrow n(B) = 56$

$n(B \cap C) = 26$

$$P({}^{C}/_{B}) = \frac{P(B \cap C)}{P(B)} = \frac{{}^{n(B \cap C)}/_{n(S)}}{{}^{n(B)}/_{n(S)}} = \frac{{}^{26}/_{120}}{{}^{56}/_{120}} = \frac{13}{28}$$

$$P\left({}^{B}/_{C}\right) = \frac{P(B\cap C)}{P(C)} = \frac{{n(B\cap C)}/{n(S)}}{{n(C)}/{n(S)}} = \frac{{26}/{120}}{{40}/{120}} = \frac{13}{20}$$

Remark

Contribution of n things taken r

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

It represents the number of different groups of size r that can be selected from a set of size n when the order of selection is not important.

Note:

- $\binom{n}{r} = \binom{n}{n-r}$ $\qquad 0! = 1 \qquad$ $\binom{n}{1} = \frac{n!}{1!(n-1)!} = \frac{n\times(n-1)\times(n-2)\ldots\times1}{1!\times(n-1)(n-2)\ldots\times1} = n$

Example: How many different groups of size 3 can be chosen from a set of 6 people?

$$\text{groups of size 3 from a set of 6 people} = \binom{6}{3} = \frac{6!}{3!\,(6-3)!} = \frac{6\times5\times4\times3!}{3\times2\times3!} = 20$$

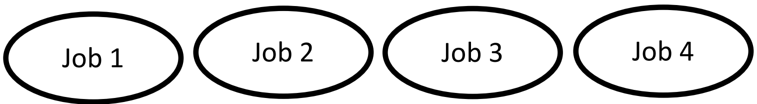
Example: A committee of 4 people is selected randomly from a group of 5 men and 7 women.

- What is the probability that the committee will consist of 2 men and 2 women?

$$\text{probability that the committee will consist of 2 men and 2 women} = \frac{\binom{5}{2}\times\binom{7}{2}}{\binom{12}{4}} = \frac{14}{33}$$

Example: if 4 workers are assigned to 4 jobs,

- How many different assignments are possible?
- How many assignments are possible if workers 1 & 2 are both able to do jobs 1 & 2 and workers 3 & 4 are able to do jobs 3 & 4?



$$\text{Possible assignments} = 4\times3\times2\times1 = 24$$

$$\text{possible assignments if workers 1 \& 2 able to do jobs 1 \& 2 and workers 3 \& 4 able to do jobs 3 \& 4} = 2\times1\times2\times1 = 4$$

Example: A delivery company has 12 trucks which 4 of them have faulty brakes, if an inspector randomly chosen 2 trucks for brake check,

- What is the probability that none of them has faulty brake?

$$\text{probability that none of them has faulty brake} = \frac{\binom{4}{0}\times\binom{8}{2}}{\binom{12}{2}} = \frac{14}{33}$$

Chapter 5

Discrete random variable

Random variable: is a function that associates a real number with each element in the sample space.

Discrete Random Variable: A random variable whose possible values constitute a sequence of disjoint on the number line.

$$\sum_{i=1}^{n} P(X = x_i) = 1$$

Example: suppose that X is a random variable that takes one of the values 1, 2, or 3. If $P(X = 1) = 0.4$ & $P(X = 2) = 0.25$, what is the $P(X = 3)$?

$P(X = 1) + P(X = 2) + P(X = 3) = 1 \rightarrow 0.4 + 0.25 + P(X = 3) = 1 \rightarrow P(X = 3) = 0.35$

Example: A shipment of parts contains 10 items of which 2 are defective. Two of these items are randomly chosen and inspected, let X denote the number of defectives. Find the probability of all possible values of X and probability mass function of X.

x_i	0	1	2
$P(X = x_i)$	$^{28}/_{45}$	$^{16}/_{45}$	$^{1}/_{45}$

$$P(X = 0) = \frac{\binom{2}{0} \times \binom{8}{2}}{\binom{10}{2}} = \frac{28}{45} \quad P(X = 1) = \frac{\binom{2}{1} \times \binom{8}{1}}{\binom{10}{2}} = \frac{16}{45} \quad P(X = 2) = \frac{\binom{2}{2} \times \binom{8}{0}}{\binom{10}{2}} = \frac{1}{45}$$

$$\text{Probability Mass Function} = \text{P.M.F} = \text{P(X = x)} = \frac{\binom{2}{x} \times \binom{8}{2-x}}{\binom{10}{2}} \quad x = 0, 1, 2$$

Example: A supervisor in a manufacturing plant has three men and three women working for him. He wants to choose two workers for a special job. Not wishing to show any biases in his selection, he decides to select the two workers at random. Let Y denote the number of women in his selection. Find the probability mass function of Y.

y_i	0	1	2
$P(Y = y_i)$	$^{3}/_{15}$	$^{9}/_{15}$	$^{3}/_{15}$

$$P(Y = 0) = \frac{\binom{3}{0} \times \binom{3}{2}}{\binom{6}{2}} = \frac{3}{15} \quad P(Y = 1) = \frac{\binom{3}{1} \times \binom{3}{1}}{\binom{6}{2}} = \frac{9}{15} \quad P(Y = 2) = \frac{\binom{3}{2} \times \binom{3}{0}}{\binom{6}{2}} = \frac{3}{15}$$

$$\text{Probability Mass Function} = \text{P.M.F} = \text{P(Y = y)} = \frac{\binom{3}{y} \times \binom{3}{2-y}}{\binom{6}{2}} \quad y = 0, 1, 2$$

Expectation value of a discrete random variable

If X is discrete random variable having a probability mass function $P(X = x_i)$, then

$$E(X) = \sum_{i=1}^{n} x_i p(X = x_i)$$

Example: in rolling a fair dice, where X is the side facing up, find $E(X)$?

x_i	1	2	3	4	5	6
$P(X = x_i)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$
$x_i P(X = x_i)$	$1/6$	$2/6$	$3/6$	$4/6$	$5/6$	$6/6$

$$E(X) = \sum_{i=1}^{n} x_i p(X = x_i) = 1/6 + 2/6 + 3/6 + 4/6 + 5/6 + 6/6 = 21/6$$

Properties of expectation value:

- $E(cX) = c.E(X)$ x is random variable and c is constant value
- $E(X + c) = c + E(X)$
- $E(X \pm Y) = E(X) \pm E(Y)$ X & Y are random variables
- $E[\sum_{i=1}^{n} x_i] = \sum_{i=1}^{n} E(x_i)$

Example: Suppose that a radio contains six transistors, two of which are defective. Three transistors are selected at random, removed from the radio, and inspected. Let Y equal the number of defectives observed, where Y = 0, 1, or 2. Find the expected value for Y.

y_i	0	1	2
$P(Y = y)$	$4/20$	$12/20$	$4/20$
$y_i P(Y = y)$	0	$12/20$	$8/20$

$$P(Y = 0) = \frac{\binom{2}{0} \times \binom{4}{3}}{\binom{6}{3}} = \frac{4}{20} \quad P(Y = 1) = \frac{\binom{2}{1} \times \binom{4}{2}}{\binom{6}{3}} = \frac{12}{20} \quad P(Y = 2) = \frac{\binom{2}{2} \times \binom{4}{1}}{\binom{6}{3}} = \frac{4}{20}$$

$$E(Y) = \sum_{i=1}^{n} y_i p(Y = y_i) = 0 + 12/20 + 8/20 = 1$$

Example: Roll a dice. If the side that comes up is odd, you win the $ equivalent of that side. If it is even, you lose $4. Find expected value of earning?

x_i	1	3	5	-4
$P(X = x_i)$	$1/6$	$1/6$	$1/6$	$3/6$
$x_i P(X = x_i)$	$1/6$	$3/6$	$5/6$	$-12/6$

$$E(X) = \sum_{i=1}^{n} x_i p(X = x_i) = 1/6 + 3/6 + 5/6 - 12/6 = -1/2$$