

MATH 104
Limits and Derivatives

Q1. Find the limits of the following functions.

a) $\lim_{x \rightarrow -1} \frac{x^2 + 2x + 1}{x + 1}$

b) $\lim_{r \rightarrow 9} \frac{4r - 3}{11}$

c) $\lim_{y \rightarrow 15} \sqrt{y + 3}$

d) $\lim_{x \rightarrow 4} \frac{x^2 - 9x + 20}{x^2 - 3x - 4}$

e) $\lim_{x \rightarrow 0} \frac{(x + 2)^2 - 4}{x}$

f) $\lim_{h \rightarrow -4} \frac{h^2 + 2h - 8}{h^2 + 5h + 4}$

Q2. Determine whether the function is continuous at the given points.

a) $f(x) = \frac{x + 4}{x - 2}$ at $x = -2$

b) $f(x) = \frac{x^2 - 4x + x}{6}$ at $x = 2$

c) $f(x) = \frac{x - 3}{x^2 - 9}$ at $x = -3, x = 3$

Q3. Find all points of discontinuity.

a) $f(x) = 10x^5 + 4x^3 + 100$

b) $f(y) = 0$

c) $g(x) = \frac{x^3 - 8}{10}$

d) $f(x) = \frac{x^2}{x^2 - 2x}$

e) $f(x) = \frac{x^{100} + 1}{x}$

f) $f(x) = \frac{x^4}{x^4 - 1}$

g) $f(x) = \sqrt{x^2 - 49}$

Q4. Determine the **first derivatives** of the following functions.

a) $f(x) = x^8 - 2x^7 - 5x^3 + 3x$

b) $f(x) = x^e + e^2 + ex$

c) $g(x) = \frac{1}{\sqrt[5]{x}} + 40x^2 - \frac{1}{x}$

d) $f(x) = \sqrt{x}(5 - 6x + 3\sqrt[4]{x})$

e) $f(y) = \frac{(8y-1)^5}{(3y-1)(7y+5)}$

f) $f(x) = \left(\frac{x-5}{2x+2}\right)^{10}$

g) $f(x) = \sqrt[3]{x^3 + 4x + 1}$

h) $f(x) = \ln x^{100} + 6 \ln \sqrt[3]{x} + \frac{x^2}{\ln x} + (2x+5)^2 \ln(2x+5)$

i) $f(x) = e^{9x^2+5x^3-6} + xe^{3x} + e^{2x}(x+6) + \frac{e^x-1}{e^x+1} - e^{1+\sqrt{x}}$

Q5. Find all higher order derivatives of $f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 + x^2 + 7$