

MATH 104 TUTORIAL
Applications of the Derivatives

Q1. Profit

For a monopolist, the cost per unit of producing a product is 3\$, and the demand equation is

$$p = \frac{10}{\sqrt{q}}$$

What price will give the greatest profit?

Q2. Profit

For a monopolist's product, the demand equation is

$$p = 42 - 4q$$

and the average-cost function is

$$\bar{c} = 2 + \frac{80}{q}$$

Find the profit-maximizing price.

Q3. Profit

A manufacturer can produce at most 120 units of a certain product each year. The demand equation for the product is

$$p = q^2 - 100q + 3200$$

and the manufacturer's average-cost function is

$$\bar{c} = \frac{2}{3}q^2 - 40q + \frac{10000}{q}$$

Determine the profit-maximizing output q and the corresponding maximum profit.

Q4. Profit

For XYZ Manufacturing Co. total fixed costs are \$1200, material and labor costs combined are \$2 per unit, and the demand equation is

$$p = \frac{100}{\sqrt{q}}$$

What level of output will maximize profit? Show that this occurs when marginal revenue is equal to marginal cost. What is the price at profit at profit maximization?

Q5. Revenue

The demand equation for a monopolist's product is $p = -5q + 30$. At what price will revenue be maximized?

Q6. Revenue

A real-estate firm owns 100 garden-type apartments. At \$400 per month, each apartment can be rented. However, for each \$10 per month increase, there will be two vacancies with no possibility of filling them. What rent per apartment will maximize monthly revenue?

Q7. Revenue

A TV cable company has 4800 subscribers who are each paying \$18 per month. It can get 150 more subscribers for each \$0.5 decrease in the monthly fee. What rate will yield maximum revenue, and what will this revenue be?

Q8. Cost

A manufacturer has determined that, for a certain product, the average cost is given

$$\bar{c} = 2q^2 - 36q + 210 - \frac{200}{q}$$

where $2 \leq q \leq 10$.

- At what level within the interval $[2,10]$ should production be fixed in order to minimize total cost? What is the minimum total cost?
- If production were required to lie within the interval $[5,10]$, what value of q would minimize total cost?

Q9. Average Cost

The total cost of producing q cases of stockings is given by

$$c = 3q^2 + 50q - 18q \ln q + 120$$

Find the number of cases that should be processed in order to minimize the average cost per case. Determine this minimum average cost.

Q10. Average Cost

If $c = 0.01q^2 + 5q + 100$ is a cost function, find the average-cost function. At what level of production q is there a minimum average cost?