

Question1) Suppose that $A_{4 \times 5}$, $B_{4 \times 5}$, $C_{5 \times 2}$, $D_{4 \times 2}$, and $E_{5 \times 4}$ are matrices. Determine which of the following matrix expression are defined. For those which are defined, give the size of the resulting matrix.

- a) BA
- b) $AC + D$
- c) $AE + B$
- d) $AB + B$
- e) $E(A + B)$
- f) $E^T A$
- g) $(A^T + E)D$

Question2) Solve the following matrix equation for a,b,c, and d.

$$\begin{bmatrix} a-b & b+c \\ 3d+c & 2a-4d \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}$$

Question3) Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ -3 & 2 & 4 \end{bmatrix}, E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

Compute the following. (if possible)

- a) $D + E$
- b) $-3(D + 2E)^T$
- c) $(C^T B)A^T$
- d) $(4B)C + 2B$
- e) $D^T E^T - (ED)^T$
- f) $(BA^T - 2C)^T$
- g) $(C^T A^T + 2E^T)^T$

Question4) Find matrices A,x, and B that express the given system of linear equations as a single matrix equation $Ax = B$.

$$\begin{aligned} 4x_1 & - 3x_3 + x_4 = 1 \\ 5x_1 + x_2 & - 8x_4 = 3 \\ 2x_1 - 5x_2 + 9x_3 - x_4 & = 0 \\ 3x_2 - x_3 + 7x_4 & = 2 \end{aligned}$$

Question5) Express the matrix equation as a system of linear equations.

$$\begin{bmatrix} 3 & -2 & 0 & 1 \\ 5 & 0 & 2 & -2 \\ 3 & 1 & 4 & 7 \\ -2 & 5 & 1 & 6 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Question6) $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ are matrices.

- Compute the inverse of the given matrices.
- Verify that the matrices A,B and C satisfy the relationships $(AB)^{-1} = B^{-1}A^{-1}$ and $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

Question7) In each part use the given information to find A.

- $A^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$
- $(7A)^{-1} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$
- $(I + 2A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$

Question8) Let A be the matrix

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

In each part, find $p(A)$

a) $p(x) = x - 2$

b) $p(x) = 2x^2 - x + 1$

c) $p(x) = x^3 - 2x + 4$

Question9) Find the inverse of

$$\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$