MATH 106– ELEMENTARY MATRICES AND METHOD FOR FINDING INVERSE OF A

2013-2014 SUMMER

Question1) Which of the following are elementary matrices?

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} -4 & 1 \\ 1 & 0 \end{bmatrix}$

Question2) Find the row operation that will restore the given elementary matrix to an identity matrix.

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & -1/7 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

Question3) Consider the matrices

$$A=\begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}, B=\begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix}, C=\begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}$$

Find elementary matrices E_1, E_2, E_3, E_4 such that

a) $E_1A=B$ b) $E_2B=A$ c) $E_3A=C$ d) $E_4C=A$

Question4) Find the inverse of the given matrix if the matrix is invertible and check your answer by multiplication.

a) $\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$ b) $\begin{bmatrix} 6 & -4 \\ -3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$ f) $\begin{bmatrix} \sqrt{2} & 3\sqrt{2} & 0 \\ -4\sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Question5) Find the inverse of the following 4×4 matrix where k is all nonzero.

$$\begin{bmatrix} k & 0 & 0 & 0 \\ 1 & k & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & k \end{bmatrix}$$

Question6) Solve the system by inverting the coefficient matrix.

a) $\begin{aligned} 4x_1 - 3x_2 &= -3 \\ 2x_1 - 5x_2 &= 9 \end{aligned}$

b) $\begin{aligned} 5x_1 + 3x_2 + 2x_3 &= 4 \\ 3x_1 + 3x_2 + 2x_3 &= 2 \\ + x_2 + x_3 &= 5 \end{aligned}$

c) $\begin{aligned} -x - 2y - 3z &= 0 \\ w + x + 4y + 4z &= 7 \\ w + 3x + 7y + 9z &= 4 \\ -w - 2x - 4y - 6z &= 6 \end{aligned}$

Question7) Find conditions that b's must satisfy for the system to be consistent.

a) $\begin{aligned} 6x_1 - 4x_2 &= b_1 \\ 3x_1 - 2x_2 &= b_2 \end{aligned}$

b) $\begin{aligned} x_1 - 2x_2 - x_3 &= b_1 \\ -4x_1 + 5x_2 + 2x_3 &= b_2 \\ -4x_1 + 7x_2 + 4x_3 &= b_3 \end{aligned}$

Question8) Determine whether the matrix is invertible, if so, find the inverse by inspection.

a) $\begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$ b) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$

Question9) If $A=\begin{bmatrix}1 & 0\\ 0 & -2\end{bmatrix}$, then find A^2, A^{-2} and A^{-k}.

Question10) Find all values of a,b,c, for which A is symmetric.

$$A=\begin{bmatrix}2 & a-2b+2c & 2a+b+c\\ 3 & 5 & a+c\\ 0 & -2 & 7\end{bmatrix}$$

Question11) Find a diagonal matrix A that satisfies

a) $A^5=\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$ b) $A^{-2}=\begin{bmatrix}9 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 1\end{bmatrix}$