

**Question1)** Evaluate the determinant of the following matrices.

$$\begin{array}{l}
 \text{a) } \begin{bmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{bmatrix} \quad \text{b) } \begin{bmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{bmatrix} \quad \text{c) } \begin{bmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 0 & 9 & -4 \end{bmatrix} \quad \text{d) } \begin{bmatrix} -5 & 6 \\ -7 & -2 \end{bmatrix} \quad \text{e) } \begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}
 \end{array}$$

**Question2)** Find all values of  $\lambda$  for which  $\det(A) = 0$ .

$$\begin{array}{l}
 \text{a) } \begin{bmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{bmatrix} \quad \text{d) } \begin{bmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{bmatrix}
 \end{array}$$

**Question3)** Evaluate the determinant

$$\begin{vmatrix} 4 & -9 & 9 & 2 \\ -2 & 5 & 6 & 4 \\ 1 & 2 & -5 & -3 \\ 1 & -2 & 0 & -2 \end{vmatrix}$$

**Question4)** Use the determinant definition to evaluate

$$\begin{array}{l}
 \text{a) } \begin{vmatrix} 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 \end{vmatrix} \quad \text{b) } \begin{vmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{vmatrix}
 \end{array}$$

**Question5)** Solve for x.

$$\begin{vmatrix} x & -1 \\ 3 & 1-x \end{vmatrix} = \begin{vmatrix} 1 & 0 & -3 \\ 2 & x & -6 \\ 1 & 3 & x-5 \end{vmatrix}$$

**Question6)** Show that the value of the determinant

$$\begin{vmatrix} \sin \theta & \cos \theta & 0 \\ -\cos \theta & \sin \theta & 0 \\ \sin \theta - \cos \theta & \sin \theta + \cos \theta & 1 \end{vmatrix}$$

does not depend on  $\theta$  .

**Question7)** Evaluate the determinant of the given matrix by reducing the matrix to row-echelon form.

a)  $\begin{bmatrix} 3 & 6 & -9 \\ 0 & 0 & -2 \\ -2 & 1 & 5 \end{bmatrix}$       b)  $\begin{bmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 5 & -2 & 2 \end{bmatrix}$       c)  $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$       d)  $\begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$

e)  $\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

**Question8)** Given that  $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$  , find,

a)  $\begin{vmatrix} d & e & f \\ g & h & i \\ a & i & c \end{vmatrix}$       b)  $\begin{vmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{vmatrix}$       c)  $\begin{vmatrix} a+g & b+h & c+i \\ d & e & f \\ g & h & i \end{vmatrix}$       d)  $\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix}$

**Question9)** Use row-reduction to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

**Question10)** Determine which of the following matrices are invertible.

$$\text{a) } \begin{bmatrix} 1 & 0 & -1 \\ 9 & -1 & 4 \\ 8 & 9 & -1 \end{bmatrix} \quad \text{b) } \begin{bmatrix} 4 & 2 & 8 \\ -2 & 1 & -4 \\ 3 & 1 & 6 \end{bmatrix} \quad \text{c) } \begin{bmatrix} -3 & 0 & 1 \\ 5 & 0 & 6 \\ 8 & 0 & 3 \end{bmatrix}$$

**Question11)** Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Assuming that  $\det(A) = -7$ , find

$$\text{a) } \det(3A) \quad \text{b) } \det(A^{-1}) \quad \text{c) } \det((3A)^{-1})$$

**Question12)** If  $\begin{vmatrix} x^2 & x & 2 \\ 2 & 1 & 1 \\ 0 & 0 & -5 \end{vmatrix} = 0$  then find x.

**Question13)** For which value(s) of k does A fail to invertible?

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 6 \\ k & 3 & 2 \end{bmatrix}$$