

## Graduate Microeconomics Longer Study Questions

**Areas:** 1. *Consumer Demand Theory*  
2. *Theory of the Firm*  
3. *Competitive Markets*

**Lecturer:** *Glenn P. Jenkins*

1. In order to keep pace with the fast changing world of economics, a professor typically reads several business publications. Consider a professor that restricts her readings to the *American Economic Review* (AER) and the *Journal of Political Economy* (JPE). Her preference for each publication is given by the utility function:

$$U(\text{AER}, \text{JPE}) = 2\text{AER} + \text{JPE}$$

Suppose that she allocates \$20 a month for the purchase of those publications and that the price of an AER per issue is \$10 and the price of the JPE is \$5 per issue.

- a. Graph the indifference curves for three different levels of utility. What does the shape of the indifference curves imply about the professor's preferences for the two journals?
  - b. Find the utility maximizing quantity of AERs and JPEs.
  - c. Suppose the price of an issue of the JPE increases to \$10. What are the new optimal values of AERs and JPEs? Explain the change in quantities, if any.
  - d. How would the demand for the AER and the JPE change if the professor had a utility function  $U(\text{AER}, \text{JPE}) = 2\text{AER} + 2\text{JPE}$  and the price of AER is \$10, the price of the JPE is \$5 per issue and the professor allocates \$20 a month to the purchase of these publications.
- 
2.
    - a. When coffee prices rise, an individual buys more tea, less coffee and fewer coffee mugs. Explain the substitution and income effects of the price change on each of the three goods. Use graphs to demonstrate your intuition.
    - b. Suppose the consumers share of his total expenditures on tea is 5 percent, 7 percent on coffee and 2 percent on coffee mugs. Also assume that the income elasticity of demand for tea is 1.0, and the compensated own-price elasticity for tea is -0.8. Furthermore, we know that the uncompensated cross price elasticity of the demand for teas with respect to coffee mugs is -0.6, and the uncompensated cross-price elasticity between the quantity of tea and the price of coffee is 0.4.
      - (i) What is the uncompensated own price elasticity of demand for tea?
      - (ii) What is the compensated cross price elasticity of demand for tea with respect to coffee mugs?

(ii) What is the compensated cross-price elasticity between the quantity of tea and the price of coffee?

3. You are given the following utility function:

$$U(X,Y)=X^{0.2}Y^{0.8}$$

- a. Derive the Marshallian demand curves for X and Y.
  - b. Find the indirect utility function and explain what it means.
  - c. Find the expenditure function and explain what it means.
  - d. Explain how you would find the Hicksian demand curves for X and Y.
  - e. Discuss the differences between the Marshallian demand and the Hicksian demand curves.
4. A government considers two policy options, (i) a cash grant of \$100 and (ii) a certificate good for \$100 worth of food. Is a wealthy person more likely than a poor person to prefer the cash grant? Explain using graphs.
5. Isobel considers peaches and nectarines to be perfect substitutes. She spends \$5 a month on these fruits. Initially, peaches are \$1 a pound and nectarines are \$1.25 a pound. Then the price of peaches increases to \$1.50 a pound. Her income allocated to fruit does not change, however.
- a. How does a consumption change when the price of peaches changes?
  - b. Show with the aid of a graph how utility changes when the price changes.
  - c. How much must her budget increase in order to return to the original utility level?
  - d. Derive and graph the demand curve for nectarines.
  - e. Derive and graph the Engel curve for nectarines (under the assumption that the price of peaches is \$1.50 a pound).
- 6.
- a. Show using well labeled diagrams how you would derive the Marshallian Demand curve for a good X for a consumer maximizing her welfare subject to a budget constraint.
  - b. Now derive the consumer's compensated demand curve for a good X. Be sure to identify the substitution and income effects of the change in the price of the good. Furthermore, please define carefully the rule you are following to measure the amount of compensation required to estimate the substitution and income effects.
  - c. Give some examples of the kind of economic questions that the Marshallian Demand curve would be the most accurate way to measure the effect of a change in the price of the commodity on the quantity demanded? Explain.

- d. Give an example of the kind of economic questions that a compensated demand function would be the most appropriate form of the demand function to use in their analyzes? Explain.
7. Suppose that a firm's fixed proportion production function is given by  $q = \min(5K, 10L)$ , and the rental rates for capital and labor are given by  $v=1, w=3$ .
- Calculate the firm's long-run total, average, and marginal cost curves.
8. Suppose that the demand for stilts is given by  $Q=1500-50P$  and that the long-run total operating costs of each stilt making firm in a competitive industry are given by  $TC=50+10q+0.5q^2$ .
- How many stilts are produced by each firm? What is the long-run equilibrium price of stilts? What is the long-run equilibrium quantity of stilts produced in the market? How many firms will there be?
9. The domestic demand for portable radios is given by  $Q=5,000-100P$  where price (P) is measured in dollars and quantity (Q) is measured in thousands of radios per year. The domestic supply curve for radios is given by  $Q=150 P$ .
- What is the domestic equilibrium in the portable radio market?
  - Suppose portable radios can be imported at a world price of \$10 per radio. If trade were unencumbered, what would the new market equilibrium be? How many portable radios would be imported?
  - If domestic portable radio producers succeeded in getting a \$5 tariff implemented, how would this change the market equilibrium? How much would be collected in tariff revenues? How much consumer surplus would be transferred to domestic producers? What would the deadweight loss from the tariff be
10. Fred has a utility function of  $U(C,T)=10C^2T^3$ .
- Calculate  $U_C$  and  $U_T$ .
  - Explain in words what these (results of part "a") represent.
  - Write the total differential for U.
  - Using the result of part "c," calculate  $dC/dT$  for  $dU=0$ .
  - Explain in words (using the language of economics) what the results of part "d" represent.
11. The CES utility function is given by the following equation when  $\delta \neq 0$ .
- $$U(X,Y) = \alpha X^{\delta/\delta} + \beta Y^{\delta/\delta}$$
- Find  $MU_X$  and  $MU_Y$  for this function.
  - Is the CES function homothetic? Explain.

12. Write down the utility function of the following people and sketch their indifference maps:
- Mr. Green likes apples (X) and melons (Y) and is always willing to give up one apple for 12 melons.
  - Mr. Roberts smokes a pipe, but is very sensitive about the blend of tobacco he uses. The only blend he will smoke has 20 parts Havana Gold (W) to 6 part Virginia Splendor (Z).
13. Venus Williams likes both tennis rackets and tennis shoes. She has many of both. Her marginal rate of substitution (MRS) of rackets for shoes is 3. Unused rackets and shoes may be returned to the local sporting goods store for a refund. The current price for a racket is \$200 and the price for a pair of shoes is \$100. Suggest a way for Venus to make herself better off.
14. Stuart's utility function for goods X and Y is represented as  $U(X,Y)=X^{0.2}Y^{0.8}$ . Assume his income is \$100 and the prices of X and Y are \$10 and \$20, respectively.
- Express his marginal rate of substitution (MRS) between goods X and Y. As the amount of X increases relative to the amount of Y along the same indifference curve, does the MRS increase or decrease? Explain.
  - What is his optimal consumption bundle  $(X^*,Y^*)$ , given income and prices of the two goods?
  - How will this bundle change when all prices double and income is held constant?
  - Derive the demand curve for good X and demand curve for good Y.
  - Now a government subsidy program lowers the price of Y from \$20 per unit to \$ 10 per unit. Calculate and graphically show the change in good Y consumption resulting from the program.
15. A government considers two policy options, (i) a cash grant of \$100 and (ii) a certificate good for \$100 worth of food. Is a wealthy person more likely than a poor person to prefer the cash grant? Explain using graphs.
16. Isobel considers peaches and nectarines to be perfect substitutes. She spends \$5 a month on these fruits. Initially, peaches are \$1 a pound and nectarines are \$1.25 a pound. Then the price of peaches increases to \$1.50 a pound. Her income allocated to fruit does not change, however.
- How does a consumption change when the price of peaches changes?
  - Show with the aid of a graph how utility changes when the price changes.
  - How much must her budget increase in order to return to the original utility level?
  - Derive and graph the demand curve for nectarines.

- e. Derive and graph the Engel curve for nectarines (under the assumption that the price of peaches is \$1.50 a pound).
17. Suppose for North Cyprus all goods and services are classified into three groups (1) Durable (D) goods (2) Non-durable (N) goods and (3) Services (C). From the household consumption survey (1999) we know that their respective shares in total consumption are  $S_D=0.3$ ,  $S_N=0.45$ ,  $S_C=0.25$ .
- If the income elasticity of demand for Durable goods (mainly housing) is 1.2 and for Non-durable goods (food, clothing) is 0.50. What is the income elasticity of demand for services?
  - If you know that the compensated own-price elasticity of demand for durable goods is -1.0, for non-durable goods (N) is -0.6, and the compensated own-price elasticity of demand for services is -1.2. What are the uncomcompensated own-price elasticities of demand for these three goods?

18. If salt has no net substitutes, does it have any net complements?

19. Digging clams by hand in Sunset Bay requires only labor input. The total number of clams obtained per hour ( $q$ ) is given by

$$q=100\sqrt{L}$$

where  $L$  is labor input per hour.

- Graph the relationship between  $q$  and  $L$ .
- What is the average productivity of labor in Sunset Bay? Graph this relationship and show that  $AP_L$  diminishes for increases in labor input.
- Show that the marginal productivity of labor in Sunset Bay is given by

$$MP_L=50/\sqrt{L}.$$

Graph this relationship and show that  $MP_L < AP_L$  for all values of  $L$ . Explain why this is so.

20. For  $q=10 L^{0.5}K^{0.4}$

- Find  $RTS_{LK}$  and determine whether it diminishes in  $L$ .
- Determine whether the function is homogeneous and, if so, determine whether returns to scale are constant, increasing, or decreasing.
- Find the elasticity of substitution ( $\sigma$ ).

21. Timothy's Toothpicks, Inc., is making plans for a new processing plant. Ernie's Engineering, Inc., has been hired to estimate the cost of producing  $t$  tons of toothpicks as a function of the area of plant,  $A$ . The engineers come up with:

$$C(t, A) = (t^3 / 3) - t^2 + (300 - 6A)t + A^2$$

Once Timothy's has chosen  $A$ , this function will be its short-run cost function.

- Find short-run marginal and average cost functions.

- b. Find the plant size,  $A$ , that will minimize costs, as a function of  $t$ .
  - c. Now use your result in b. And the short-run cost function to find the “long-run” cost function, assuming that in the long-run  $A$  is adjusted to minimize costs.
  - d. Find long-run marginal and average cost functions.
22. The boiler in the heating plant at arctic College is designed to burn either fuel oil or coal. Suppose that one ton of coal,  $C$ , produces as many BTUs as 100 gallons of fuel oil,  $F$ .
- a. Draw a typical isoquant for BTU production.
  - b. What is the elasticity of substitution between fuel oil and coal?
  - c. Suppose that fuel oil is \$1.00 per gallon and coal is \$90 per ton. Draw the output expansion path on the diagram and describe the marginal and average cost curves.
  - d. Repeat c., assuming that the price of coal has increased to \$110 per ton.

23. Suppose that a firm’s short-run total cost function for T-shirts is:

$$STC=0.05q^2+5q+125$$

- a. Find the firm’s short-run supply function.
  - b. How many T-shirts will the firm produce if  $P=\$10$ ?
  - c. What’s the firm’s short-run producer surplus at  $P=\$10$ ?
  - d. Is this amount larger, smaller, or equal to the firm’s profit at this price? Explain briefly.
  - e. Develop a formula for this firm’s short-run producer surplus as a function of  $P$ .
24. The bird seed industry is perfectly competitive. The demand function for bird seed is  $Q= 200,000-20,000P$ . There are 100 identical firms in the bird seed industry. In a particular year each firm produced 1,000 tons of seed. Since seed takes a year to produce and rots if it is not used within a year, the supply of seed in this particular year is fixed at 100,000 tons.
- a. Calculate the equilibrium price.
  - b. Calculate the elasticity of demand at the market equilibrium point.
  - c. Suppose that one firm has an extra ton of seed to sell while all other firms continue to supply 1,000 tons. What will the price be then?
  - d. Now, suppose that the short-run supply curve of each firm is  $q=200P$ . Assuming no interactions between the 100 firms, what is market supply?
  - e. Calculate equilibrium price and quantity.
  - f. Suppose that one firm decides to sell one more ton, whatever the price. Its supply function is now  $q=200P+1$ . Calculate the new equilibrium price and quantity, assuming that the supply functions of other firms remain unchanged.
  - g. What is the elasticity of demand facing the individual firm now?
  - h. Suppose that, in the long run, the bird seed industry exhibits constant costs. What will the elasticity of demand facing an individual firm be then?
  - i. Under what condition(s) is it reasonable to assume that a firm faces an infinitely elastic demand for its product?

25. The inverse demand curve for baby-sitting in Suburbia is given by:  $P_D=10-H/20$  where H is number of hours demanded per week and P is price, in dollars per hour. The supply curve for baby-sitting is:  $P_S=H/20$ . Assume that the market is competitive.
- Draw the supply and demand curves and indicate the market equilibrium price and quantity.
  - Calculate consumer surplus and producer surplus in the competitive equilibrium.
  - The members of the Parent and Teachers Association (PTA) at Suburbia High School think that high school students –their own teenagers- are spending too much time baby-sitting and not enough time studying. They agree to restrict the hours that each of their teenagers can baby-sit per week to 5 hours. The effect of this restriction on the market is to reduce the total quantity of hours supplied to 80. Find the hourly price that consumers (parents of young children) are willing to pay for this amount of baby-sitting.
  - What is the hourly price that teenagers, independent of their parents' preferences, are willing to supply this amount of care.
  - Since there is no restriction on price, how much will the market price increase or decrease per hour?
  - What is the amount of consumer surplus that is transferred to producers?

26. Consider the following demand curve for the quantity of boxes of paper clips per capita, C, where  $P_C$  is the price per box,  $P_S$  is the price of staples per box, and I is per capita income:

$$C=100-\frac{1000P_C}{2I+5P_S}$$

- If  $I=10$ ,  $P_S=1$ , and  $P_C=0.5$ , what will the quantity demanded of C be?
- What will quantity demanded be if  $I=20$ ,  $P_S=2$  and  $P_C=1$ ?
- Is this demand function homogeneous of degree zero in  $P_C$ ,  $P_S$ , and I? Explain.
- Write down the expressions for the elasticities:  
 $\epsilon_{C,P_C}$ ,  $\epsilon_{C,P_S}$ ,  $\epsilon_{C,I}$
- Calculate their respective values where  $P_C=0.50$ ,  $P_S=1$ , and  $I=10$ .
- As in d., assume  $P_S=1.00$  and  $I=10$ , so  $C=100-40P_C$ . Find the value of  $P_C$  for which  $\epsilon_{C,P_C}=-1$ .
- Suppose that the demand function was  $C=a-bP_C$ . In terms of a and b, what value of  $P_C$  would yield  $\epsilon_{C,P_C}=-1$ ?

27. An economist estimates the following demand function for beef:

$$\ln(Q)=-8.30-0.68\ln(P_{\text{beef}})+0.35\ln(P_{\text{poultry}})+0.49\ln(P_{\text{pork}})+1.21\ln(I)$$

where Q is per capita consumption of beef and I is per capita income.

- What is the own-price elasticity of demand for beef?
- What is the cross-price elasticity with respect to poultry?
- What is the cross price elasticity with respect to pork?
- Is demand for beef elastic or inelastic?

- e. If the price of beef increased, would expenditures on beef increase or decline?
- f. Is beef a gross substitute or complement for poultry?
- g. Is beef a gross substitute or complement for pork?
- h. Is beef a normal good?
- i. The economist also found that the average consumer spends about one percent of income on beef and half a percent each on poultry and pork. What is the compensated own-price elasticity of demand for beef?
- j. What is the compensated cross-price elasticity with respect to poultry?
- k. Are the differences between the compensated and uncompensated elasticities large? Why or why not?
- l. Is this demand function homogeneous of degree zero in prices and income? Explain.
- m. Suppose that each of the three prices and per capita income are redefined to be nominal prices and income divided by Consumer Price Index. Would this function then be homogeneous of degree zero in prices-including prices of all other goods- and income? Explain.

28. Explain what the following items mean:

- a. A composite good.
- b. The demand curve for a good X that is derived keeping the production possibilities for the economy held constant. (Show in a diagram how it is derived).
- c. Consumer surplus.
- d. The Slutsky equation (in elasticity form).

31.

- a. What is the relationship between the uncompensated own-price elasticity of demand for good X and the compensated own-price elasticity of demand for good X?
- b. Show using a diagram, or algebraically, how technological progress changes the relationship between inputs and outputs in the production process.
- c. Show on a well labeled diagram the dead weight losses created by levying a tariff on the importation of a good into a country. Explain how these deadweight losses are created.

32. David N. gets \$3 per week as an allowance to spend any way he pleases. Since he only likes peanut butter and jelly sandwiches, he spends the entire amount on peanut butter (at \$0.05 per ounce) and jelly (at \$0.10 per ounce). Bread is provided free of charge by a concerned neighbor. David is a particular eater and makes his sandwiches with exactly 1 ounce of jelly and 2 ounces of peanut butter. He is set in his ways and will never change these proportions.

- a. How much peanut butter and jelly will David buy with his \$3 allowance in a week?

- b. Suppose the price of jelly were to rise to \$0.15 an ounce. How much of each commodity would be bought?
  - c. By how much should David's allowance be increased to compensate for the rise in the price of jelly in part (b)?
  - d. Graph your results in parts (a) to (c).
33. For a normal good show using graphs how the compensated demand curve for a good X is constructed by (a) holding Utility Constant, (b) holding real income constant so that after the price change one can purchase the original bundle on goods, (c) holding production possibilities in the country constant. Discuss their relationships in terms of which demand function is more or less elastic than the other. Without redrawing the graphs, please explain how these relationships would change if the good were an inferior good, why?
34. Explain what is meant by the following concepts..
- a. The marginal rate of substitution of x for y
  - b. Perfectly complementary goods in consumption
  - c. The degree of homogeneity of a demand function.
  - d. The adding up property of the income elasticities of demand for all goods in the economy.
  - e. A Giffen good
35. Suppose that the market for mushrooms is characterized as being perfectly competitive. Assume that the long-run total cost function for a typical mushroom producer is given by
- $$TC = wq^2 - 10q + 100,$$
- Where q is the output of the typical firm, and w represents the hourly wage rate paid to workers in the firm. Suppose also that the market demand for mushrooms is given by
- $$Q = -1,000P + 40,000,$$
- where Q is total quantity demanded and P is the market price of mushrooms.
- a. If the wage rate for worker is \$1, what will be the long-run equilibrium output for the typical mushroom producer?
  - b. Assuming the mushroom industry exhibits constant costs and that all firms are identical, what will be the long-run equilibrium price of mushrooms, and how many mushroom producers will there be?
36. Suppose there are a fixed number of 1000 identical firms in a perfectly competitive concrete pipe industry. Each firm produces the same fraction of total market output, and each firm's production function for pipe is given by  $q = \sqrt{KL}$ . Suppose the demand for concrete pipe is given by  $Q = 400,000 - 100,000 P$ , where Q is the total quantity of concrete pipe.
- a. If  $w = v = \$1$ , in what ratio will the typical firm use K and L?

- b. What will be the long run average and marginal cost of pipe?
  - c. In long run equilibrium what will be the market equilibrium price and quantity of pipe?
  - d. How much will each firm produce?
  - e. How much labor will be hired by each firm and in the market as a whole?
  - f. If  $w$  rose to \$2 while  $v$  remained constant at \$1, how will this change the capital labor ratio for the typical firm?
37. Suppose the own-price elasticity of demand for student admissions in English language universities in Turkey plus North Cyprus is  $-0.5$  for all the university applicants in Turkey. It has been estimated that the long-run price elasticity of supply of places in English Language university places in Turkey plus North Cyprus is  $1.0$ . Suppose also that the universities in North Cyprus admit 20 percent of the total of university students in Turkey.

**Questions:**

- a. What is the own-price elasticity of demand for university admission places in North Cyprus universities by Turkish applicants.
  - b. Suppose at the present time there are 20 thousand students studying at the university level in North Cyprus and the annual tuition fees are US\$ 4000 per year. What would be the change in the quantity demanded of places in North Cyprus Universities if the universities increased their tuition by \$400 per year?
- 38.
- a. Using a well labeled diagram show how a sales tax on a single good affects the quantity supplied, the quantity demanded and the tax revenues collected. Show also the deadweight loss created by the tax.
  - b. Suppose the initial price of a good is \$10, and the initial quantity bought and sold is 200 units per week. Also assume that the own price elasticity of demand is  $-1.0$  and the elasticity of supply of the commodity is  $2.0$ . What is the deadweight loss (economic loss) per week from imposing a 10 percent tax on the sales of this good?
  - c. Using a well labeled diagram show how a tariff imposed on imports creates dead weight (economic losses). Also show the impact of the tariff on the quantity of the good demanded, domestically produced and the quantity of imports. Show also the amount of tariff revenues generated by the tariff.
  - d. How will the dead weight losses created by levying a tariff on the importation of a good into a country change as the consumer's elasticity of demand for the importable goods is increased.
39. Jen, Eric, and Kurt are all buyers of chain saws. Jen's demand function is  $Q_j = 520 - 13p$ ; Eric's demand function is  $Q_e = 400 - 5p$ ; and Kurt's demand function is  $Q_k = 200 - 5p$ : Together, these three constitute the entire demand for chainsaws. At what price will the price elasticity of market demand be  $-1$ ? (Be sure to explain your answer).

40. Mike Teevee likes to watch television and to eat candy. In fact his utility function is  $u(x; y) = x^2y$  where  $x$  is the number of hours he spends watching television and  $y$  is the number of dollars per week he spends on candy. Mike's mother doesn't like him to watch so much television. She limits his television watching to 36 hours a week and in addition she pays him \$1 an hour for every hour that he reduces his television watching below 36 hours a week. If this is Mike's only source of income to buy candy, how many hours of television does he watch per week?
41. The bicycle industry is made up of 100 firms with the long run cost curve  $c(y) = 2+y^2/2$  and 80 firms with the long run cost curve  $c(y) = y^2/6$ . No new firms can enter the industry. What is the long run industry supply function?
42. The inverse demand function for mangos is defined by the equation,  $p = 91-5q$ , where  $q$  is the number of crates that are sold. The inverse supply function is defined by  $p = 3+6q$ . In the past there was no tax on mangos but now a tax of \$44 per crate has been imposed. (a) What are the quantities produced before and after the tax was imposed? (b) What are the demand and supply prices after the tax? (c) What is the total amount of tax revenue collected? (d) What is the magnitude of the deadweight loss of the tax?
43. In a certain kingdom, the demand function for rye bread was  $q = 381-3p$  and the supply function was  $q = 5+7p$  where  $p$  is the price in zlotys and  $q$  is loaves of bread. The king made it illegal to sell rye bread for a price above 32 zlotys per loaf. To avoid shortages, he agreed to pay bakers enough of a subsidy for each loaf of bread so as to make supply equal demand. [If the price of a loaf of bread is  $p$  and the subsidy is  $s$ , then the bakers receive  $p + s$  per loaf of bread sold:  $p$  from the consumers and  $s$  from the king]. How much would the subsidy per loaf have to be?
44. Suppose the utility function for two goods,  $X$  and  $Y$ , has the Cobb-Douglas form  $U(X,Y) = \sqrt{X \cdot Y}$
- Graph the  $U=10$  indifference curve associated with this utility function.
  - If  $X=5$ , what must  $Y$  equal to be on the  $U=10$  indifference curve? What is the MRS at this point?
  - In general, develop an expression for the MRS for this utility function. Show how this can be interpreted as the ratio of the marginal utilities for  $X$  and  $Y$ .
45. Prudence maximized her utility subject to her budget constraint. Then prices changed. After the change, she became better off. Therefore we can conclude that the new bundle costs more at the old prices than the old bundle did. True or false? Explain.
46. Show that if there are only two goods ( $X$  and  $Y$ ) to choose from, both cannot be inferior goods. If  $X$  is inferior, how do changes in income affect the demand for  $Y$ .
47. Constant returns-to-scale production functions are sometimes called homogeneous of degree 1. A production function would be said to be homogeneous of degree  $k$  if
- $$f(tK, tL) = t^k f(K, L)$$
- Show that if a production function is homogeneous of degree  $k$ , its marginal productivity functions are homogeneous of degree  $k-1$ .

- b. Use the result from part (a) to show that marginal productivities for any constant returns-to-scale production functions depend only on the ratio  $K/L$ .
  - c. Use the result from part (b) to show that the RTS for a constant returns-to-scale production function depend only on the ratio  $K/L$ .
  - d. More generally, show that the RTS for any homogeneous function is independent of the scale of operation – all isoquants are radial expansions of the unit isoquant. Hence, such a function is homothetic.
  - e. Show that the results from part (d) apply to any monotonic transformation of a homogeneous function. That is, show that any such transformation of a homogeneous function is homothetic.
48. Suppose market demand is given by  $Q = 5,000 - eP$ , where  $e$  is some constant. The market is perfectly competitive and there are a total of 10 firms in the market, each with an identical cost function given by  $c(q) = 100 + 5q^2$ .
- a. Find the market's very short run equilibrium price and quantity.
  - b. Suppose the government offers a per unit subsidy of  $s$  per unit. Find the new equilibrium price.
  - c. What is the sign of  $\partial^2 P / \partial s \partial e$ ? Explain.
  - d. How does the welfare of each of the actors in the market change as a result of the subsidy and how does overall welfare change. Give values.
- 49.
- a. Jack buys two types of computers for his company, IBMs and clones. His budget for computers is \$20,000. He buys 8 clones at a price of \$2,000 each, and 1 IBM at a price of \$4,000. Draw Jack's budget set and optimal bundle.
  - b. Jack's company raises his budget to \$50,000. Now he chooses to buy 5 clones and 10 IBMs. (Prices have not changed.) Draw Jack's new budget set and optimal bundle. Then draw Jack's income-consumption path.
  - c. When Jack's budget is between \$20,000 and \$50,000, are IBMs normal or inferior? Are clones normal or inferior? Justify your answer.
  - d. Find an income at which both types of computers are normal. (Hint: you don't need to do any math.)
50. Bart's utility function is given by  $U = (X+1)*Y$ . His marginal utility for  $X$  is therefore  $Y$ : his marginal utility for good 2 is therefore  $X+1$ . Prices are  $p_X$  and  $p_Y$ ; Bart's income is  $I$ .
- a. Find Bart's MRS. As Bart gets more  $X$  and less  $Y$ , does his MRS rise or fall? Are Bart's preferences convex or not?
  - b. Use the tangency equation to solve for  $Y$  in terms of  $X$ .

- c. Now use your solution for Y to eliminate Y in the budget constraint. Then find Bart's individual demand function for X. If  $p_X=5$ ,  $p_Y=5$ , and  $I=65$ , how many X does Bart consume? How many does he consume if  $I=75$  (and the prices are the same)? Is X normal or inferior?
- d. Plot Bart's individual demand curve for when  $p_Y=5$  and  $I=75$ , using at least four points on it. Is X ordinary or Giffen? Prove your answer.
51. Earl sells lemonade at his sidewalk lemonade stand. His production function is given by  $Q = 20x_1^{1/2}x_2^{1/4}$ , where  $x_1$  is the hours of labor he puts into the lemonade stand and  $x_2$  is the number of lemons he squeezes.
- a. Does Earl have constant returns to scale, decreasing returns to scale, or increasing returns to scale.  
(Give an answer that applies to the entire function, not to a single point on it.)
- b. Find Earl's marginal rate of technical substitution between hours of labor and lemons. If Earl is working for 9 hours and squeezing 81 lemons, how much output is he making? What is his MRTS at that point? If he wanted to work one less hour, how many more lemons would he have to squeeze to keep his output the same?
- c. The price of lemons is 50 cents and Earl's time is worth \$15 per hour. Is Earl producing output in the lowest-cost way or not? How do you know? If not, should he be using more lemons or more hours of labor?
- d. Find Earl's cost-minimizing bundle of inputs.
52. Mr. Otto Carr is a used car dealer. He buys old cars for \$p each, and those are his only costs of doing business.
- a. What is his total cost if he sells 10 cars? What is total cost if he sells 20 cars? Write down his total cost function if he sells Q cars.
- b. What are Otto's fixed costs? Find his variable cost function.
- c. Find Otto's average cost function. Then find his marginal cost function. (Hint: If he sells one more car, how much do his costs go up?)
- d. Now suppose Otto finds that he has to spending \$100 on television commercials, in addition to his costs of buying old cars. What is his total cost function now? What is his average cost function now? His marginal cost function?
- e. On a graph with Q on the horizontal axis and \$/unit on the vertical axis, graph Otto's average cost function and marginal cost function when p, the price of old cars that he buys, is \$20.

53. Otto's brother, Dent Carr, is a mechanic. His cost function of repairing cars is given by  $C = 2Q^2 + 100$ .
- Find Dent's average cost, average variable cost, and marginal cost functions. Graph them on a graph with  $Q$  on the horizontal axis and \$/unit on the vertical axis.
  - If the market price is \$20, in the long run, how many cars will Dent repair? If the price is \$40, how many will he repair? If the price is \$30, how many? Draw Dent's long run supply curve on your graph.
  - When the price is \$40, draw boxes on your graph that represent Dent's total cost, total revenue, and profit.
54. Suppose welfare pays \$100 per week and requires that recipients do not work. Assume that the typical poor person can earn \$5/hour and can work up to a 40 hour week. The typical poor person may or may not be on welfare.
- Derive the equation for the budget constraint for the typical poor person who might receive welfare payments.
  - Suppose that the following reform is implemented: if poor people choose to work they may get welfare but \$0.25 will be deducted from their grants per dollar earned. Derive an equation for the typical poor person's budget constraint after the reform.
  - Graph the optimum consumption-leisure choice for a person who chooses not to receive welfare under the old program.
  - Graph the optimum choice for a poor person who chooses to receive welfare under the old program.
  - Assume that job opportunities and preferences of poor people are not changed by the reform. Will the number of poor people who get welfare change? Will the program increase poor people's labor supply? Do these questions have certain answers?
55. Define an inferior, normal, luxury and necessity good. Can a good be both inferior and normal? How?
56. Neville's passion is fine wine. When the prices of all other goods are fixed at current levels, Neville's demand function for high-quality claret is  $q = .02m - 2p$ , where  $m$  is his income,  $p$  is the price of claret (in British pounds), and  $q$  is the number of bottles of claret that he demands. Neville's income is 7500 pounds, and the price of a bottle of suitable claret is 30 pounds.
- How many bottles of claret will Neville buy?
  - If the price of claret rose to 40 pounds, how much income would Neville have to have in order to be exactly able to afford the amount of claret and the amount of other goods that he bought before the price change?

- c. At the income level you mentioned in part (b) and the higher price of claret of 40 pounds, how many bottles would Neville buy?
- d. At the original income of 7500 pounds and a price of 40, how much claret would Neville demand?
- e. Decompose the total price effect into the substitution and income effect.
57. Suppose that preferences are not convex. Is it still the case that the substitution effect is negative?
58. Suppose that the inverse demand function for wool is  $P=A/q$  for some constant  $A$ . Suppose that 1/4 of the world's wool is produced in Australia. If Australian wool production increases by 1% and the rest of the world holds its output constant, what will be the effect on the world price of wool?
59. Jerry's monthly income is \$1000. He spends 40% of his income on food and the rest on all other goods. The City Council thinks it is unfair that people spend more than 35% of their income for food. In order to lower the proportion of income going to food, the City Council gives Jerry \$200. If Jerry's income elasticity of food is 2, will the City Council accomplish its goal? Explain.
60. Assume downward sloping or flat demand and a U-shaped LRAC curve. In each of the following situations, determine graphically and/or verbally:
- Does the firm have the cost-minimizing amount of capital given its output level? If not, should the firm increase or decrease its amount of capital given its output?
  - Does the firm have the profit maximizing level of output given its amount of capital? If not, should the firm increase or decrease its level or output, given its capital?
- If the situation is impossible, state why.*
- $SRAC > LRAC, SRMC > LRMC, MR = SRMC$
  - $SRAC > LRAC, SRMC < LRMC, MR = SRMC$
  - $SRAC < LRAC, SRMC > LRMC, MR = SRMC$
  - $SRAC > LRAC, SRMC > LRMC, MR > SRMC$
  - $SRAC > LRAC, SRMC < LRMC, MR > SRMC$
  - $SRAC = LRAC, SRMC = LRMC, MR > SRMC$
  - $SRAC = LRAC, SRMC > LRMC, MR = SRMC$
61. A firm has a production technology given by:  $Q = L^{0.25} * K^{0.25}$ . Initial input prices are given by  $w = 1$  and  $r = 1$ . Suppose for the moment that the amount of capital is fixed at  $K = 4$ .
- Is the marginal product of labor diminishing? Why?
  - Find the short run total cost function, and then the short-run marginal and average cost curves.

- c. Suppose now that the period is long enough that both inputs can be adjusted.
  - d. Find the capital-labor ratio.
  - e. Find the returns to scale of this production technology.
  - f. If the rental price of capital is doubled, what happens to the profit maximizing output level?
62. Suppose a firm faces a cost function of  $C = 8 + 4q + q^2$ , so that its marginal cost is  $MC = 4 + 2q$ .
- a. What is the firm's fixed cost,  $F$ ?
  - b. What is the formula for the firm's variable cost,  $VC$ ?
  - c. What is the formula for the average cost,  $AC$ ?
  - d. What is the formula for average variable cost,  $AVC$ ?
  - e. On a diagram, draw the  $AC$ ,  $AVC$ , and  $MC$  curves.