MENG203 EXPERIMENTAL METHODS FOR ENGINEERS

ENGINEERING MEASUREMENTS

By: Assoc.Prof.Dr. Hasan Hacışevki

Standardization

- **Standardization** or **standardisation** is the process of implementing and developing technical standards based on the consensus of different parties that include firms, users, interest groups, standards organizations and governments[1] Standardization can help to maximize compatibility, interoperability, safety, repeat ability, or quality. It can also facilitate commoditization of formerly custom processes.

Standards organizations

- **TSE : Turkish Standards Organisation**
- **EN : Europian Norms**
- **ISO : International Organization for Standardization**
- **DIN : Deutsches Institut für Normung**
- **BS : British Standards**
- **SASO : Saudi Arabian Standards organization**
- **JIS : Japanese Industrial Standards**
- **ANSI : American National Standards Institute**
- **ASTM : American Society for Testing and Materials**
- **Others—ASME, NFPA, ASHRAE, etc.**

EXAMPLES OF BRITISH STANDARDS

- BS 79 Report on Dimensions of Special Trackwork for Tramways
- BS 80 Magnetos for Automobile Purposes
- BS 81 Specification for Instrument Transformers
- BS 82 Specification for Starters for Electric Motors
- BS 83 Standard of Reference for Dope and Protective Covering for Aircraft
- BS 84 Report on Screw Threads (British Standard Fine), and their Tolerances (Superseding parts of Reports Nos. 20 and 33)
- BS 86 Report on Dimensions of Magnetos for Aircraft Purposes
- BS 87 Report on Dimensions for Airscrew Hubs

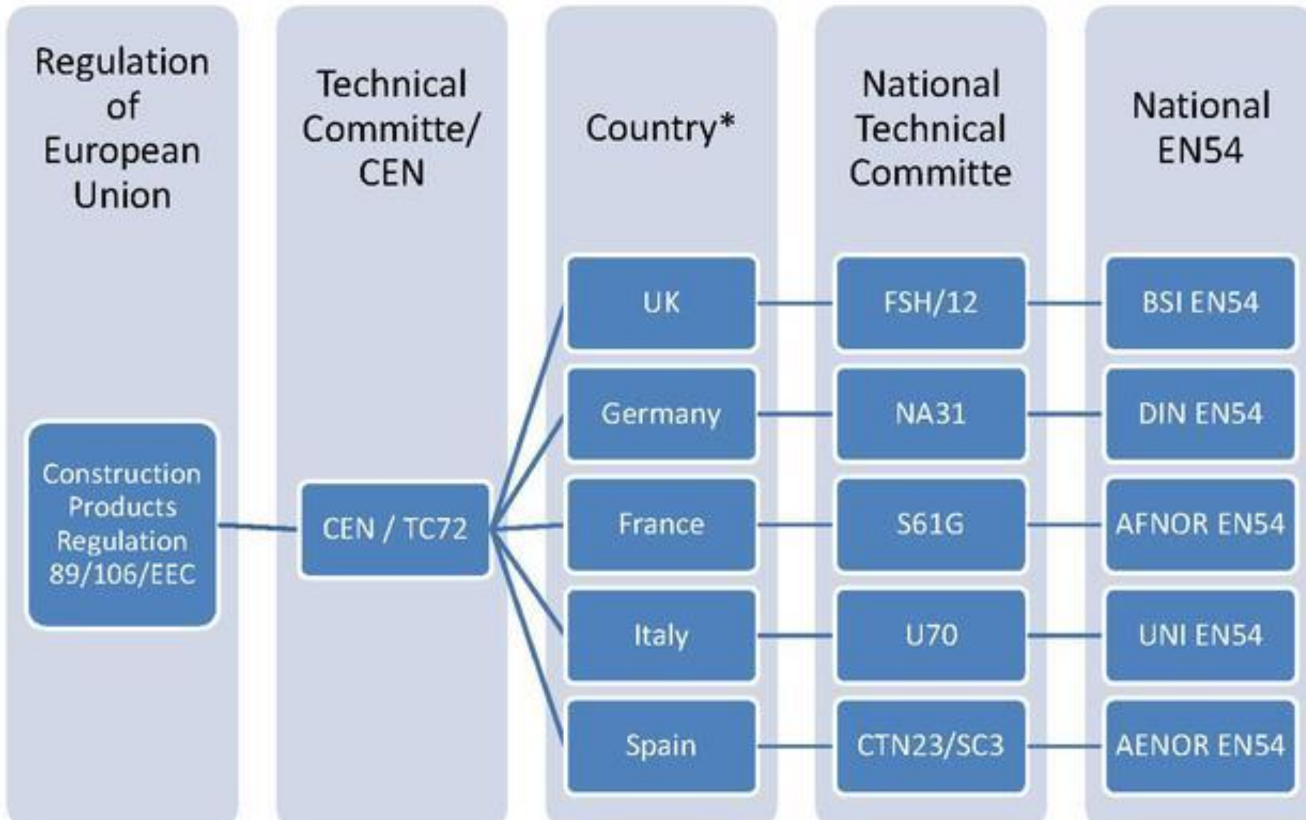
EXAMPLES OF EUROPIAN STANDARDS

- EN 1: Flued oil stoves with vaporizing burners
- EN 2: Classification of fires
- EN 3: Portable fire extinguishers
- EN 54: Fire detection and fire alarm systems
- EN 71: Safety of toys
- EN 81: Safety of lifts
- EN 115: Safety of escalators & Moving walks
- EN 166: Personal eye protection. Specifications
- EN 196: Methods for testing cement (10 parts)
- EN 590: Specification for automotive diesel

Example : EN 54

- The **EN 54 Fire detection and fire alarm systems** is a mandatory standard that specifies requirements and laboratory test for every component of fire detection and fire alarm system and it allows the free movement of construction products between countries of the European Union market.
- This standard is widely recognized around the world for several countries outside of European Union. It is recognized in Latin American countries, Brasil, African and Asian countries and several islands in the Pacific Ocean.

How EN54 are made?



*Each country of the European Union has its own committe with its EN54 following directives of CEN/TC72

EN 54 Standard Family Parts

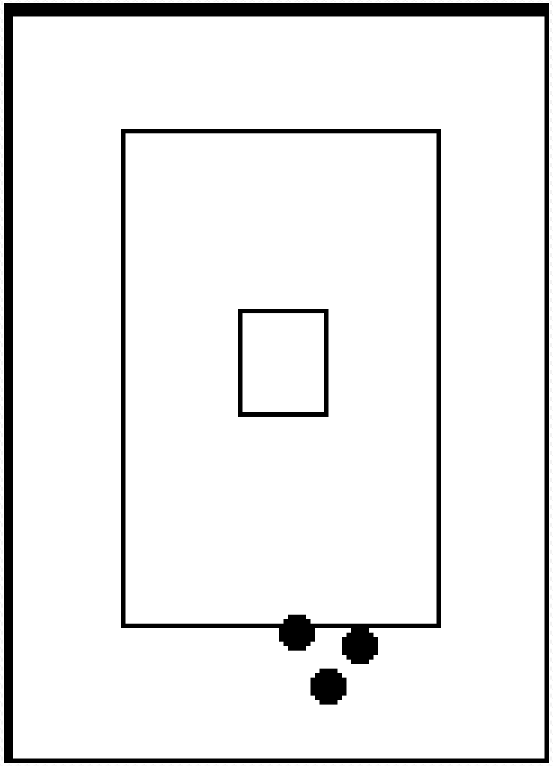
- The standard has been published in a number of parts:
- EN 54 part 1 Fire detection and fire alarm systems. Introduction[10]
- EN 54 part 2 Fire detection and fire alarm systems. Control and indicating equipment (Fire alarm control panel)
- EN 54 part 3 Fire detection and fire alarm systems. Fire alarm devices. Sounders
- EN 54 part 4 Fire detection and fire alarm systems. Power supply equipment
- EN 54 part 5 Fire detection and fire alarm systems. Heat detectors. Point detectors
- EN 54 part 6a Fire detection and fire alarm systems heat detectors; Rate-of-Rise point detectors without a static element {WITHDRAWN}
- EN 54 part 7 Fire detection and fire alarm systems. Smoke detector. Point detectors using scattered light, transmitted light or ionization
- EN 54 part 8 Components of automatic fire detection systems. Specification for high temperature heat detectors {WITHDRAWN}
- EN 54 part 9 Components of automatic fire detection systems. Methods of test of sensitivity to fire
- EN 54 part 10 Fire detection and fire alarm systems. Flame detector. Point detectors
- EN 54 part 11 Fire detection and fire alarm systems. Manual call point

- EN 54 part 12 Fire detection and fire alarm systems. Smoke detectors. Line detectors using an optical light beam
- EN 54 part 13 Fire detection and fire alarm systems. Compatibility assessment of system components
- EN 54 part 14 Fire detection and fire alarm systems. Planning, design, installation, commissioning, use and maintenance.
- EN 54 part 15 Fire detection and fire alarm systems. Point detectors using a combination of detected fire phenomena.
- EN 54 part 16 Fire detection and fire alarm systems. Components for fire alarm voice alarm systems. Voice alarm control and indicating equipment
- EN 54 part 17 Fire detection and fire alarm systems. Short circuit isolators
- EN 54 part 18 Fire detection and fire alarm systems. Input/output devices
- EN 54 part 20 Fire detection and fire alarm systems. Aspirating smoke detector

- EN 54 part 21 Fire detection and fire alarm systems. Alarm transmission and fault warning routing equipment
- EN 54 part 22 Fire detection and fire alarm systems. Line type heat detectors
- EN 54 part 23 Fire detection and fire alarm systems. Fire alarm devices. Visual alarms
- EN 54 part 24 Fire detection and fire alarm systems. Voice alarms - Loudspeakers
- EN 54 part 25 Fire detection and fire alarm systems. Components using radio links and system requirements
- EN 54 part 26 Fire detection and fire alarm systems. Point fire detectors using Carbon Monoxide sensors
- EN 54 part 27 Fire detection and fire alarm systems. Duct smoke detectors

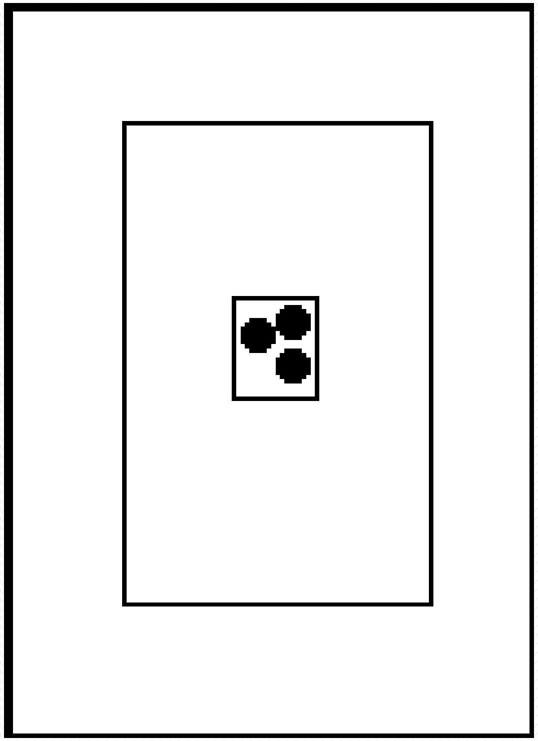
Experimental results may be described in terms of precision and accuracy.

- **Precision: Is a measure of how closely clustered a group of measurements of the same**
- - reproducibility.
- - high precision means a number of readings or trials result in values close to the same number.



Precise but not accurate

- **Accuracy:** Refers to how close a measurement is to the true or accepted value.
  - Relatively low determinate error.
  - Close to a 'true' value.



Accurate and precise

Fundamental Methods of Measurements

There are two basic methods of measurement:

- ***Direct comparison:*** **with a primary or secondary standard**
- ***Indirect comparison*** **conversion of measurand input into an analogous form which can be processed and presented as known function of input**

- A transducer is required to convert the measurand into another form

Reliability of Measurements

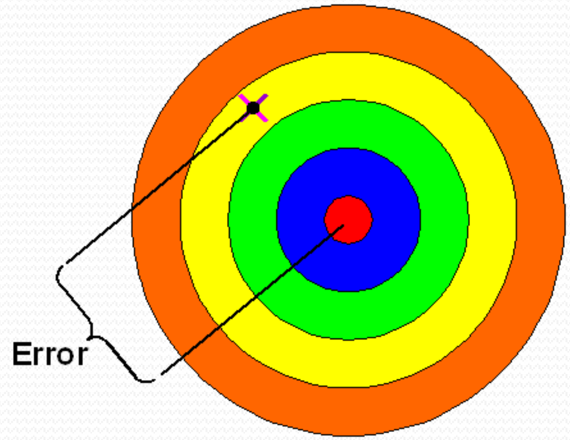
A measure is said to have a high reliability if it produces similar results under consistent conditions.

- **Measurements must be reliable to be useful**
- **Incorrect information is more damaging than no information**
- **There is no *perfect* measurement**
- **Accuracy of measurements**
- **Precision of measurements**
- **Uncertainty of measurements**
- **Do not accept data without questioning the source and uncertainty of the measurements**

- A procedure is said to be *reliable* if it may be completed with a high degree of accuracy <u>and</u> precision.

ERROR DEFINITIONS

- Error – The deviation of a measured result from the correct or accepted value of the quantity being measured.



- There are two basic types of errors, systematic and random.

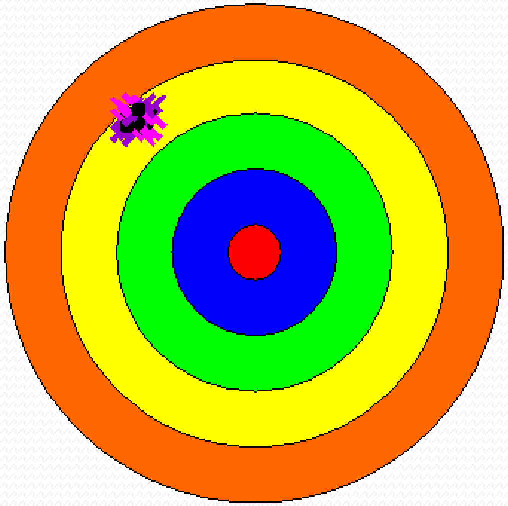
Errors in measurements are due to:

1) Systematic Errors:

Procedural errors made by the experimenter. They cause the measured value to deviate from the "accepted" value in the same direction i.e. always higher or always lower. Examples of causes include miscalibration of instruments and parallax errors. These errors can be detected and corrected. Systematic errors are calculated as Percent Error. AKA the accuracy of the work

- *Systematic Errors* – cause the measured result to deviate by a fixed amount in one direction from the correct value. The distribution of multiple measurements with systematic error contributions will be centered some fixed value away from the correct value.

- Some Examples:
  - Mis-calibrated instrument
  - Unaccounted cable loss



Systematic Errors

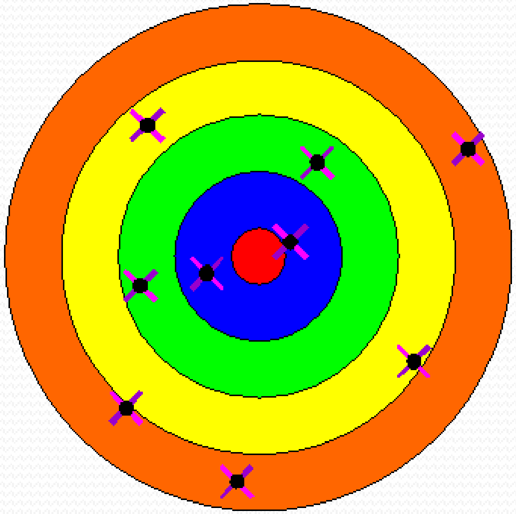
2) Random Errors

These errors arise due to uncertainties in the measuring instruments. They result in measured values that are either too high or too low. They are found commonly in mass, volume and temperature readings. These errors cannot be determined and

eliminated, however, they can be treated with statistics. The effect of these errors can be minimized by taking multiple measurements of the same thing so that the random errors cancel out. They are always written +/-. Random errors are calculated as Uncertainty.

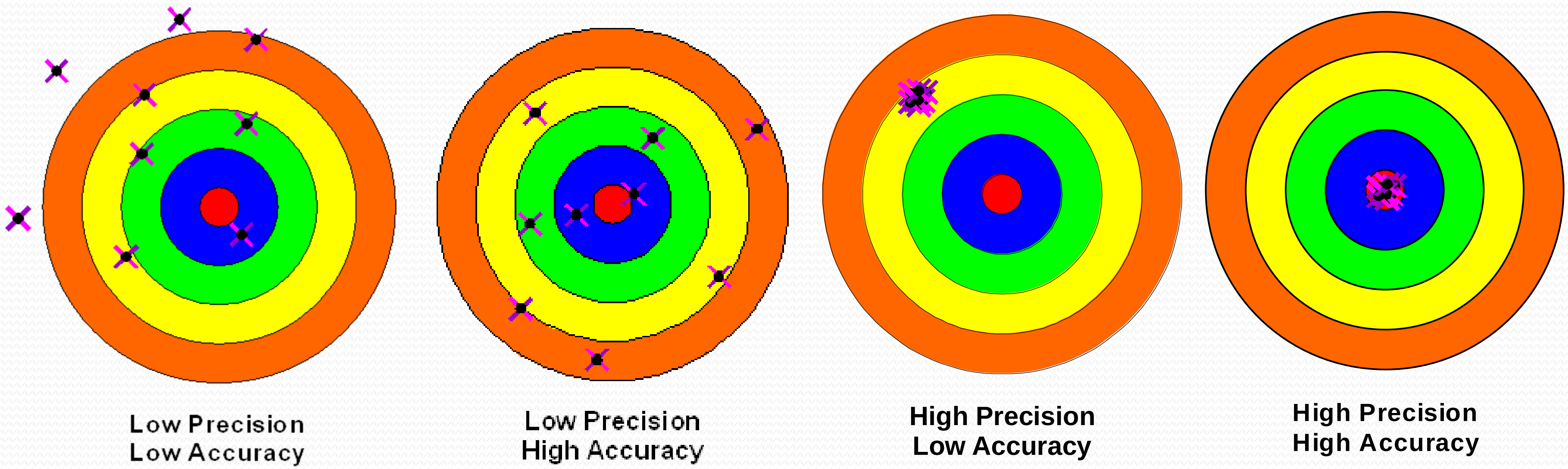
- *Random Errors* – cause the measured result to deviate randomly from the correct value. The distribution of multiple measurements with only random error contributions will be centered around the correct value.

- Some Examples
  - Noise (random noise)
  - Careless measurements
  - Low resolution instruments
  - Dropped digits



Random Errors

- Measurements typically contain some combination of random and systematic errors.
- *Precision* is an indication of the level of random error.
- *Accuracy* is an indication of the level of systematic error.
- Accuracy and precision are typically *qualitative* terms.



EXAMPLE: The boiling point of water is measured using:

A Hg thermometer TBP = 99.5 C (+/-0.5)

A data probe TBP = 98.15 (+/-0.05)

The Hg thermometer yields an answer that is more accurate because it is closer to the right answer of 100 C. The data probe is more precise because the measurement contains more significant figures. Because the fluctuation is always in the last decimal place, uncertainty in the hundredths place (+/-0.05) is less than uncertainty in the tenths place (+/-0.5).

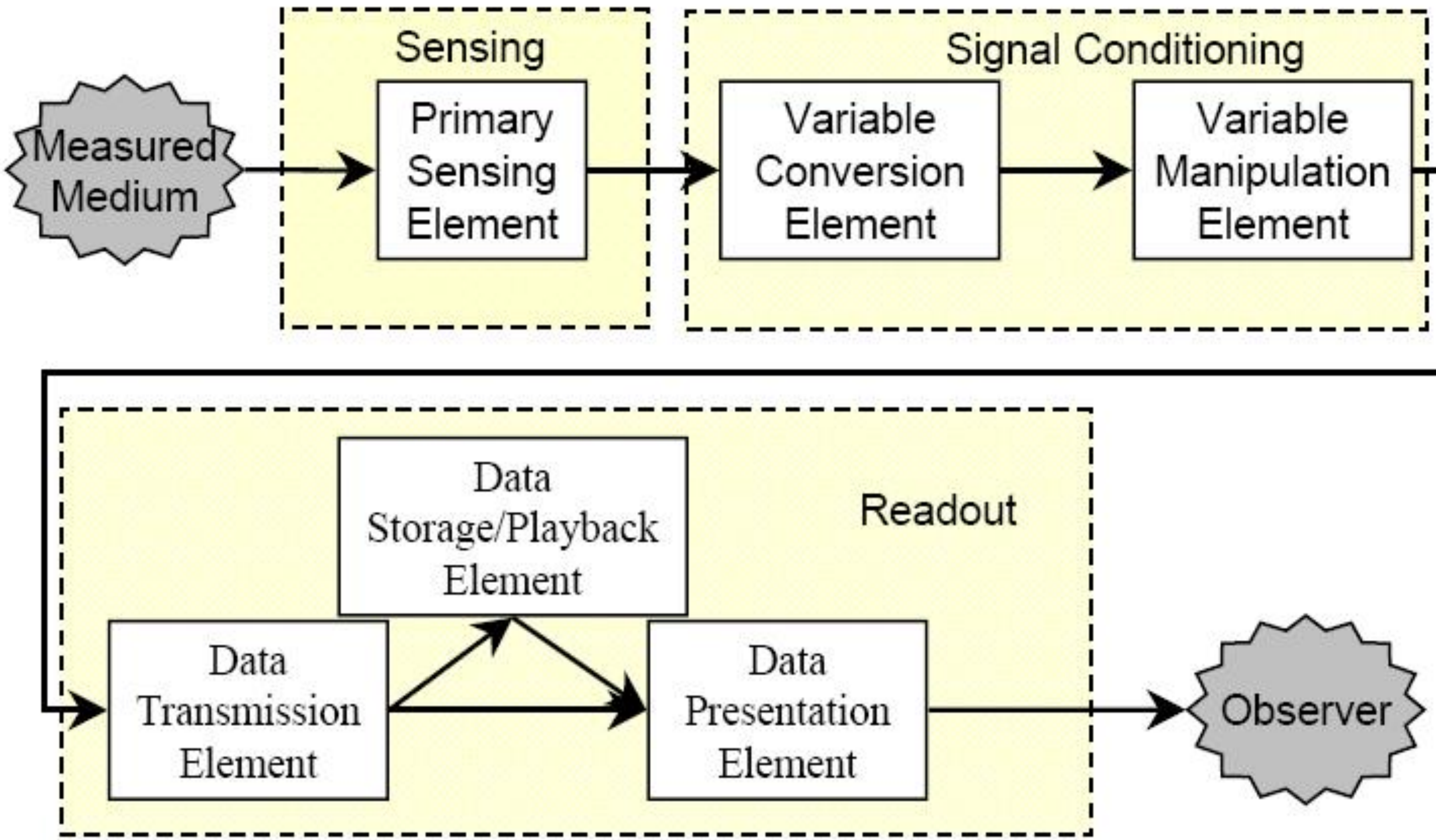
Generalized Measurement System

- **Sensor or transducer stage to detect measurand and Convert input to a form suitable for processing e.g. :**

  - Temp. to voltage - Force to distance
- **Signal conditioning stage to modify the transduced signal e.g. :**

  Amplification, Attenuation, Filtering, Encoding
- **Terminating readout stage to present desired output (Analog or Digital form)**

Generalized Measurement System



GENERALIZED MEASUREMENT SYSTEM

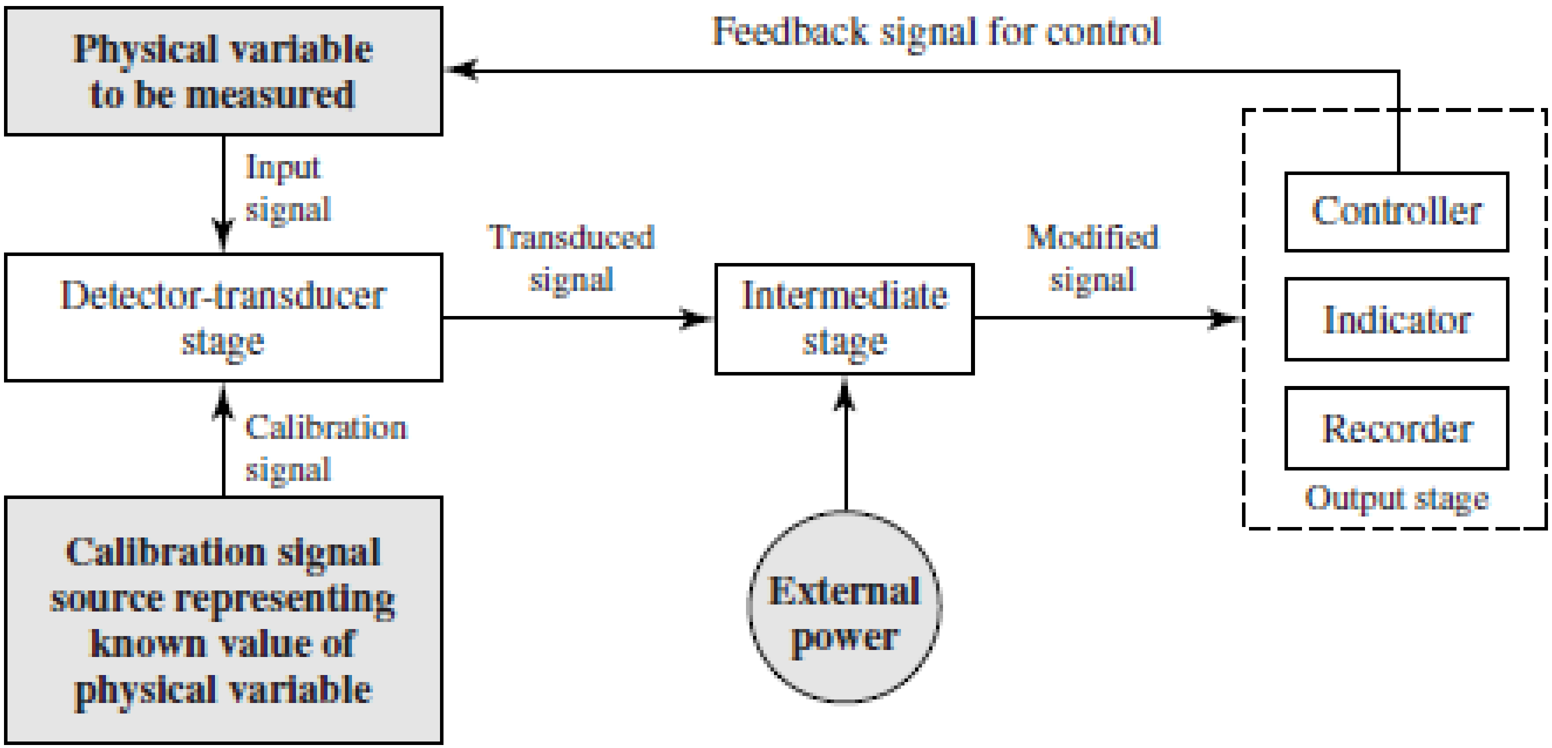
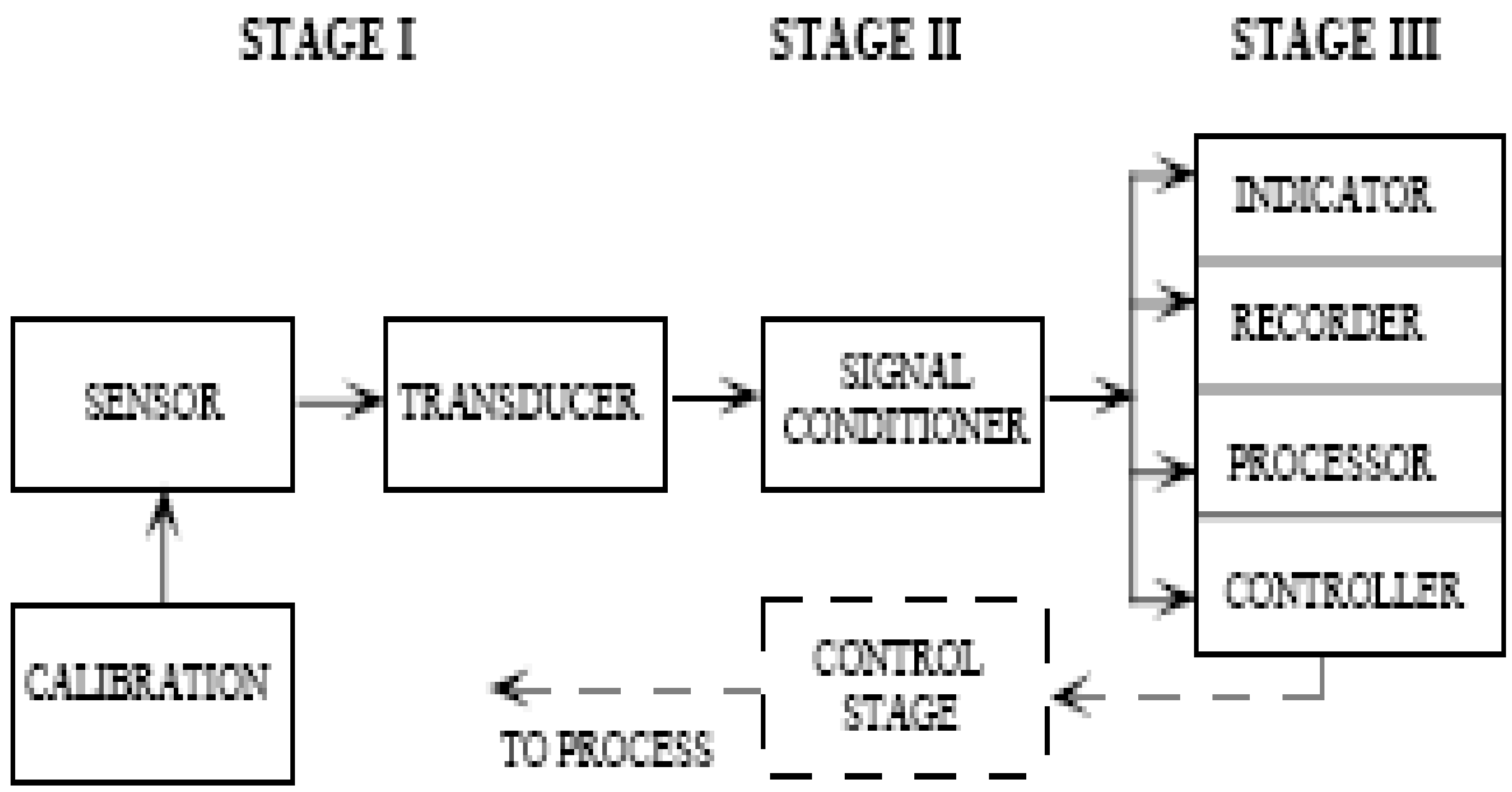


Figure 2.2 Schematic of the generalized measurement system.

Measuring System Stages



MEASUREMENT STAGES

- Primary Sensing (Strain gage, thermometer)
  - Retrieves energy from the measured system
  - Produces some form of output
- Variable conversion
  - Changes data from one physical form to another
    - Elongation to resistance, temperature to volume change
- Variable manipulation
  - Performs mathematical operation on data
    - Amplifier, filter

MEASUREMENT STAGES

- Data transmission
  - Gets data between measurement elements
    - Wire, speedometer cable, satellite downlink system
- Data storage/playback
  - Stores data for later retrieval
    - Hard drive, RAM
- Data presentation
    - Indicators, alarms, analog recording, digital recording

Objectives of Mechanical Measurements

- **Measurement of physical variables:** Force vector (N), Velocity vector (m/sec.), T(°C), P (Pascal), Frequency (Hz=cycle/sec)..
- Measurement of Mechanical Parameters: ***Re=ρvd/μ, Mach No.= v/c,*** $P_D=0.5\,\rho\,V^2$
- Accurate and Reliable Measurements: ***Real value – vs – Measured value***

Mechanical Measurements

- **Act of measurement**
- **— the quantitative comparison between a predefined standard and a measurand to produce a measured result**
- **Standard:** An acknowledged measure of comparison for quantitative or qualitative value; a criterion.
- **Measurand: physical parameter or variable to be measured**

Application of Mech. Measurements

- **Monitoring and operation of process.**
- **Control of a process** *(accurate control f^n measurement acc.)*
- **Experimentation:**

- Testing and performance operation

- Verification of properties or theory

- Information needed for analysis

e.g. Checking or evaluation of:

Oil viscosity variation with temp.

Pump performance curve

piping head loss

Lift and drag of new airfoil shape…….etc.

Calibration

- Calibration involves the determination of the relationship between the input and output of a measurement system
- Eliminate Bias error
- The proving of a measurement system's capability to quantify the input accurately
- Calibration is accomplished by applying known magnitudes of the input and observing the measurement system output
- The indirect measuring system must be calibrated.

CALIBRATION

- Once a measurement device is selected, it must be calibrated
  - Calibration –Comparison of instrument's reading to a calibration standard
  - Calibration standard created from a measurement
    - Inherent error
- Basic issue is how do we know that what we record has any relation to what we wish to measure?

Calibration using Primary or/and Secondary Standards

- **Known input signal and find the output.**
  - To establish the correct output scale.
  - To find instrument reliability.
  - To eliminate bias error (systematic error)
- **For linear relation O/P $\propto$ I/P needs single point calibration.**
- **For non-linear relation needs multi-point calibrations.**
- **Static calibration – vs – Dynamic calibration**

Primary Standards For Comparison and Calibration

- **SI System:** Meter – Kg - Sec. – Kelvin – volt - Mole – Ampere – Radian
- **LENGTH [meter]:** Distance traveled by light in vacuum during *1/299792458* of a sec.
- **MASS [Kg.]:** International prototype *(alloy of platinum and iridium)* kept near Paris.
- **TIME [Sec.]:** Duration of ***9192631770*** periods of the radiation emitted between two excitation levels of Cesium-133
- **TEMPERATURE [Kelvin]:** $K = {}^{o}C + 273$

What is Uncertainty?

- Uncertainty is essentially lack of information to formulate a decision.
- Uncertainty may result in making poor or bad decisions.
- As living creatures, we are accustomed to dealing with uncertainty – that's how we survive.
- Dealing with uncertainty requires reasoning under uncertainty along with possessing a lot of common sense.

Dealing with Uncertainty

- Deductive reasoning – deals with exact facts and exact conclusions
- Inductive reasoning – not as strong as deductive – premises support the conclusion but do not guarantee it.
- There are a number of methods to pick the best solution in light of uncertainty.
- When dealing with uncertainty, we may have to settle for just a good solution.

Uncertainty of Measurements

- **Measurement error = Measured result - T*rue* value**
- **The true value of a measurand is Unknown (Error is unknown)**
- **The potential value of error can be estimated (uncertainty)**
- **Two types of error:**

a) **Systematic errors (bias) e.g.** Failure to calibrate or check zero of instrument, Instrument drift, Lag time and hysteresis

b) **Random errors (Statistics to estimate random errors)** e.g. Instrument resolution, Physical variations

UNCERTAINTY IN PLANING

During the design of the experiment

- Identify all possible sources of error:
  - Experiment set up: facility effects, environmental effects, human , …..
  - Measurement system: velocity, temperature,...
- Estimate possible severity of each source
  - Discuss with advisor.
- For those that are considered “important”, identify strategies.
  - Experimental design and/or test protocols (e.g. repeat tests)
- Plan for quantitative analysis of reduced data
  - Quantitative analysis relies on math model of the system
  - Often good for measurement systems: pitot probe, strain gauge,...

UNCERTAINTY STAGES

- **During the experiment**
  - Execute experiment with replications
  - Record notes in lab notebook
  - Check for mistakes and Bias errors
- **During data reduction**
  - Calculate error bars for measurements
  - Check for outlier points
- **During data interpretation/reporting**
  - Consider errors when interpreting data 1st order &Nth order
    Assure findings are beyond uncertainty of experiment
  - Display error bars in way that aids in understanding findings

Propagation of Uncertainty

- When a measurement model is used to estimate the value of the measurand, the uncertainty of the output estimate is usually obtained by mathematically combining the uncertainties of the input estimates
- The mathematical operation of combining the uncertainties is called ***propagation*** of uncertainty

Standard Uncertainty

- Before propagating uncertainties of input estimates, you must express them in comparable forms
- The commonly used approach is to express each uncertainty in the form of an estimated standard deviation, called a ***standard uncertainty***
- The standard uncertainty of an input estimate x_i is denoted by $u(x_i)$

Combined Standard Uncertainty

- The standard uncertainty of an output estimate obtained by uncertainty propagation is called the ***combined standard uncertainty***
- The combined standard uncertainty of the output estimate y is denoted by $u_c(y)$

m = 75 ± 5 g
What is the meaning of ± 5 ?

- Best guess by experimenter
- Half the smallest division of measurement
- Standard deviation: σ
- Standard error: $\sigma_m = \sigma/\sqrt{n}$
- Expanded uncertainty of $\pm 2\sigma$ or $\pm 3\sigma$ (95% or 99% confidence interval)
- Standard uncertainty: u
- Combined standard uncertainty: u_c

Experimental Data and Measures of Uncertainty

$$\text{Average} = \frac{\text{Sum of all values}}{\text{Number of measurements}} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Quantities that give some measure of experimental **precision** are **Deviation** (individual values)

$$\text{deviation of the } i^{th} \text{ value} = d_i = x_i - \bar{x}$$

Average deviation

$$\text{a.d.} = \frac{\text{sum of deviations}}{\text{number of measurements}} = \frac{\sum_{i=1}^{N} \left| x_i - \bar{x} \right|}{N} = \frac{\sum_{i=1}^{N} \left| d_i \right|}{N}$$

Average Deviation of the Mean (Standard Average Deviation)

$$\text{A.D.} = \frac{\text{a.d.}}{\sqrt{N}}$$

Sample standard deviation

$$s = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{(N - 1)}} = \sqrt{\frac{\sum_{i=1}^{N} d_i^2}{(N - 1)}}$$

Standard error

$$S_x = \frac{s}{\sqrt{N}} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}}$$

It is customary to report experimental results with an uncertainty in the following form

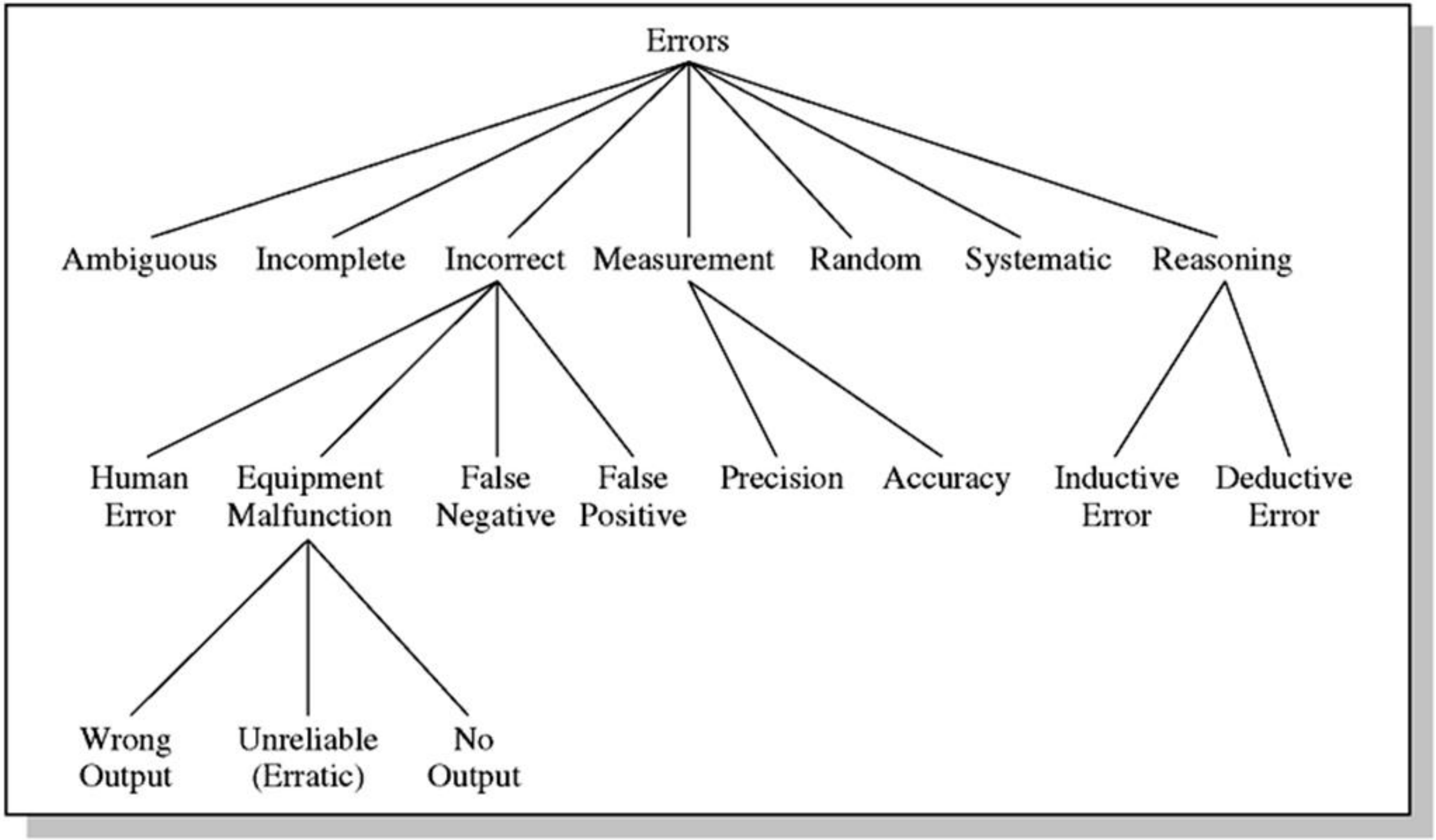
The uncertainty is one of the measures of precision given above (a.d., A.D., s, or S_x).
For our present cases we will use standard error and report results as

$$\bar{x} \pm S_x$$

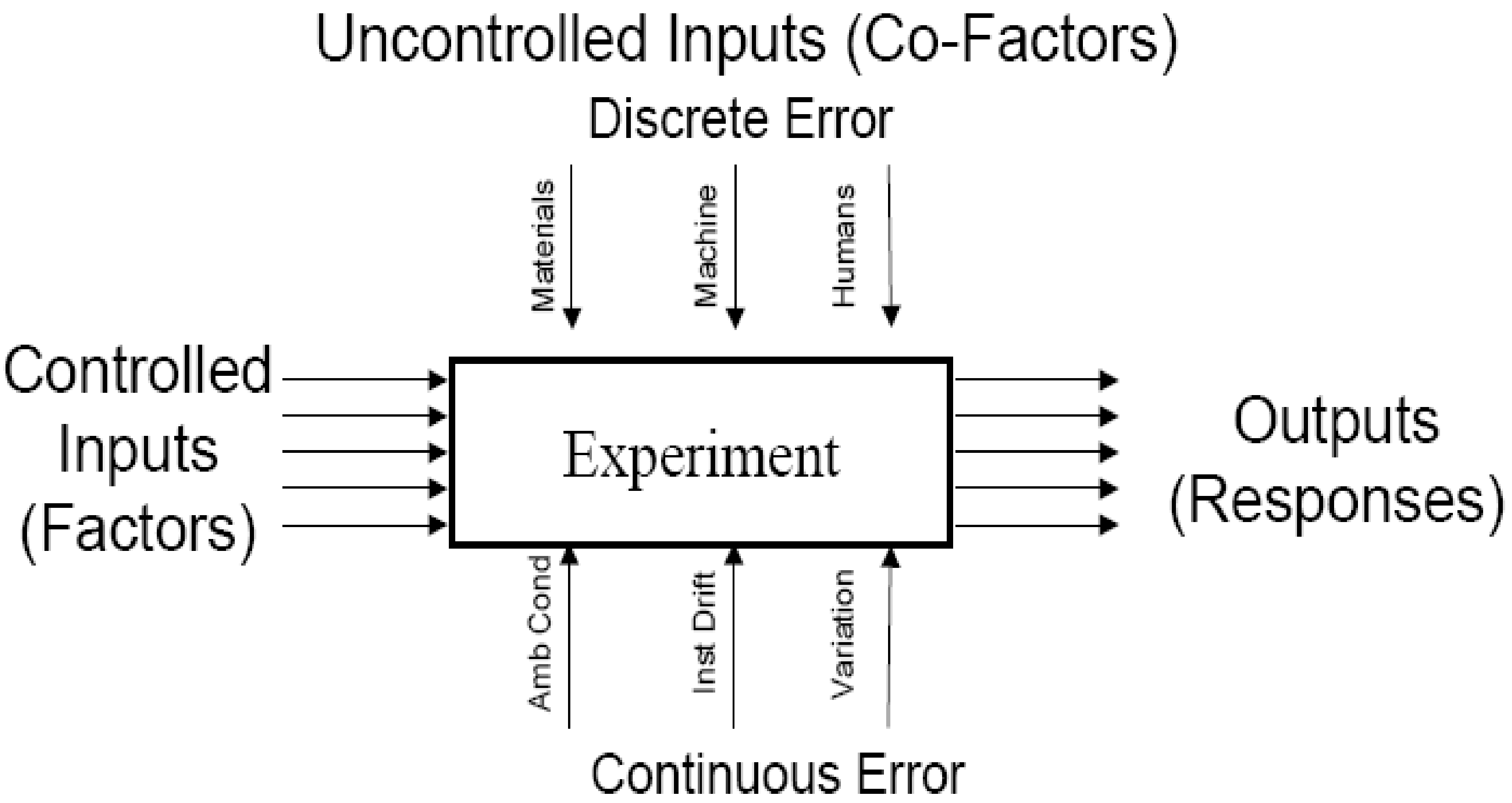
Errors Related to Measurement

- Errors of precision – how well the truth is known
- Errors of accuracy – whether something is true or not
- Unreliability stems from faulty measurement of data – results in erratic data.
- Random fluctuations – termed random error
- Systematic errors result from bias

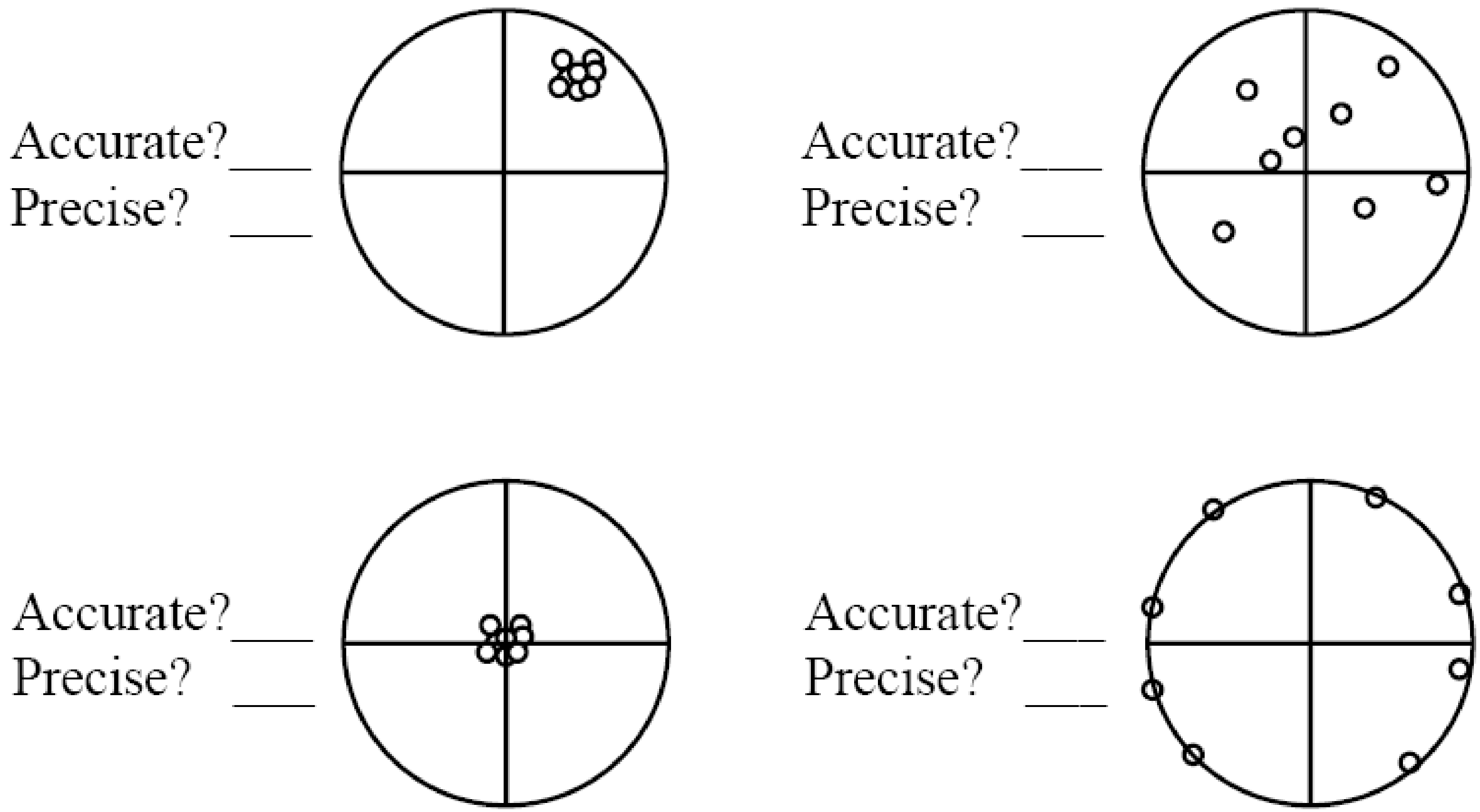
Types of Errors



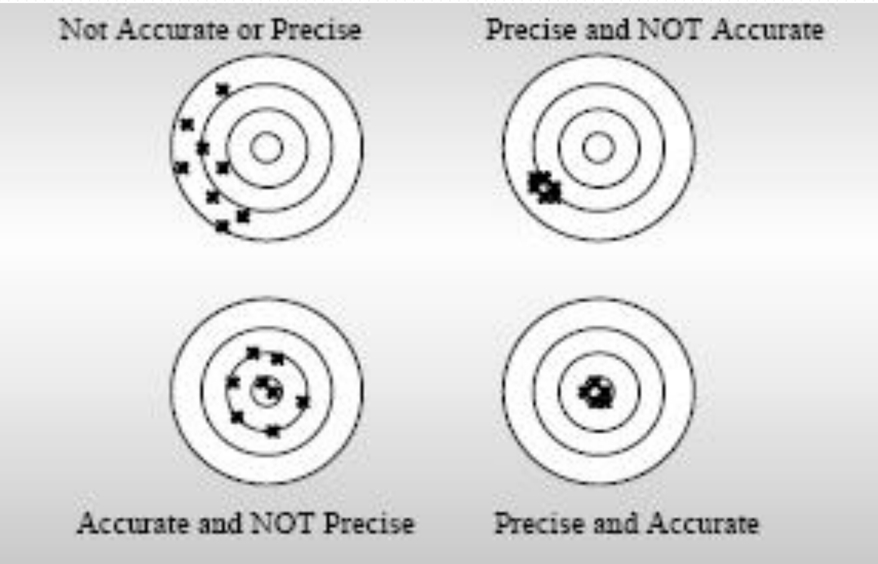
SOURCE OF ERRORS

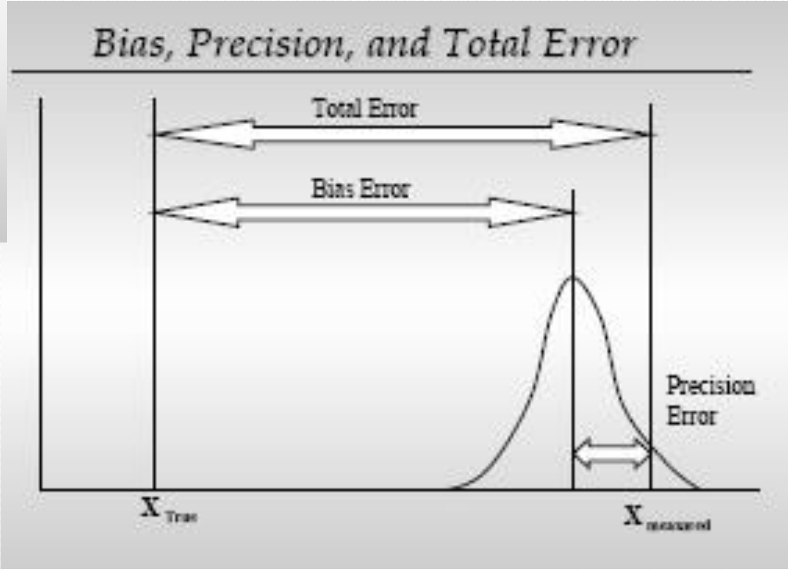


BIAS AND RANDOM ERRORS

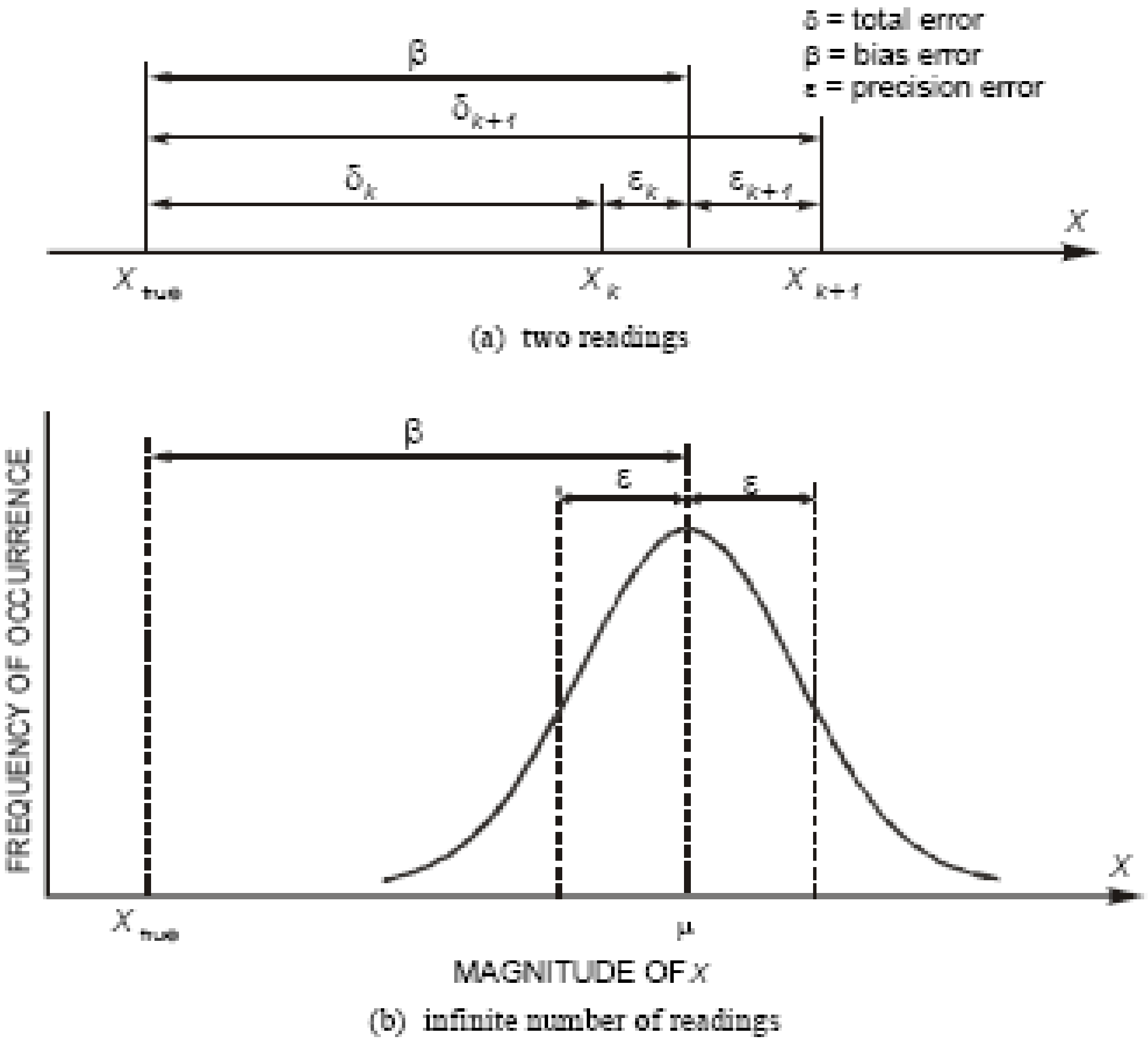


Bias (systematic) and Random (precise) Errors



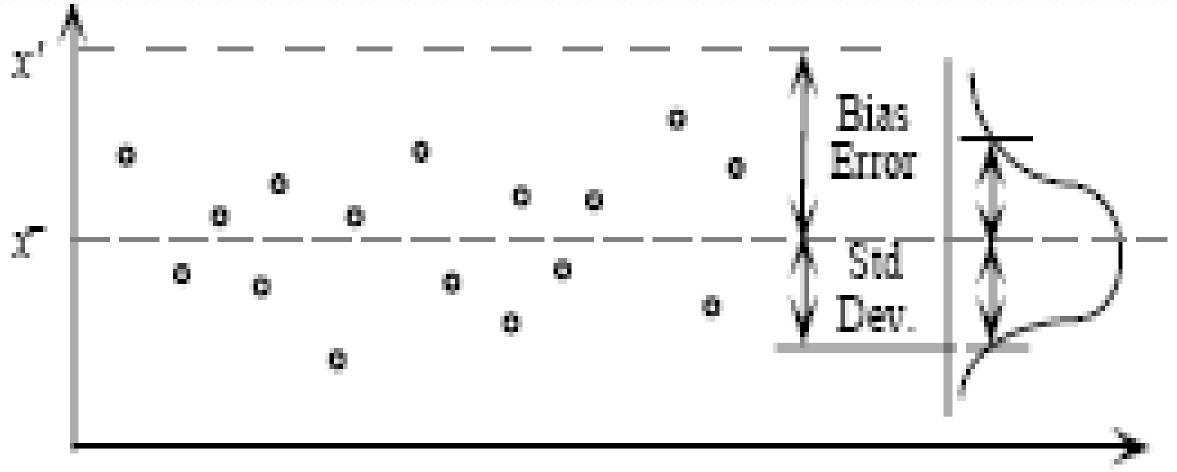


Measurement errors

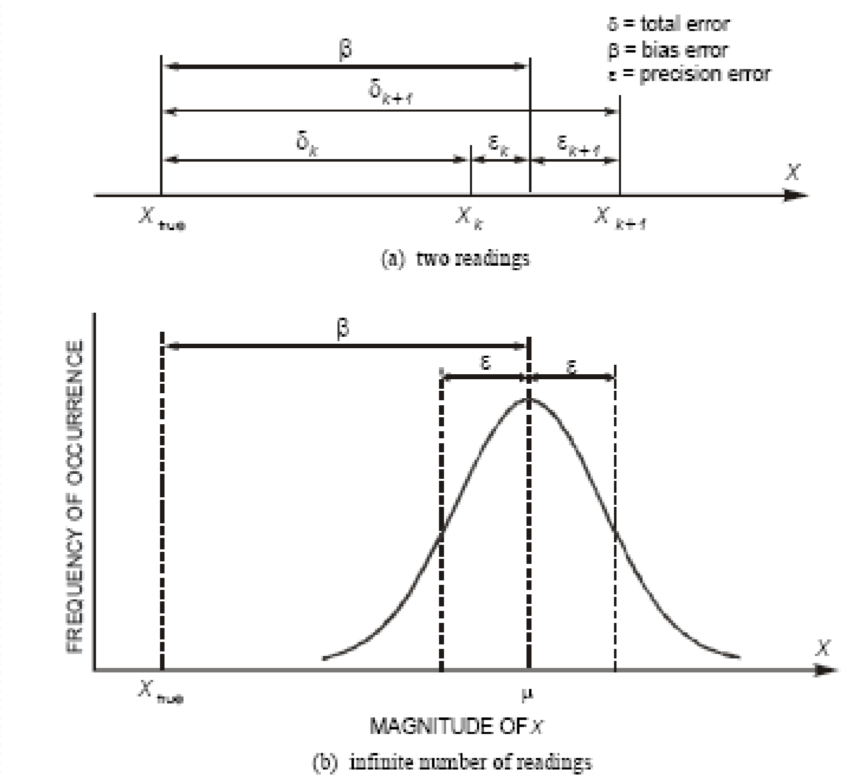


Bias and Random Errors

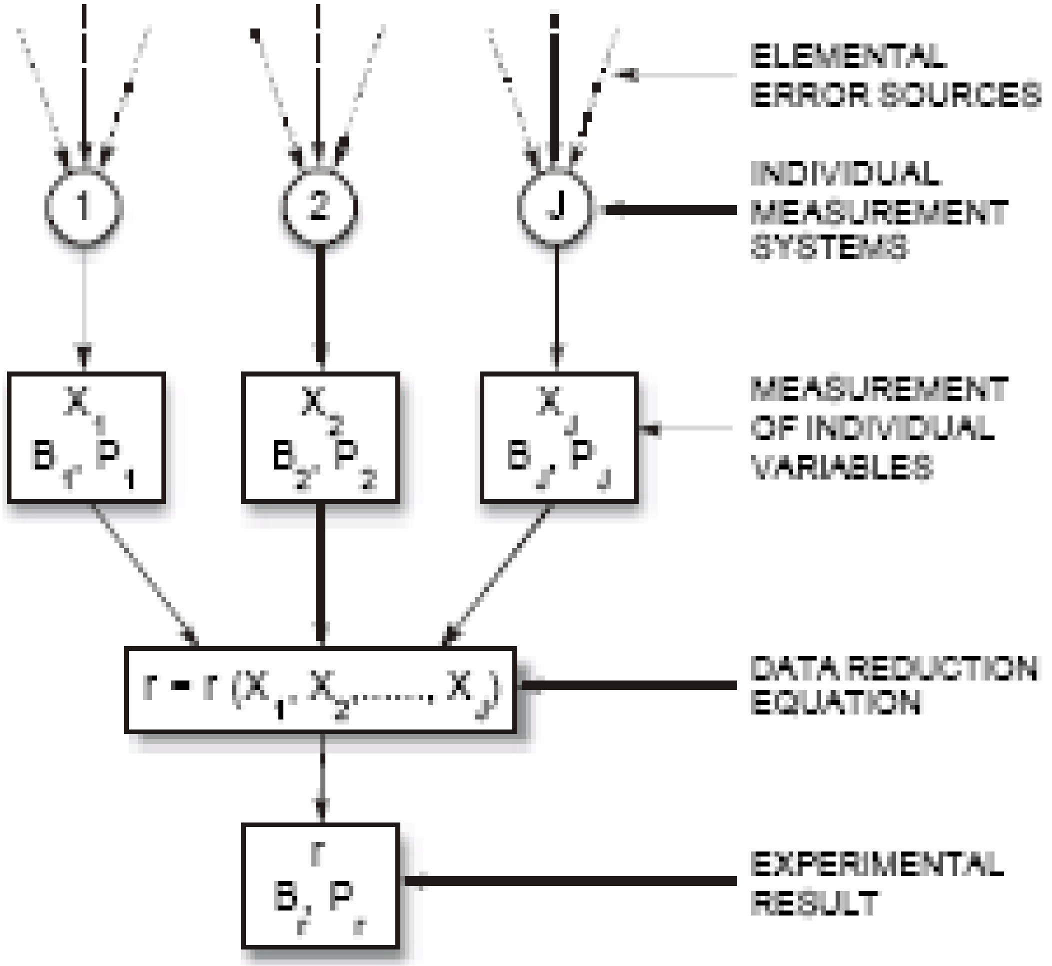
Error	Error type
Accuracy	Bias
Common-mode voltage	Bias
Hysteresis	Bias
Installation	Bias
Linearity	Bias
Loading	Bias
Spatial	Bias
Repeatability	Precision
Noise	Precision
Resolution/scale/quantization	Precision
Thermal stability (gain, zero, etc.)	Precision



Errors in Measuring a Variable



Propagation of Errors



Combination of Errors

- If $Z = F(X_1, X_2, X_3, X_4)$ is quantity we want
- The error in Z, dz, is given by our rule from before
- So, if the error F due to X_1 can be estimated as

$$dF_1 = \frac{\partial F}{\partial X_1} dx_1$$

Error in X_1

Influence coeff.

and so on

- $$dz = \sqrt{\left(\frac{\partial F}{\partial X_1}\right)^2 dx_1^2 + \left(\frac{\partial F}{\partial X_2}\right)^2 dx_2^2 + \cdots \left(\frac{\partial F}{\partial X_n}\right)^2 dx_n^2}$$
- The important consequence of this is that generally one or few of these factors is the main player and others can be ignored

Types of Input Signals

- **Static**
- **Dynamic (Time dependence)**

  - Steady periodic, complex periodic

  - Nonperiodic: nearly periodic or transient

  - Single pulse.

  - Random
- **Analog or digital:**

  - Analog; continuous signal,

  - Digital; distinct values, step changes.

A *static* measurement of a physical quantity is performed when the quantity is not changing with time. The deflection of a beam under a constant load would be a static deflection. However, if the beam were set in vibration, the deflection would vary with time, and the measurement process might be more difficult. Measurements of flow processes are much easier to perform when the fluid is in a nice steady state and become progressively more difficult to perform when rapid changes with time are encountered.

Many experimental measurements are taken under such circumstances that ample time is available for the measurement system to reach steady state, and hence one need not be concerned with the behavior under non-steady-state conditions. In many other situations, however, it may be desirable to determine the behavior of a physical variable over a period of time. Sometimes the time interval is short, and sometimes it may be rather extended. In any event, the measurement problem usually becomes more complicated when the transient characteristics of a system need to be considered. In this section we wish to discuss some of the more important characteristics and parameters applicable to a measurement system under dynamic conditions.

Zeroth-, First-, and Second-Order Systems

A system may be described in terms of a general variable $x(t)$ written in differential equation form as

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} + \cdots + a_1 \frac{dx}{dt} + a_0 x = F(t) \qquad \textbf{[2.5]}$$

where $F(t)$ is some forcing function imposed on the system. The *order* of the system is designated by the order of the differential equation.

A *zeroth-order* system would be governed by

$$a_0 x = F(t) \qquad \textbf{[2.6]}$$

a *first-order* system by

$$a_1 \frac{dx}{dt} + a_0 x = F(t) \qquad \textbf{[2.7]}$$

and a *second-order* system by

$$a_2 \frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = F(t) \qquad \textbf{[2.8]}$$

We shall examine the behavior of these three types of systems to study some basic concepts of dynamic response. We shall also give some concrete examples of physical systems which exhibit the different orders of behavior.

The zeroth-order system described by Eq. (2.6) indicates that the system variable $x(t)$ will track the input forcing function instantly by some constant value: that is,

$$x = \frac{1}{a_0} F(t)$$

The constant $1/a_0$ is called the *static sensitivity* of the sytem. If a constant force were applied to the beam mentioned above, the static deflection of the beam would be F/a_0.

The first-order system described by Eq. (2.7) may be expressed as

$$\frac{a_1}{a_0} \frac{dx}{dt} + x = \frac{F(t)}{a_0} \qquad \textbf{[2.9]}$$

The ratio a_1/a_0 has the dimension of time and is usually called the *time constant* of the system. If Eq. (2.9) is solved for the case of a sudden constant (step) input $F(t) = A$ at time zero, we express the condition as

$$F(t) = 0 \qquad \text{at } t = 0$$

$$F(t) = A \qquad \text{for } t > 0$$

along with the initial condition

$$x = x_0 \qquad \text{at } t = 0$$

The solution to Eq. (2.9) is then

$$x(t) = \frac{A}{a_0} + \left(x_0 - \frac{A}{a_0}\right)e^{-t/\tau} \qquad \textbf{[2.10]}$$

where, now, we set $\tau = a_1/a_0$. The *steady-state response* is the first term on the right, or the value of x, which will be obtained for large values of time. The second term, involving the exponential decay term, represents the *transient response* of the system. Designating the steady-state value as x_∞, Eq. (2.10) may be written in dimensionless form as

$$\frac{x(t) - x_\infty}{x_0 - x_\infty} = e^{-t/\tau} \qquad \textbf{[2.11]}$$

When $t = \tau$, the value of $x(t)$ will have responded to 63.2 percent of the step input, so the time constant is frequently called the time to achieve this value. The *rise time* is the time required to achieve a response of 90 percent of the step input. This requires

$$e^{-t/\tau} = 0.1$$

STEP RESPONSE OF FIRST-ORDER SYSTEM. A certain thermometer has a time constant of 15 s and an initial temperature of 20°C. It is suddenly exposed to a temperature of 100°C. Determine the rise time, that is, the time to attain 90 percent of the steady-state value, and the temperature at this time.

Solution

The thermometer is a first-order system which will follow the behavior in Eq. (2.11). The variable in this case is the temperature, and we have

$$T_0 = 20^\circ\text{C} = \text{temperature at } t = 0$$

$$T_\infty = 100^\circ\text{C} = \text{temperature at steady state}$$

$$\tau = 15 \text{ s} = \text{time constant}$$

For the 90 percent rise time

$$e^{-t/\tau} = 0.1$$

and

$$\ln(0.1) = \frac{-t}{15}$$

so that

$$t = 34.54 \text{ s}$$

Then, at this time Eq. (2.11) becomes

$$\frac{T(t) - 100}{20 - 100} = 0.1$$

and

$$T(t) = 92^\circ\text{C}$$

EXAMPLE

PHASE LAG IN FIRST-ORDER SYSTEM. Suppose the thermometer in Example (2.1) was subjected to a very slow harmonic disturbance having a frequency of 0.01 Hz. The time constant is still 15 s. What is the time delay in the response of the thermometer and how much does the steady-state amplitude response decrease?

Solution

We have

$$\omega = 0.01 \text{ Hz} = 0.06283 \text{ rad/s}$$
$$\tau = 15 \text{ s}$$

so that

$$\omega\tau = (0.06283)(15) = 0.9425$$

From Eq. (2.14) the phase angle is

$$\phi(\omega) = -\tan^{-1}(0.9425) = -43.3^\circ = -0.756 \text{ rad}$$

so that the time delay is

$$\Delta t = \frac{\phi(\omega)}{\omega} = \frac{-0.756}{0.06283} = -12.03 \text{ s}$$

The amplitude response decreases according to

$$\frac{1}{[1+(\omega\tau)^2]^{1/2}} = \frac{1}{[1+(0.9425)^2]^{1/2}} = 0.7277$$

Dimensional Analysis

- **Data presented in dimensionless form.**
- **Reducing № of experimental variables.**

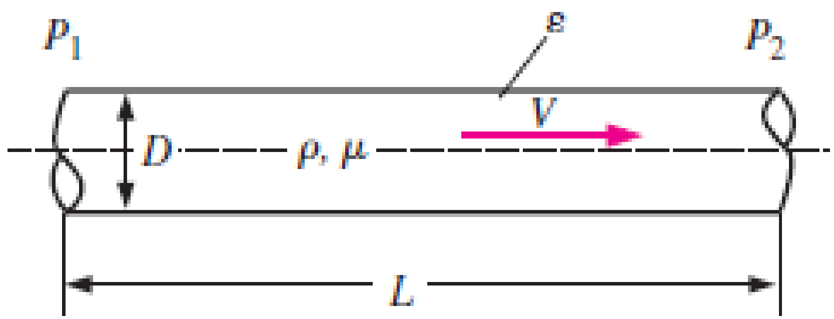
№ of variables - № of dims.= № of $\boldsymbol{\pi}$ *groups*

- **Use pi method or by inspection**
- **Basic dimensions: M L T θ (kg, m, sec, K)**
- Saving (time & \$) (*10 tests – vs - 10^4 tests for $F = f^{n}(L, V, \rho, \mu)$*)

Force coef. $\boldsymbol{F/\rho V^2 L^2} = f^{n}$ (Reynolds number $\boldsymbol{\rho VL/\mu}$)

- Helping in exp. Planning, insight, and similitude.

EX.2.1. An incompressible fluid of density ρ and viscosity μ flows at average speed V through a long, horizontal section of round pipe of length L, inner diameter D, and inner wall roughness height ε (Fig. P7–66). The pipe is long enough that the flow is fully developed, meaning that the velocity profile does not change down the pipe. Pressure decreases (linearly) down the pipe in order to "push" the fluid through the pipe to overcome friction. Using the method of repeating variables, develop a nondimensional relationship between pressure drop $\Delta P = P_1 - P_2$ and the other parameters in the problem. Be sure to modify your Π groups as necessary to achieve established nondimensional parameters, and name them. (Hint: For consistency, choose D rather than L or ε as one of your repeating parameters.) *Answer:* Eu $= f$ (Re, ε/D, L/D)



Solution We are to generate a nondimensional relationship between the given parameters.

Assumptions **1** The flow is fully developed. **2** The fluid is incompressible. **3** No other parameters are significant in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters.

Step 1 All the relevant parameters in the problem are listed in functional form:

List of relevant parameters: $\Delta P = f\left(V, \varepsilon, \rho, \mu, D, L\right) \qquad n = 7$

Step 2 The primary dimensions of each parameter are listed:

$$\begin{array}{ccccccc} \Delta P & V & \varepsilon & \rho & \mu & D & L \\ \{m^1L^{-1}t^{-2}\} & \{L^1t^{-1}\} & \{L^1\} & \{m^1L^{-3}\} & \{m^1L^{-1}t^{-1}\} & \{L^1\} & \{L^1\} \end{array}$$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $j = 3$

If this value of j is correct, the expected number of Πs is

Number of expected Π*s:* $k = n - j = 7 - 3 = 4$

Step 4 We need to choose three repeating parameters since $j = 3$. Following the guidelines listed in Table 7-3, we cannot pick the dependent variable, ΔP. We cannot choose any two of parameters ε, L, and D since their dimensions are identical. It is not desirable to have μ or ε appear in all the Πs. The best choice of repeating parameters is thus V, D, and ρ.

Repeating parameters: V, D, and ρ

Step 5 The dependent Π is generated:

$$\Pi_1 = \Delta P V^{a_1} D^{b_1} \rho^{c_1} \qquad \{\Pi_1\} = \left\{\left(\mathrm{m^1 L^{-1} t^{-2}}\right)\left(\mathrm{L^1 t^{-1}}\right)^{a_1}\left(\mathrm{L^1}\right)^{b_1}\left(\mathrm{m^1 L^{-3}}\right)^{c_1}\right\}$$

mass: $\{\mathrm{m^0}\} = \{\mathrm{m^1 m^{c_1}}\}$ $\quad 0 = 1 + c_1 \quad$ $c_1 = -1$

time: $\{\mathrm{t^0}\} = \{\mathrm{t^{-2} t^{-a_1}}\}$ $\quad 0 = -2 - a_1 \quad$ $a_1 = -2$

length: $\{\mathrm{L^0}\} = \{\mathrm{L^{-1} L^{a_1} L^{b_1} L^{-3c_1}}\}$ $\quad 0 = -1 + a_1 + b_1 - 3c_1 \quad$ $b_1 = 0$

$$0 = -1 - 2 + b_1 + 3$$

The dependent Π is thus

Π_1: $$\Pi_1 = \frac{\Delta P}{\rho V^2}$$

From Table 7-5, the established nondimensional parameter most similar to our Π_1 is the **Euler number** Eu. No manipulation is required.

We form the second Π with μ. By now we know that we will generate a **Reynolds number**,

$$\Pi_2 = \mu V^{a_2} D^{b_2} \rho^{c_2} \qquad \Pi_2 = \frac{\rho V D}{\mu} = \text{Reynolds number} = \text{Re}$$

The final two Π groups are formed with ε and then with L. The algebra is trivial for these cases since their dimension (length) is identical to that of one of the repeating variables (D). The results are

$$\Pi_3 = \varepsilon V^{a_3} D^{b_3} \rho^{c_3} \qquad \Pi_3 = \frac{\varepsilon}{D} = \text{Roughness ratio}$$

$$\Pi_4 = L V^{a_4} D^{b_4} \rho^{c_4} \qquad \Pi_4 = \frac{L}{D} = \text{Length-to-diameter ratio or aspect ratio}$$

Step 6 We write the final functional relationship as

Relationship between Πs:

$$\boxed{\text{Eu} = \frac{\Delta P}{\rho V^2} = f\left(\text{Re}, \frac{\varepsilon}{D}, \frac{L}{D}\right)} \qquad (1)$$

INSTRUMENT SELECTION. The power measurement in Example 3.2 is to be conducted by measuring voltage and current across the resistor with the circuit shown in the accompanying figure. The voltmeter has an internal resistance R_m, and the value of R is known only approximately. Calculate the nominal value of the power dissipated in R and the uncertainty for the following conditions:

$$R = 100\,\Omega \qquad \text{(not known exactly)}$$
$$R_m = 1000\,\Omega \pm 5\%$$
$$I = 5\text{ A} \pm 1\%$$
$$E = 500\text{ V} \pm 1\%$$

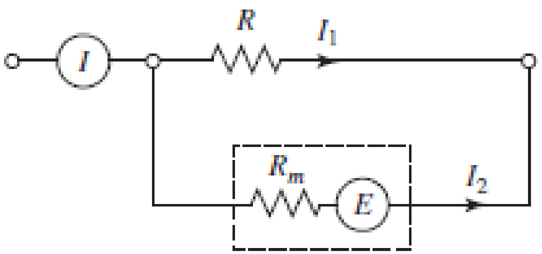


Figure Example 3.4 Effect of meter impedance on measurement.

Solution

A current balance on the circuit yields

$$I_1 + I_2 = I$$
$$\frac{E}{R} + \frac{E}{R_m} = I$$

and

$$I_1 = I - \frac{E}{R_m} \qquad \textbf{[a]}$$

The power dissipated in the resistor is

$$P = EI_1 = EI - \frac{E^2}{R_m} \qquad \textbf{[b]}$$

The nominal value of the power is thus calculated as

$$P = (500)(5) - \frac{500^2}{1000} = 2250\ \text{W}$$

In terms of known quantities the power has the functional form $P = f(E, I, R_m)$, and so we form the derivatives

$$\frac{\partial P}{\partial E} = I - \frac{2E}{R_m} \qquad \frac{\partial P}{\partial I} = E$$

$$\frac{\partial P}{\partial R_m} = \frac{E^2}{R_m^2}$$

The uncertainty for the power is now written as

$$w_P = \left[\left(I - \frac{2E}{R_m}\right)^2 w_E^2 + E^2 w_I^2 + \left(\frac{E^2}{R_m^2}\right)^2 w_{R_m}^2\right]^{1/2} \qquad \textbf{[c]}$$

Inserting the appropriate numerical values gives

$$\begin{aligned} w_P &= \left[\left(5 - \frac{1000}{1000}\right)^2 5^2 + (25 \times 10^4)(25 \times 10^{-4}) + \left(25 \times \frac{10^4}{10^6}\right)^2 (2500)\right]^{1/2} \\ &= [16 + 25 + 6.25]^{1/2}(5) \\ &= 34.4\ \text{W} \end{aligned}$$

or

$$\frac{w_P}{P} = \frac{34.4}{2250} = 1.53\%$$

WAYS TO REDUCE UNCERTAINTIES. A certain obstruction-type flowmeter (orifice, venturi, nozzle), shown in the accompanying figure, is used to measure the flow of air at low velocities. The relation describing the flow rate is

$$\dot{m} = CA\left[\frac{2g_c p_1}{RT_1}(p_1 - p_2)\right]^{1/2} \qquad \textbf{[a]}$$

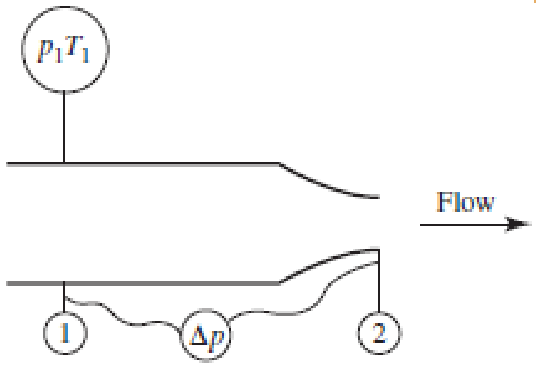


Figure Example 3.5 Uncertainty in a flowmeter.

where
C = empirical-discharge coefficient
A = flow area
p_1 and p_2 = upstream and downstream pressures, respectively
T_1 = upstream temperature
R = gas constant for air

Calculate the percent uncertainty in the mass flow rate for the following conditions:

$C = 0.92 \pm 0.005$ (from calibration data)
$p_1 = 25$ psia ± 0.5 psia
$T_1 = 70°\text{F} \pm 2°\text{F}$ $T_1 = 530°\text{R}$
$\Delta p = p_1 - p_2 = 1.4$ psia ± 0.005 psia (measured directly)
$A = 1.0\ \text{in}^2 \pm 0.001\ \text{in}^2$

Solution

In this example the flow rate is a function of several variables, each subject to an uncertainty.

$$\dot{m} = f(C, A, p_1, \Delta p, T_1) \qquad \textbf{[b]}$$

Thus, we form the derivatives

$$\frac{\partial \dot{m}}{\partial C} = A\left(\frac{2g_c p_1}{RT_1}\Delta p\right)^{1/2}$$

$$\frac{\partial \dot{m}}{\partial A} = C\left(\frac{2g_c p_1}{RT_1}\Delta p\right)^{1/2}$$

$$\frac{\partial \dot{m}}{\partial p_1} = 0.5CA\left(\frac{2g_c}{RT_1}\Delta p\right)^{1/2} p_1^{-1/2} \qquad \textbf{[c]}$$

$$\frac{\partial \dot{m}}{\partial \Delta p} = 0.5CA\left(\frac{2g_c p_1}{RT_1}\right)^{1/2} \Delta p^{-1/2}$$

$$\frac{\partial \dot{m}}{\partial T_1} = -0.5CA\left(\frac{2g_c p_1}{R}\Delta p\right)^{1/2} T_1^{-3/2}$$

The uncertainty in the mass flow rate may now be calculated by assembling these derivatives in accordance with Eq. (3.2). Designating this assembly as Eq. (c) and then dividing by Eq. (a) gives

$$\frac{w_{\dot{m}}}{\dot{m}} = \left[\left(\frac{w_C}{C}\right)^2 + \left(\frac{w_A}{A}\right)^2 + \frac{1}{4}\left(\frac{w_{p_1}}{p_1}\right)^2 + \frac{1}{4}\left(\frac{w_{\Delta p}}{\Delta p}\right)^2 + \frac{1}{4}\left(\frac{w_{T_1}}{T_1}\right)^2\right]^{1/2} \qquad \textbf{[d]}$$

We may now insert the numerical values for the quantities to obtain the percent uncertainty in the mass flow rate.

$$\frac{w_{\dot{m}}}{\dot{m}} = \left[\left(\frac{0.005}{0.92}\right)^2 + \left(\frac{0.001}{1.0}\right)^2 + \frac{1}{4}\left(\frac{0.5}{25}\right)^2 + \frac{1}{4}\left(\frac{0.005}{1.4}\right)^2 + \frac{1}{4}\left(\frac{2}{530}\right)^2\right]^{1/2}$$

$$= [29.5 \times 10^{-6} + 1.0 \times 10^{-6} + 1.0 \times 10^{-4} + 3.19 \times 10^{-6} + 3.57 \times 10^{-6}]^{1/2}$$

$$= [1.373 \times 10^{-4}]^{1/2} = 1.172\% \qquad \textbf{[e]}$$

Comment

The main contribution to uncertainty is the p_1 measurement with its basic uncertainty of 2 percent. Thus, to improve the overall situation the accuracy of this measurement should be attacked first. In order of influence on the flow-rate uncertainty we have:

1. Uncertainty in p_1 measurement (± 2 percent)
2. Uncertainty in value of C
3. Uncertainty in determination of T_1
4. Uncertainty in determination of Δp
5. Uncertainty in determination of A

Sensors

- A **sensor** is a converter that measures a physical quantity and converts it into a signal which can be read by an observer or by an (today mostlyelectronic) instrument. For example, a mercury-in-glass thermometer converts the measured temperature into expansion and contraction of a liquid which can be read on a calibrated glass tube. A thermocouple converts temperature to an output voltage which can be read by a voltmeter. For accuracy, most sensors are calibrated against known standards.

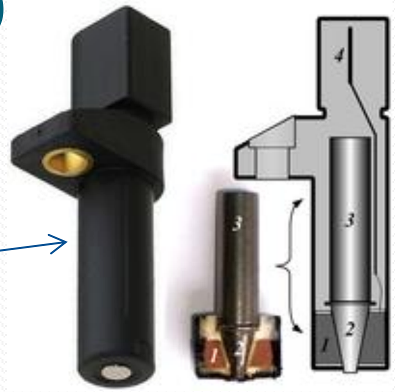
List of sensors

- 1 Acoustic, sound, vibration
- 2 Automotive, transportation
- 3 Chemical
- 4 Electric current, electric potential, magnetic, radio
- 5 Environment, weather, moisture, humidity
- 6 Flow, fluid velocity
- 7 Ionizing radiation, subatomic particles
- 8 Navigation instruments
- 9 Position, angle, displacement, distance, speed, acceleration
- 10 Optical, light, imaging, photon
- 11 Pressure
- 12 Force, density, level
- 13 Thermal, heat, temperature
- 14 Proximity, presence
- 15 Sensor technology
- 16 Other sensors and sensor related properties and concepts

Examples: Acoustic, sound, vibration

- Geophone
- Hydrophone
- Lace Sensor a guitar pickup
- Microphone
- Seismometer

Automotive, transportation



- Air–fuel ratio meter
- Blind spot monitor
- Crankshaft position sensor
- Curb feeler, used to warn driver of curbs
- Defect detector, used on railroads to detect axle and signal problems in passing trains
- Engine coolant temperature sensor, or ECT sensor, used to measure the engine temperature
- Hall effect sensor, used to time the speed of wheels and shafts
- MAP sensor, Manifold Absolute Pressure, used in regulating fuel metering.
- Mass flow sensor, or mass airflow (MAF) sensor, used to tell the ECU the mass of air entering the engine

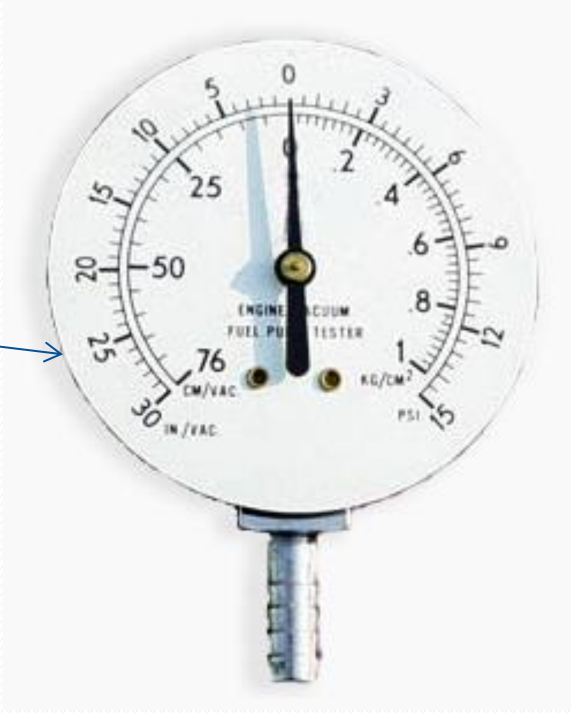
Flow, fluid velocity

- Air flow meter
- Anemometer
- Flow sensor
- Gas meter
- Mass flow sensor
- Water meter

THE ROBINSON ANEMOMETER.

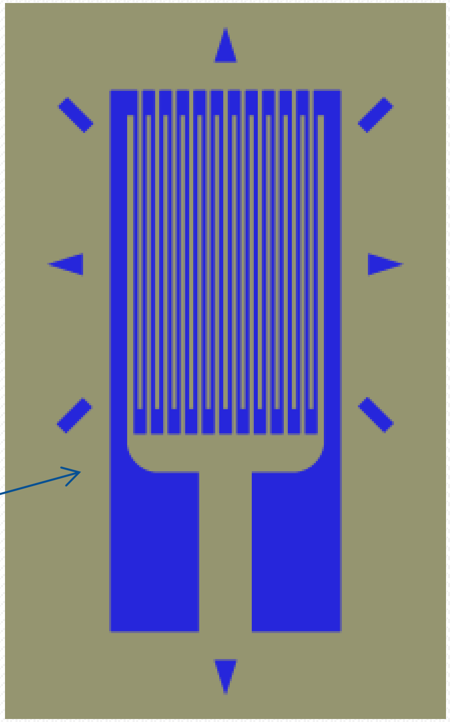
Pressure

- [Barometer]()
- [Boost gauge]()
- [Bourdon gauge]()
- [Permanent Downhole Gauge]()
- [Piezometer]()
- [Pirani gauge]()
- [Pressure sensor]()
- [Pressure gauge]()

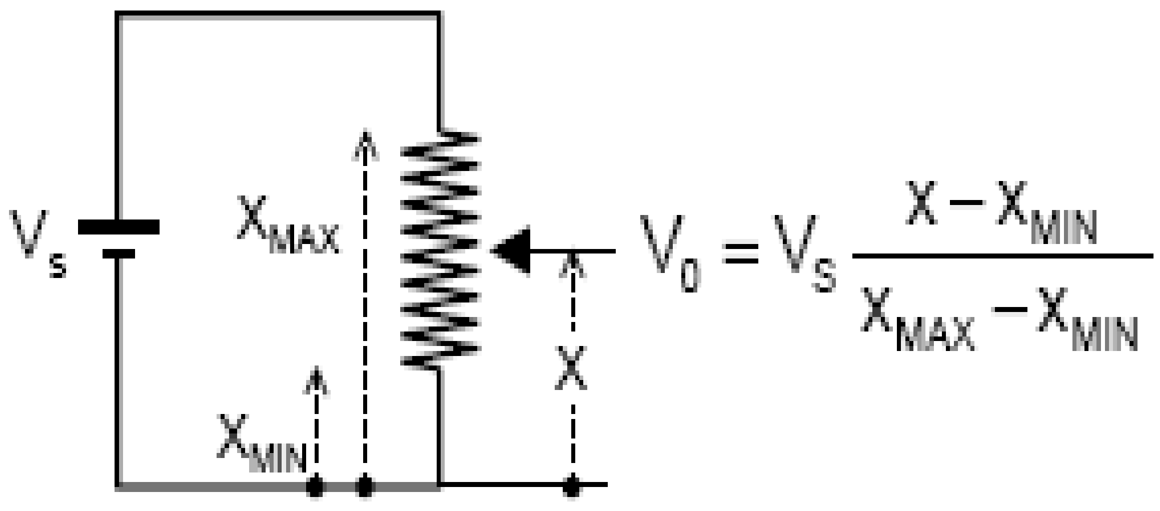


Force, density, level

- Hydrometer
- Force gauge
- Level sensor
- Load cell
- Magnetic level gauge
- Strain gauge
- Torque sensor
- Viscometer



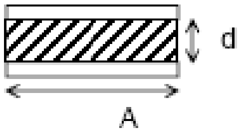
Resistive Displacement Sensor

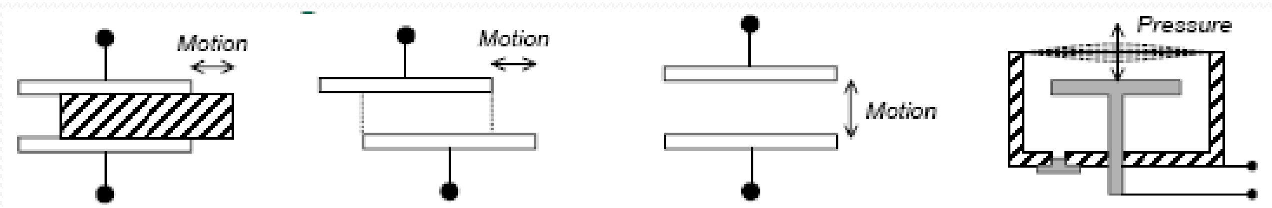


Capacitive Displacement Sensor

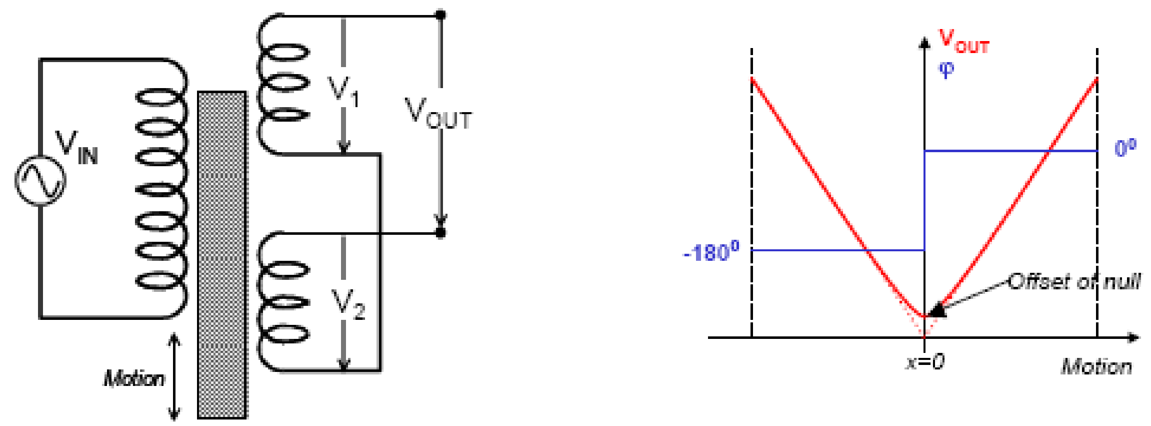
C= Capacitance, ε_o &ε_r =Permittivity of air and Dielectric

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$





Linear Variable differential Transformer (LVDT)



Linear Variable differential Transformer (LVDT)

- Primary coil voltage: $V_S \sin(\omega t)$
- Secondary coil induced emf:

  $V_1 = k_1 \sin(\omega t + \phi)$ and $V_2 = k_2 \sin(\omega t + \phi)$

  k_1 and k_2 proportional to the position of the coil
- When the coil is in the central position, $k_1 = k_2$

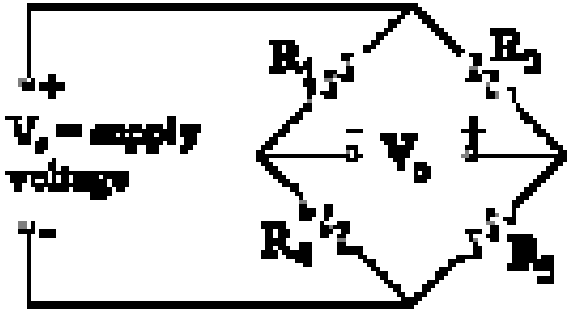
$$V_{OUT} = V_1 - V_2 = 0$$

- When the coil is is displaced , $k_1 \neq k_2$

$$V_{OUT} = (k_1 - k_2)\sin(\omega t + \phi)$$

Wheatstone Bridge

Consists of four resistors (R_1,R_2,R_3,R_4) with an excitation voltage, V_s, applied to the circuit.



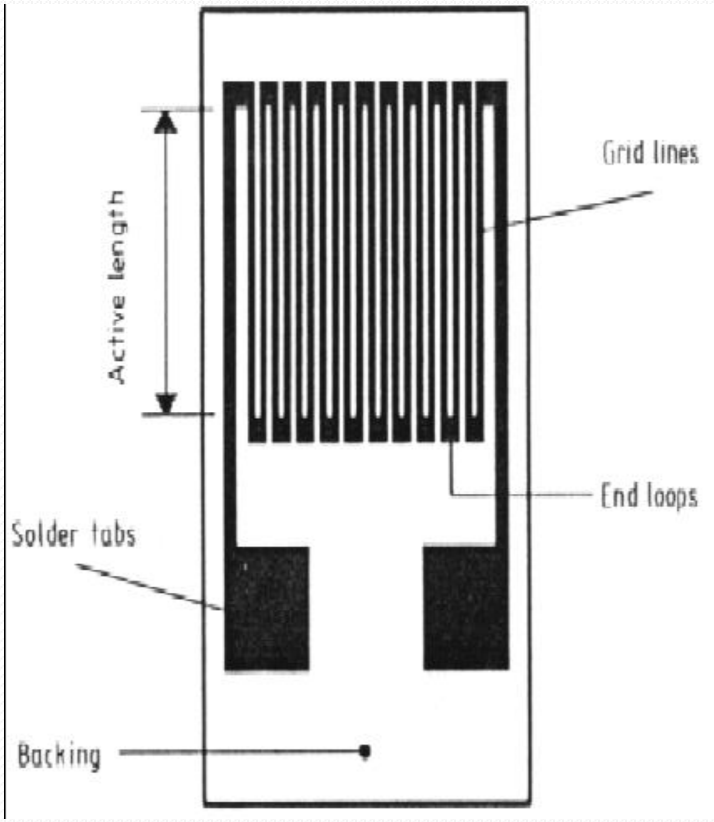
- Assume measuring instrument for V_o has infinite impedance and does not affect circuit.
- Output voltage calculated using Ohm's Law.

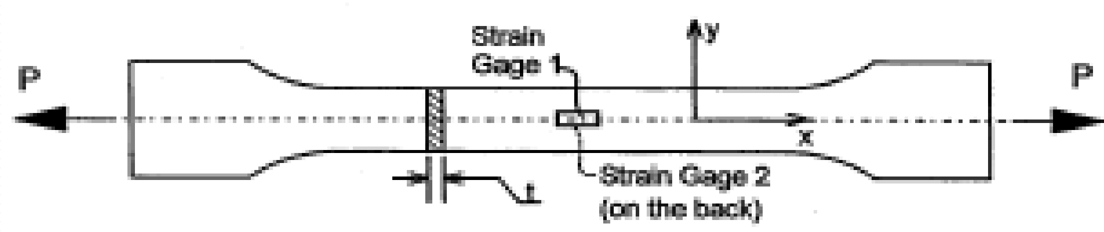
$$V_o = V_s \frac{R_3 R_1 - R_4 R_2}{(R_2 + R_3)(R_1 + R_4)}$$

- If all four resistors are equal the bridge is considered balanced, V_o=0.
  - Also balanced if: $R_3 R_1 = R_4 R_2$ or $\frac{R_1}{R_2} = \frac{R_4}{R_3}$

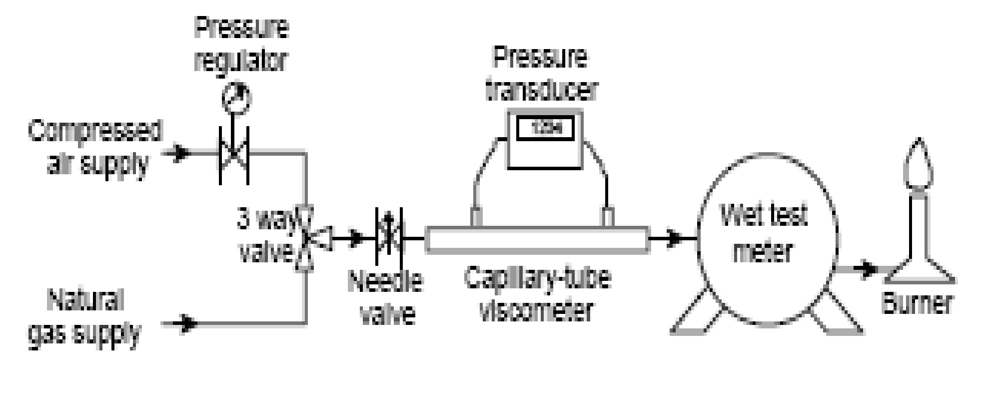
Strain Gage

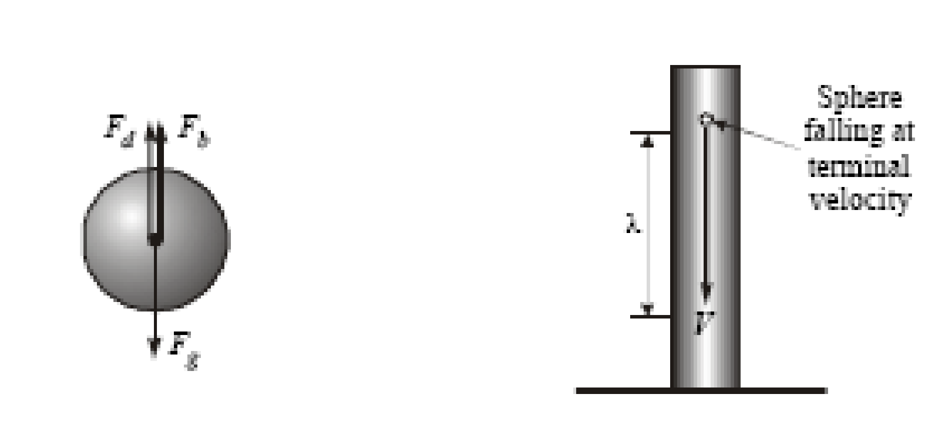
[Gage Factor = (ΔR/R)/(ΔL/L) & Young's Modulus = (P/A) / (ΔL/L)]



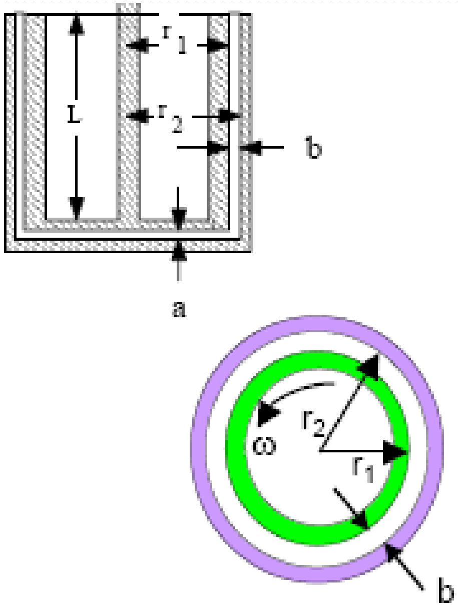


Viscosity Measurements

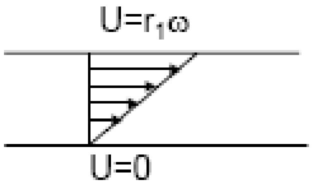




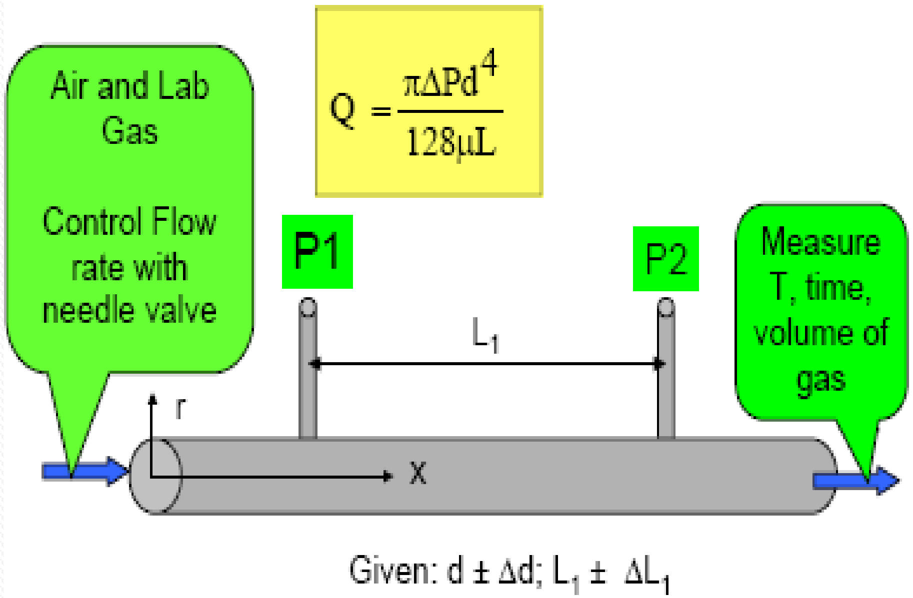
Fluid Viscosity



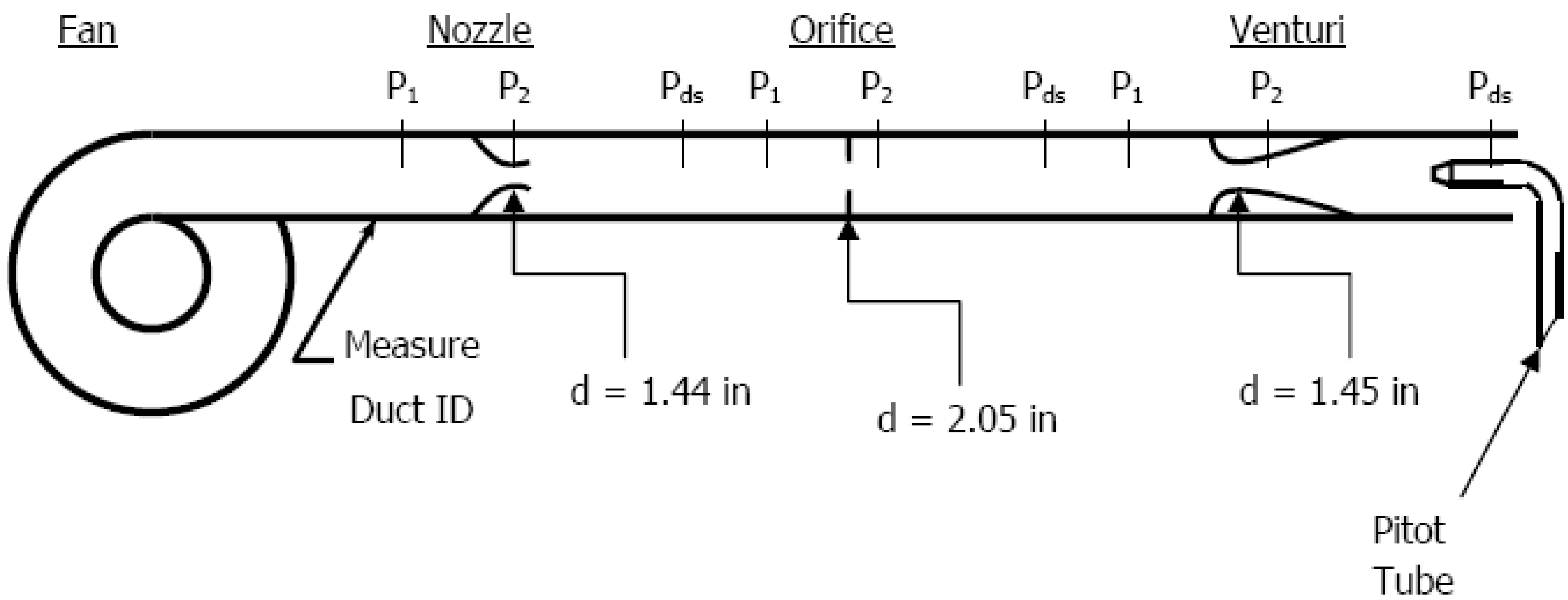
The Torque required to develop the rotational speed is measured and related to viscosity



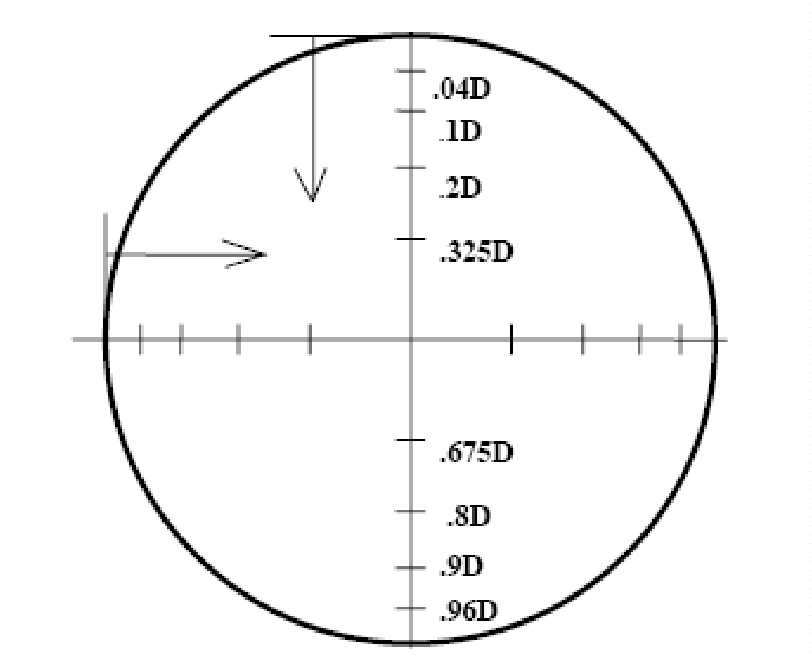
$$\frac{du}{dy} = \frac{r_1\omega}{b}$$



Flow Rate Measurements

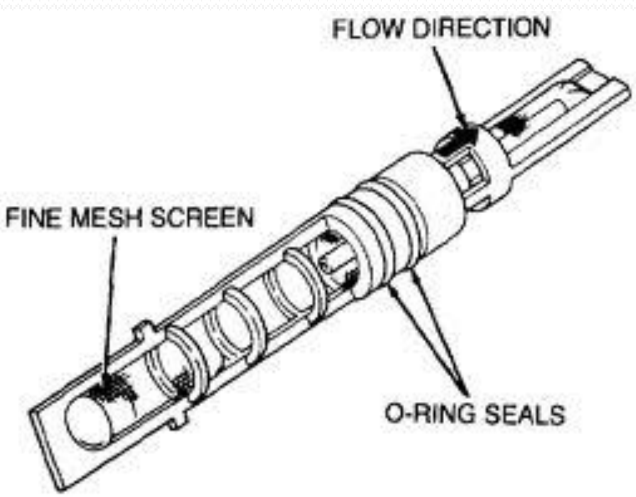


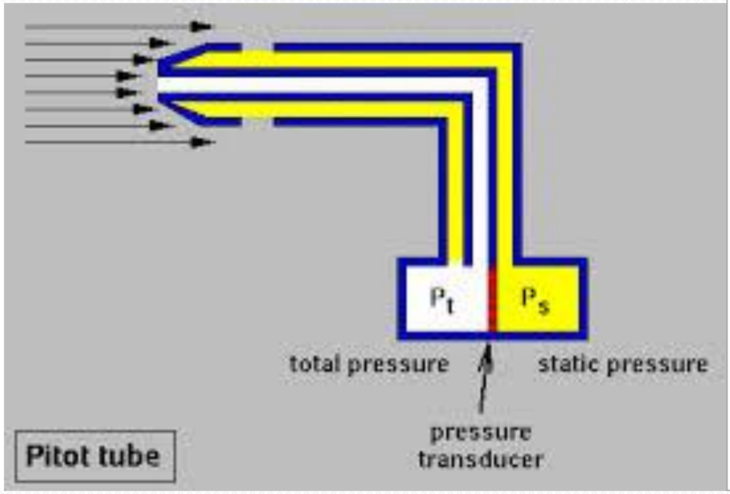
Pitot Tube Traverse Points



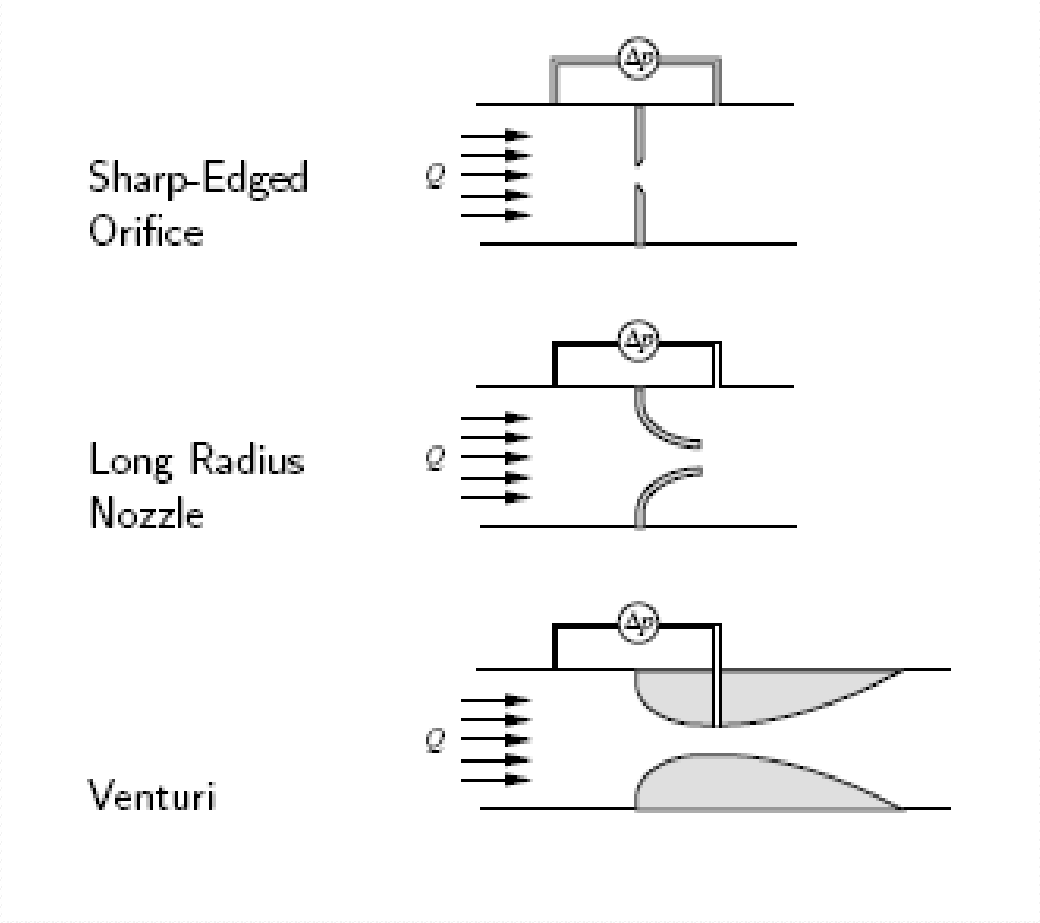
Flow Instrumentation

- Orifice, venturi tube, flow tube, flow nozzles.
- Pitot tubes, elbow-tap meters, target meters.
- Rotameter and Nutating disk





Obstruction Flow Meter

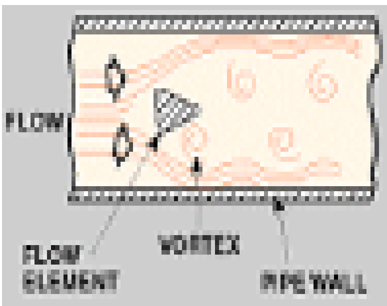


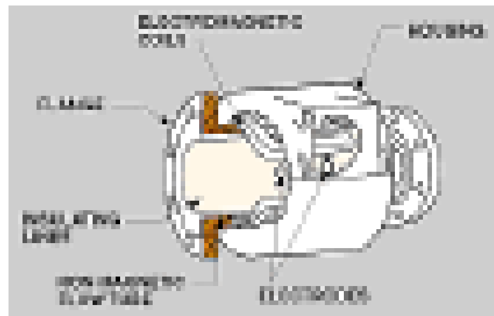
Miscellaneous Flow Meters

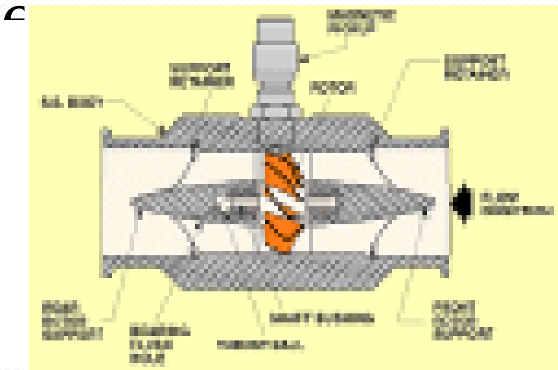
- Turbine, vortex shedding flow meters.
- Mass meters include Coriolis and thermal types.
- Hot-Wire Anemometer: Electrically heated, fine platinum wire immersed in flow Wire is cooled as flow is increased Measure either change in wire resistance or heating current to determine flow
- Electromagnetic Flow meter:Electromotive force induced in fluid as it flows through magnetic field and measured with electrodes which is proportional to flow rate
- Ultrasonic Flow equipment: Uses Doppler frequency shift of ultrasonic signals reflected off discontinuities in fluid
- Laser Doppler Anemometer which employ Doppler effect and Hetrodyning of two signals

Flow Meters

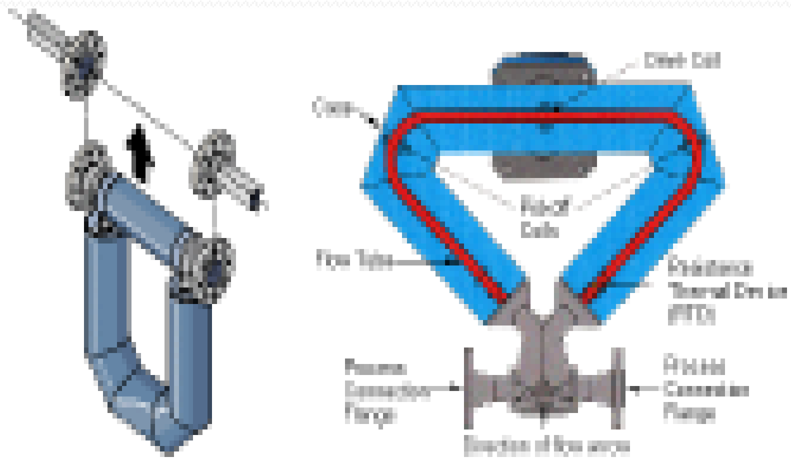
Vortex magnetic Turbine





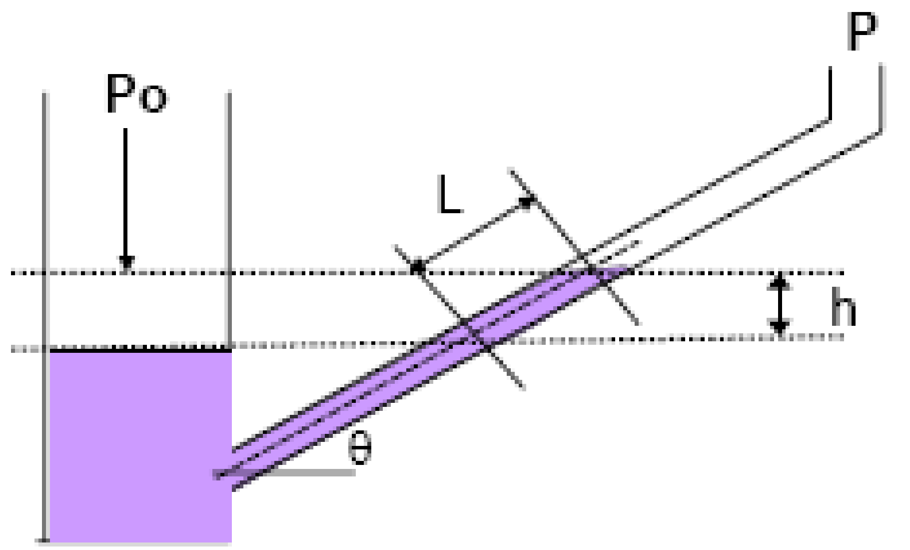


Coriolis mass flow meter



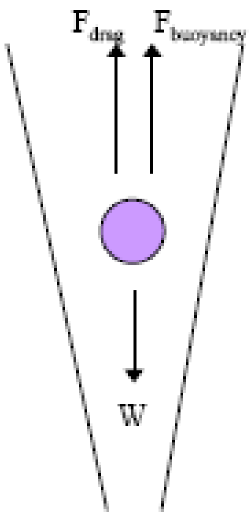
Flow velocity measurement

PRESSURE MEASUREMENT WITH INCLINED MANOMETER



$$\boxed{P_o - P = \rho_m g h = \rho_m g L \sin\theta}$$

Rotameter



$$F_{drag} + F_{buoyancy} = W_{float} \quad (Force\ Balance)$$

$$F_{drag} = C_D \frac{\rho V^2 A_{frontal}}{2}$$

$$F_{drag} + \rho_{fluid} V_{float} g = \rho_{float} V_{float} g$$

$$F_{drag} = (\rho_{float} - \rho_{fluid}) V_{float} g$$

When operating a rotameter at densities different than those for which it calibrated, a correction to the reading is required.

$$\dot{m}_{use} = \dot{m}_{cal} \sqrt{\frac{\rho_{use}}{\rho_{cal}}}$$

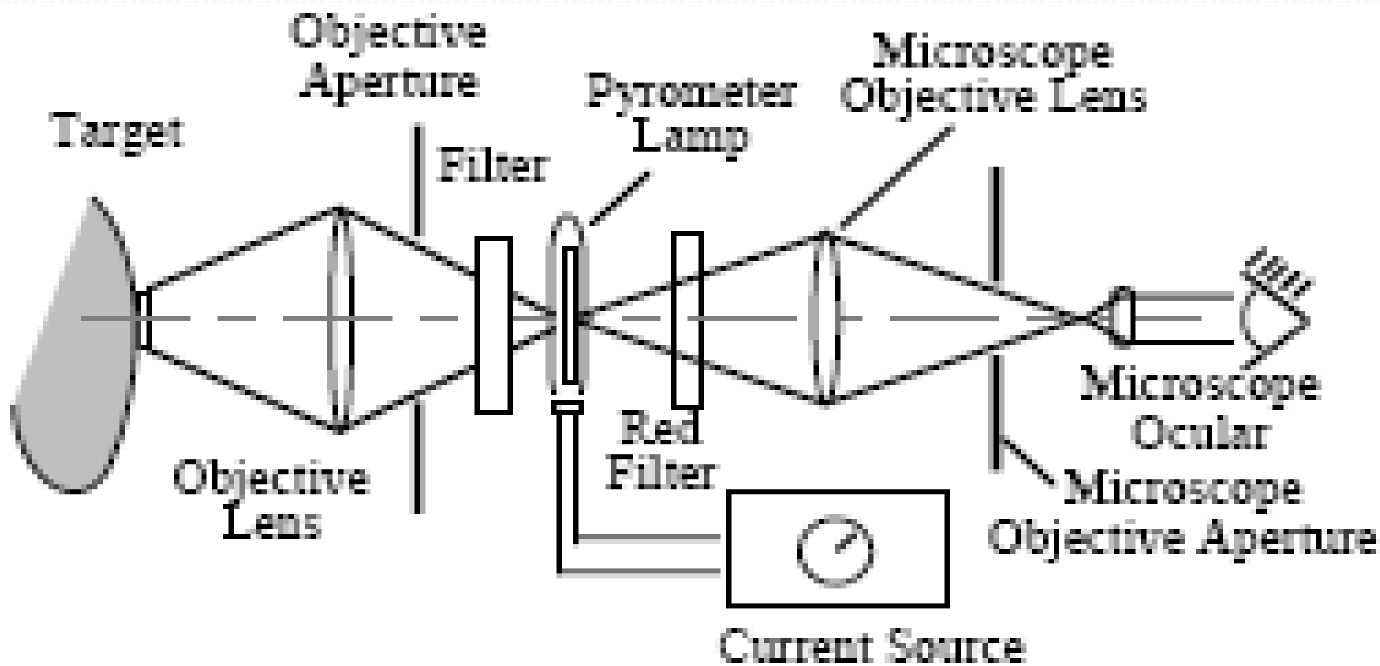
$$\dot{m} = \rho \cdot Q$$

$$\boxed{\rho_{use} \cdot Q_{use} = \rho_{cal} \cdot Q_{cal} \sqrt{\frac{\rho_{use}}{\rho_{cal}}}}$$

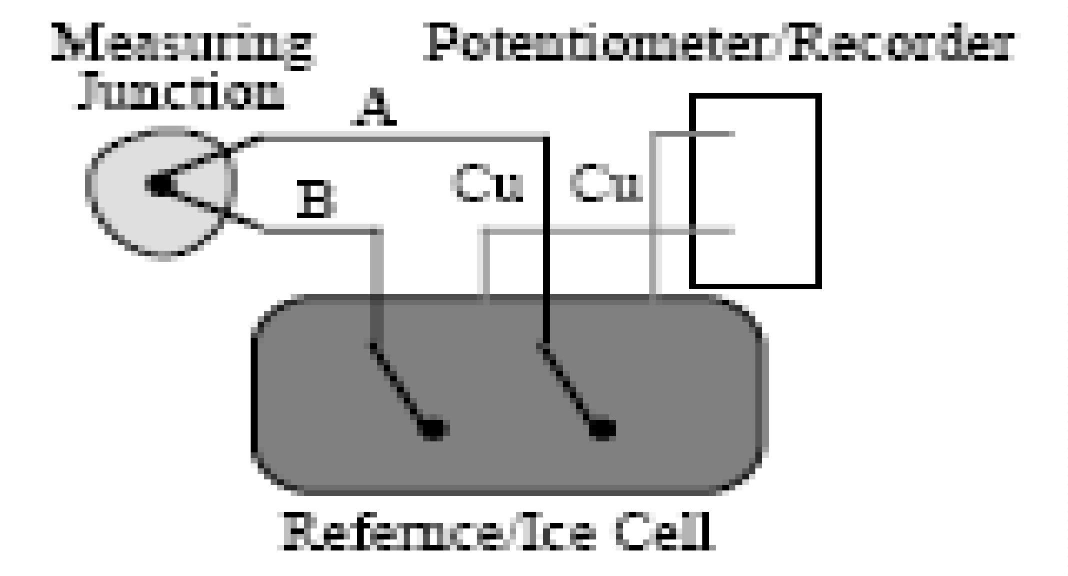
FLOWMETER SELECTION

Flowmeter element	Recommended Service	Range	Pressure loss	Typical Accuracy, %	L (Dia.)	Cost
Orifice	Clean, dirty liquids; some slurries	4 to 1	Medium	±2 to ±4 of full scale	10 to 30	Low
Wedge	Slurries and Viscous liquids	3 to 1	Low tomedium	±0.5 to ±2 of full scale	10 to 30	High
Venturi tube	Clean, dirty and viscous liquids;	4 to 1	Low	±1 of full scale	5 to 20	Medium
Flow nozzle	Clean and dirty liquids	4 to 1	Medium	±1 to ±2 of full scale	10 to 30	Medium
Pitot tube	Clean liquids	3 to 1	Very low	±3 to ±5 of full scale	20 to 30	Low
Elbow meter	Clean, dirty liquids; some slurries	3 to 1	Very low	±5 to ±10 of full scale	30	Low
Target meter	Clean, dirty viscous liquids;	10 to 1	Medium	±1 to ±5 of full scale	10 to 30	Medium
Variable area	Clean, dirty viscous liquids	10 to 1	Medium	±1 to ±10 of full scale	None	Low
Positive Displacement	Clean, viscous liquids	10 to 1	High	±0.5 of rate	None	Medium
Turbine	Clean, viscous liquids	20 to 1	High	±0.25 of rate	5 to 10	High
Vortex	CLean, dirty liquids	10 to 1	Medium	±1 of rate	10 to 20	High
Electromagnetic	Clean, dirty viscous conductive liquids& slurries	40 to 1	None	±0.5 of rate	5	High
Ultrasonic (Doppler)	Dirty, viscous liquids and slurries	10 to 1	None	±5 of full scale	5 to 30	High
Ultrasonic(Travel Time)	Clean, viscous liquids	20 to 1	None	±1 to ±5 of full scale	5 to 30	High
Mass (Coriolis)	Clean, dirty viscous liquids; some slurries	10 to 1	Low	±0.4 of rate	None	High
Mass (Thermal)	Clean, dirty viscous liquids; some slurries	10 to 1	Low	±1 of full scale	None	High
Weir (V-notch)	Clean, dirty liquids	100 to 1	Very low	±2 to ±5 of full scale	None	Medium
Flume (Parshall)	Clean, dirty liquids	50 to 1	Very low	±2 to ±5 of full scale	None	Medium

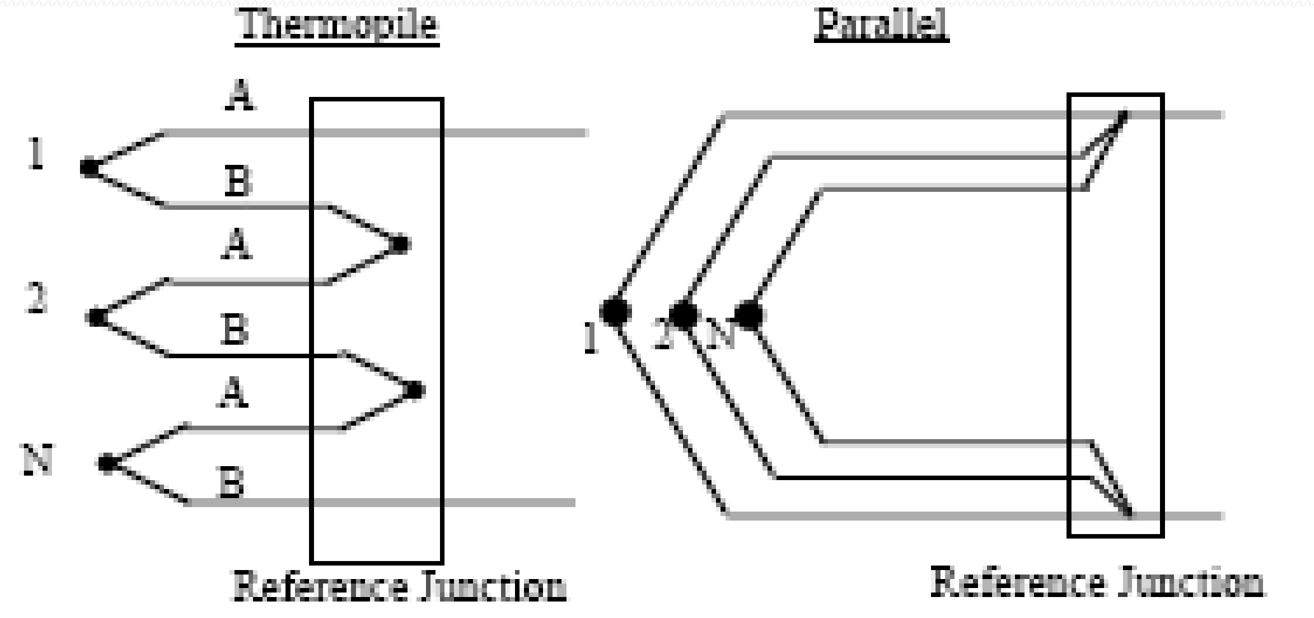
Optical Pyrometer



Thermocouple



Thermocouples in Series and in Parallel



THERMOCOUPLE TIME CONSTANT

- The conservation of energy:

$$\mathbf{m\, c_p\, dT / dt = h\, A\, (T_o - T)}$$

m : mass of thermocouple junction, C_p: specific heat of thermocouple junction

h : heat transfer coefficient , A : surface area of thermocouple

T : junction temperature , T_o : environs temperature

$$\boldsymbol{\theta = T - T_o / T_i - T_o}$$

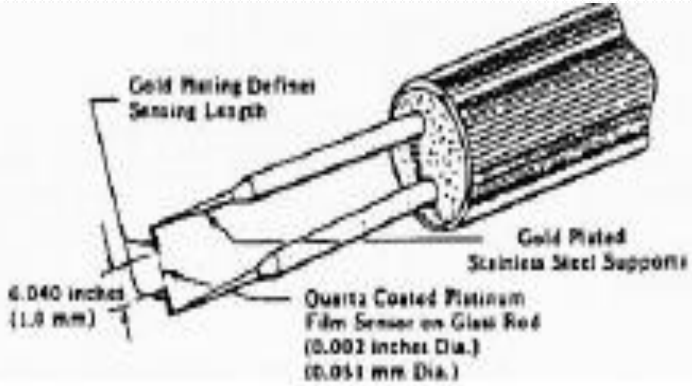
T_i = initial measurement junction temperature, then the solution is

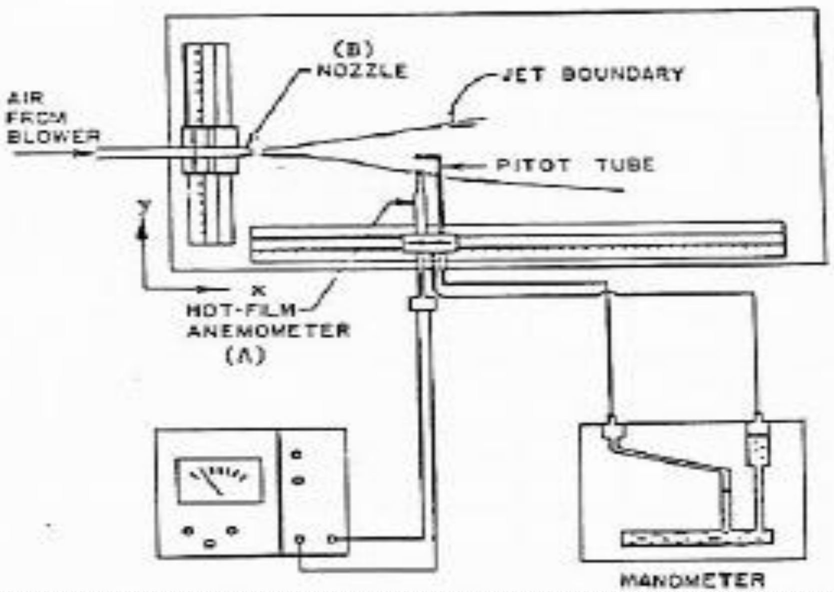
$$\boldsymbol{\theta = e^{(-t / \tau)}}$$

where we have defined the time constant for this process as

$$\boldsymbol{\tau = m\, c_p / h\, A}$$

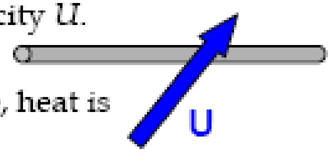
Hot Wire


Gold Plating Defines Sensing Length
Gold Plated Stainless Steel Supports
0.040 inches (1.0 mm)
Quartz Coated Platinum Film Sensor on Glass Rod (0.002 inches Dia.) (0.051 mm Dia.)


(B) NOZZLE
JET BOUNDARY
AIR FROM BLOWER
PITOT TUBE
Y
X
HOT-FILM ANEMOMETER (A)
MANOMETER

King's Law

- Consider a thin wire exposed to a velocity U.



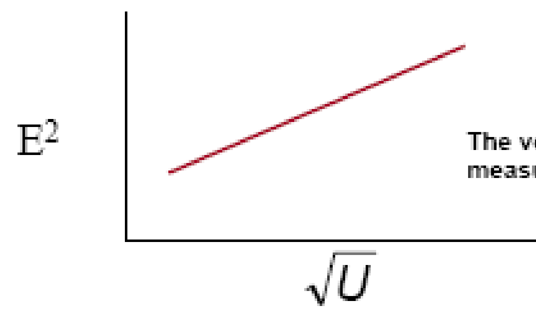
- When a current is passed through wire, heat is generated (I^2R_w).
- Heat generated is balanced by heat loss (primarily convective) to the surroundings.

$$I^2 R = hA(T_{wire} - T_{fluid})$$

- If velocity changes, convective heat transfer coefficient (h) will change, wire temperature will change and eventually reach a new equilibrium.

$$I^2 R = hA(T_{wire} - T_{fluid})$$

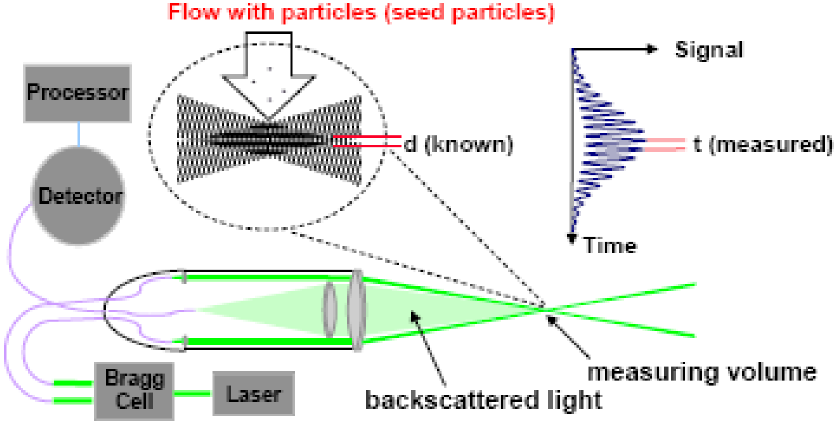
$$\boxed{I^2 R = (A + B\sqrt{U})(T_{wire} - T_{fluid})}$$



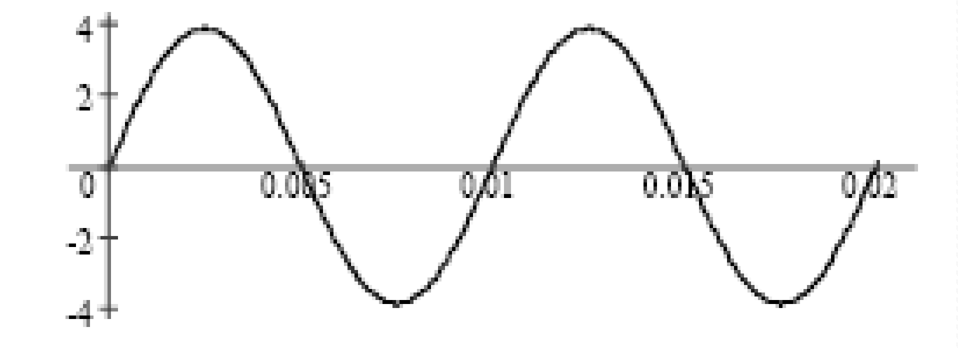
The voltage drop is used as a measure of velocity.

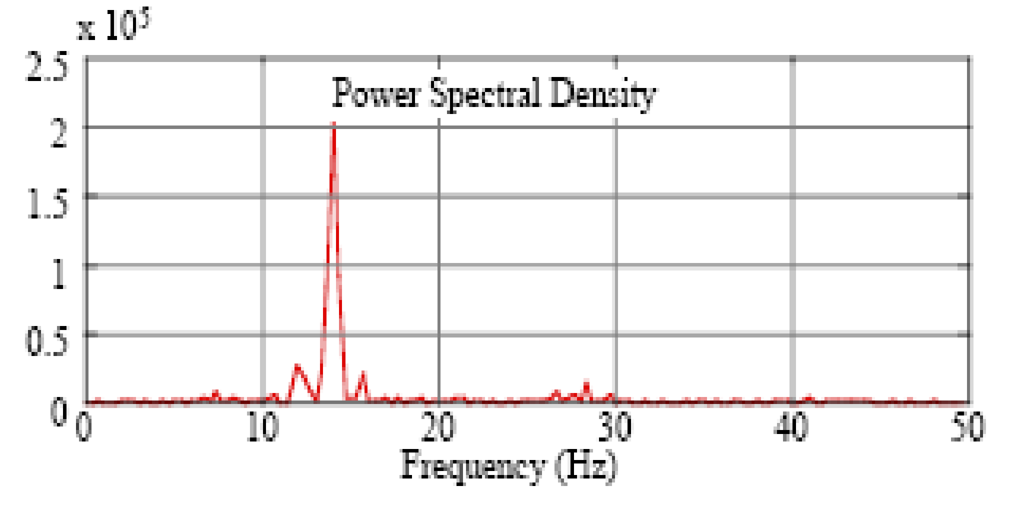
Laser Doppler Anemometer

- Laminar and turbulent flows
- Investigations on aerodynamics
- Supersonic flows (to Mach 8)
- Boundary layer flows
- Turbines, automotive etc.
- Liquid flows
- Surface velocity and vibration measurement
- Hot environments (Flames, Plasma etc.)
- Highly corrosive environments
- Velocity of particles

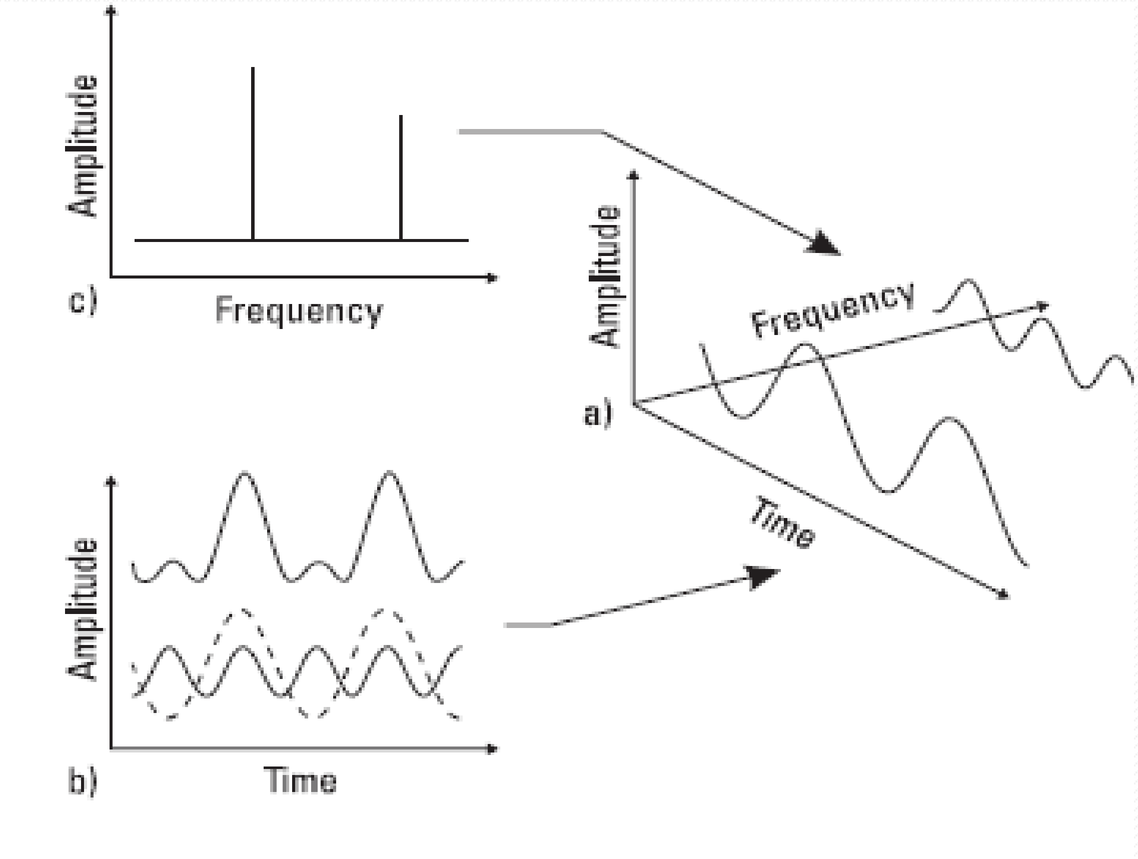


Periodic Wave and its Spectrum

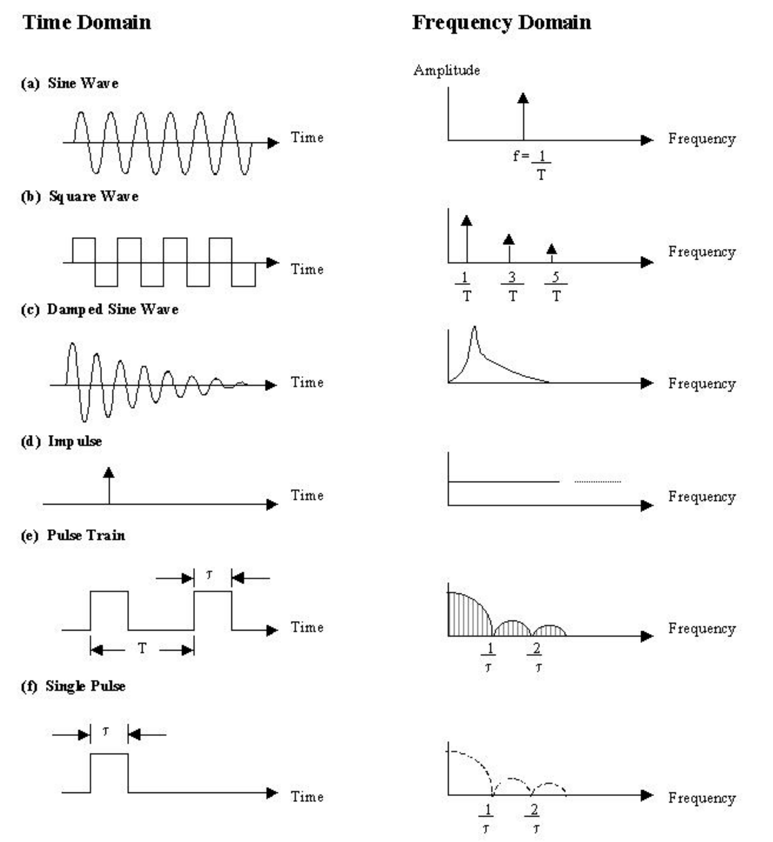




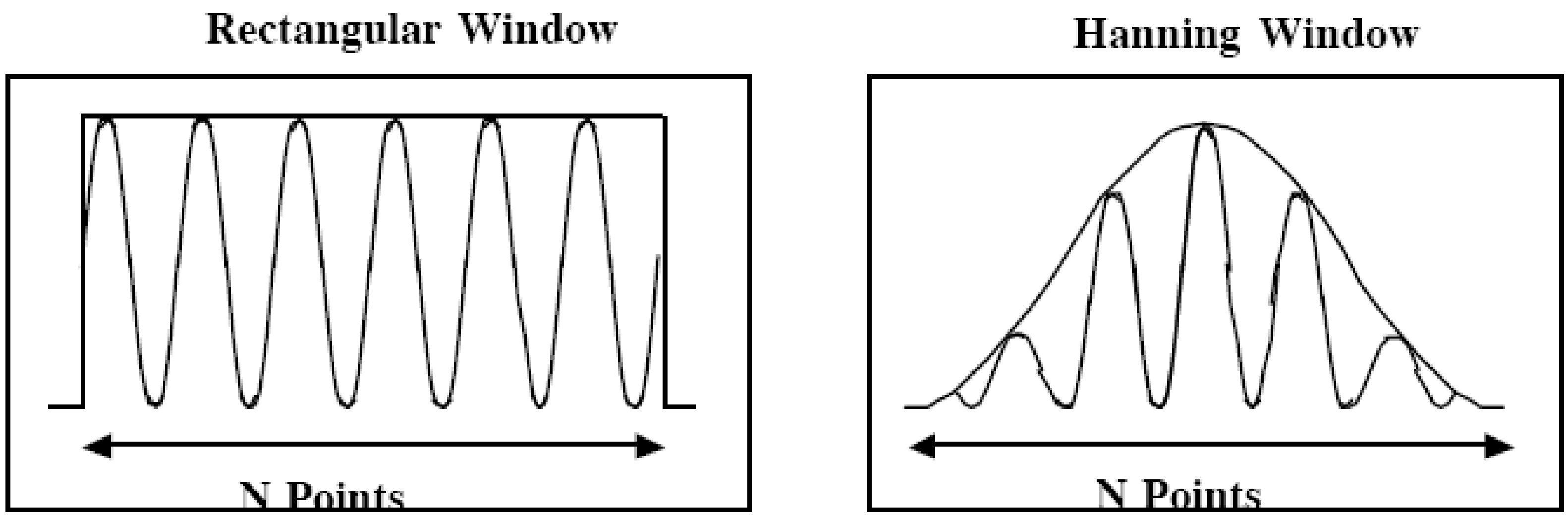
Time Domain & Freq. Domain



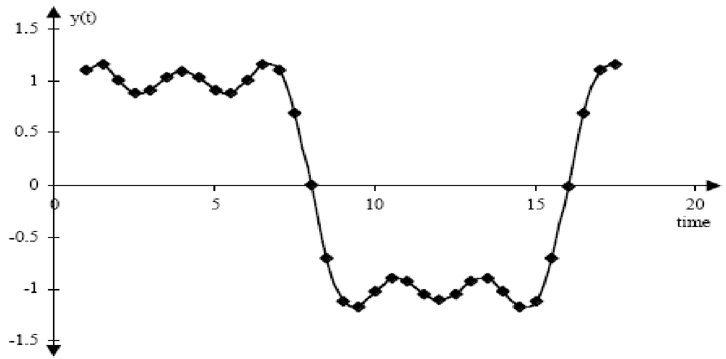
frequency spectrum examples

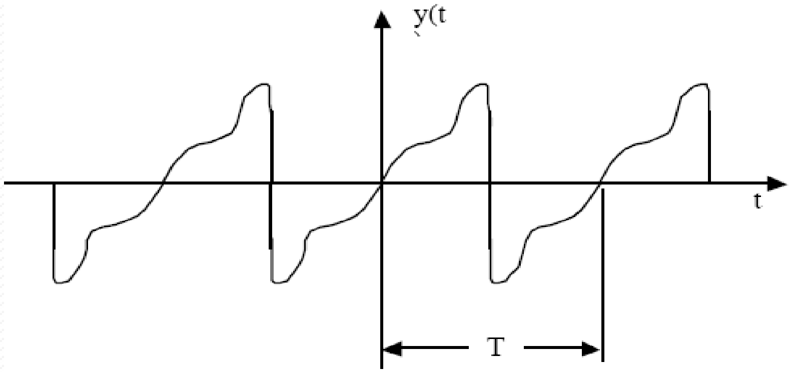


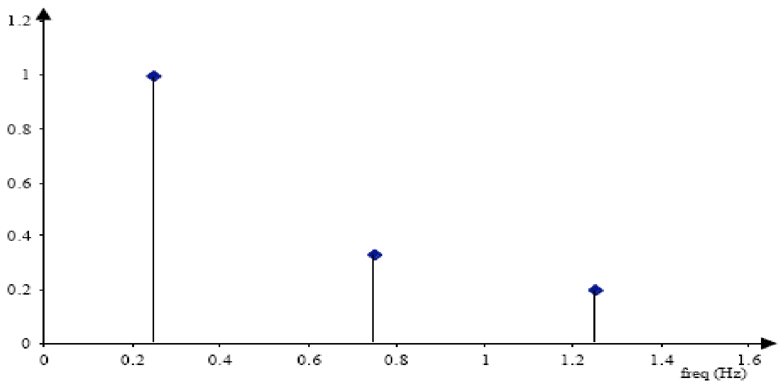
Square and Hanning window functions

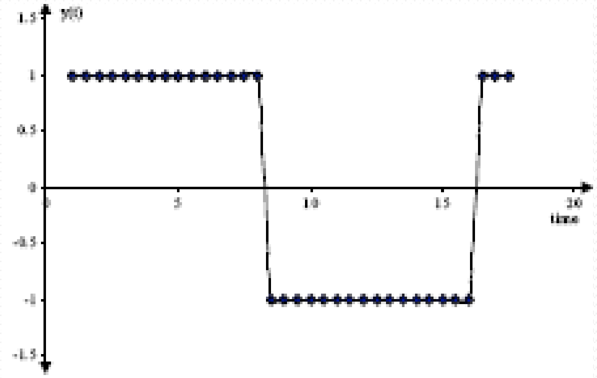


Periodic Signals

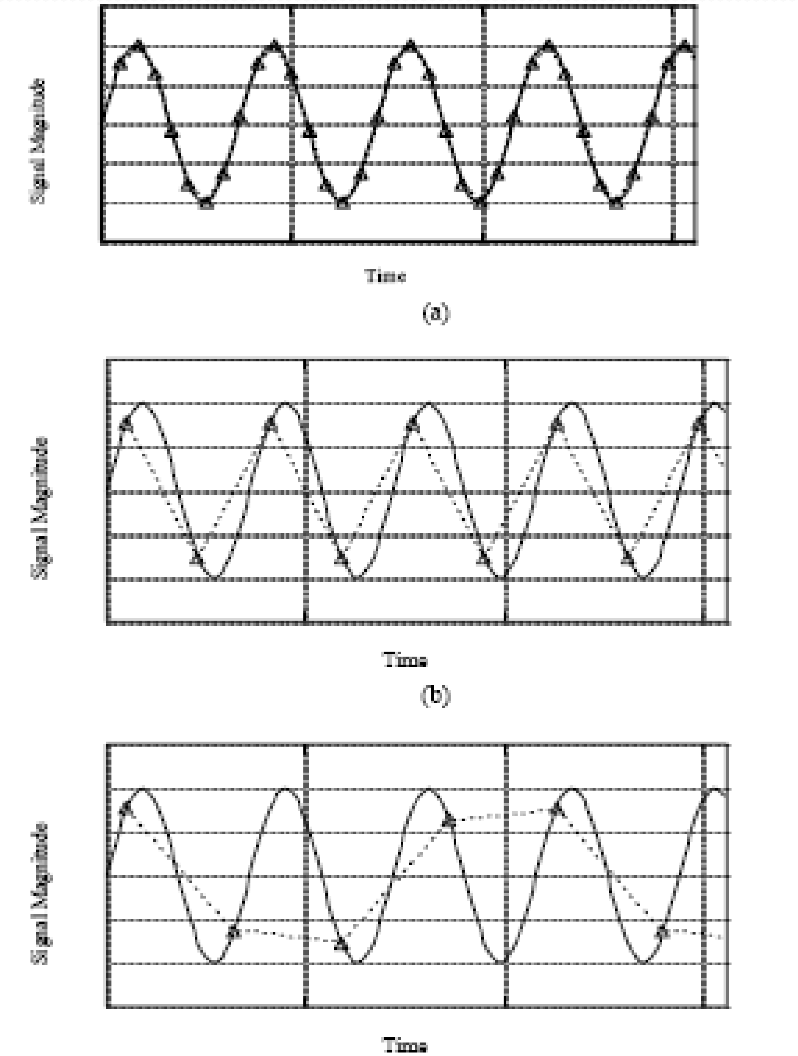




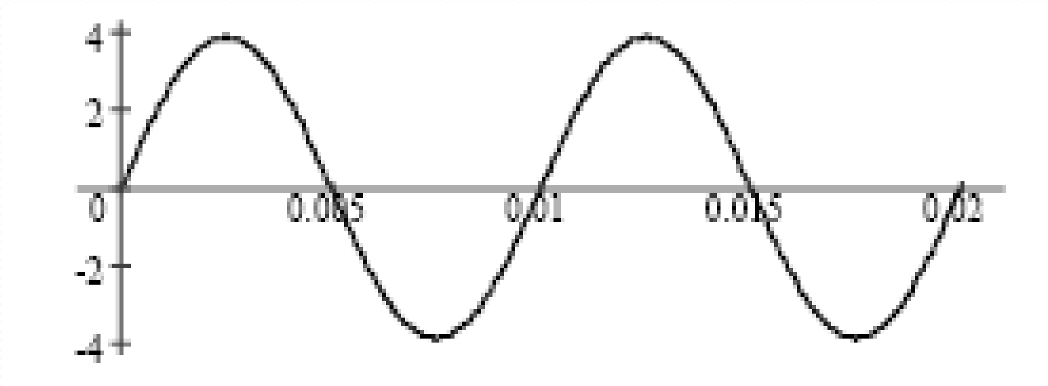


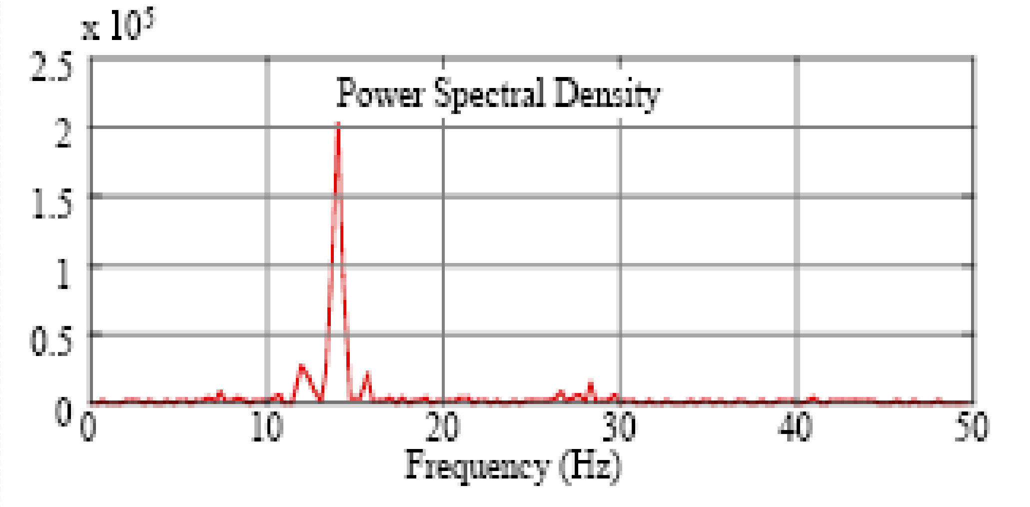


Sine Wave Digitising

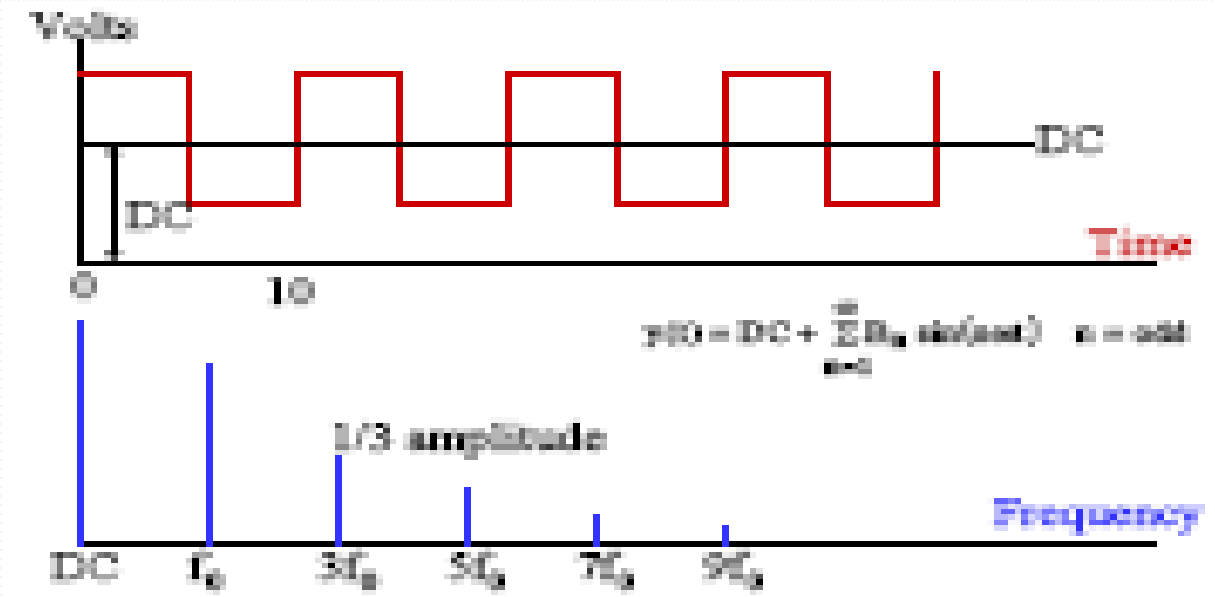


Periodic Wave and its Spectrum

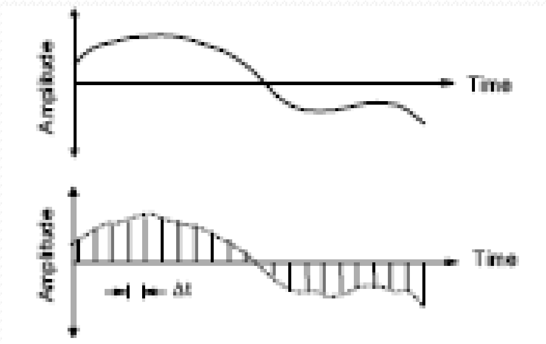




Square Wave and its Spectrum

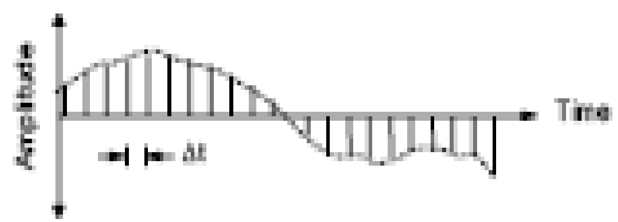


Analog and Digital Signals



Digitization of an analog signal requires two separate operations.

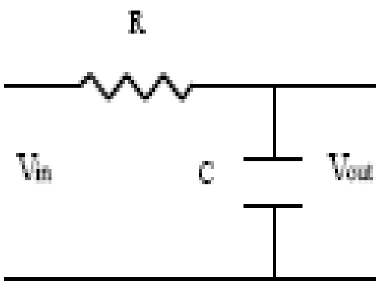
- **Sampling**-rate at which data is acquired.
- **Quantization**-conversion into useful form.



- Alternatively, the resolution can be expressed as half of the bin value on either side.
  - **Quantization error/Quantizing error**

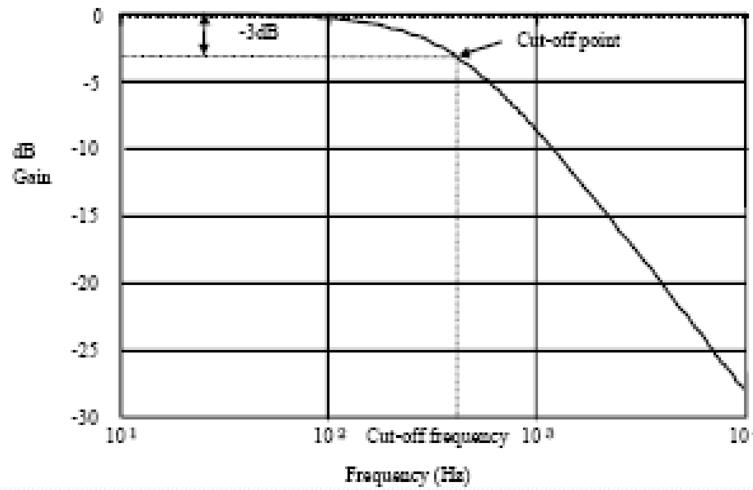
$$\text{Quantization error} = \pm\frac{1}{2}\frac{(V_{max}-V_{min})}{2^M}$$

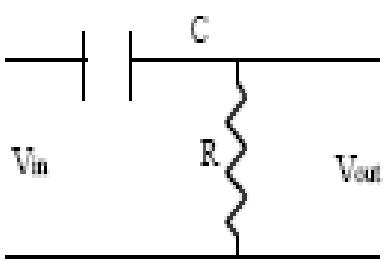
Analog RC Filtering



$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

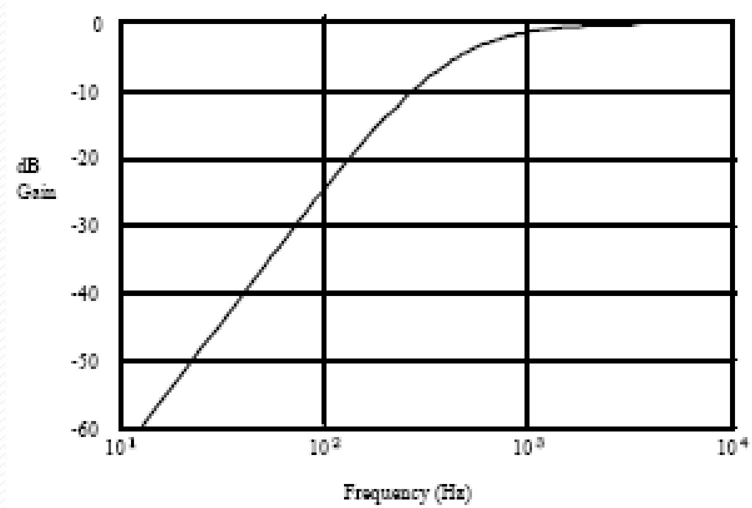
(a) Low pass filter





$$\left|\frac{V_o}{V_i}\right| = \sqrt{\frac{(\omega RC)^2}{1+(\omega RC)^2}}$$

(b) High pass filter



Sampling and Aliasing error

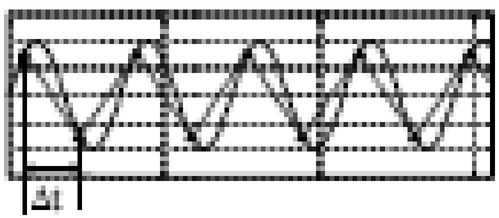
- Nyquist Criterion

$$\Delta t = 1/f_s = 1/2f_{max}$$

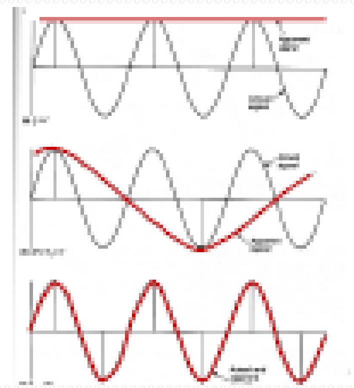
- If we sample too slow then the high and low frequency data may be confused--aliasing.
  - Recall old westerns when wheel appear to be turning backwards. Aliasing problem of camera.

$$f_{sample} \geq 2 \cdot f_{max}$$

$$\Delta t = \frac{1}{2f_{max}}$$



- Looks like DC
- Aliasing Error- frequency appears lower than actual.
- JUST RIGHT



Resolution of an A/D Converter

- Resolution of an A/D converter is a function of the number of bits (M) and the range.

$$\boxed{\text{Resolution} = \frac{(V_{max} - V_{min})}{2^M} = \frac{\text{Full Scale Range}}{2^M}}$$

Number of bins=2^M

- 12-bit resolution for 0-10 V range $\frac{10\text{V}}{2^{12}} = \frac{10\text{V}}{4096} = 2.44\,\text{mV}$
- 16-bit resolution -10 to 10 V range $\frac{20\text{V}}{2^{16}} = 0.305\,\text{mV}$

- Alternatively, the resolution can be expressed as half of the bin value on either side.
  - Quantization error/Quantizing error

$$\text{Quantization eror} = \pm\frac{1}{2}\frac{(V_{max} - V_{min})}{2^M}$$

Experimental Design and Analysis

- Simple Comparative Experiment.

One Factor: t-Test (2-levels or treatments)

- Used to tell you if a significant difference exists between two treatments.
- Assumes each sample is independent, variances are not equal ($\sigma_1^2 \neq \sigma_2^2$), and normally distributed.
- Test the hypothesis
  - H_0 (null) →Samples are from the same population
  - **Same until proven different!**

H_0(null)	Criteria to Reject
$H_0 : \mu_1 = \mu_2$	$\lvert t_0 \rvert > \lvert t_{\alpha,\nu} \rvert$

- Is there enough evidence to say there are differences between cases or treatments?
  - Is the variation in means, caused by treatments, unexpectedly large, relative to what you would expect from random error?
- Analysis of Variance (ANOVA)
  - Assume no difference, the "null hypothesis"
  - Determine what observations look like when the null hypothesis is true.
    - This is the "reference distribution" ("t" or "F" Distribution)
  - Compute appropriate test statistic to compare current observations to reference distribution
    - t-test or F-test

- $\bar{x}$, s, and n are the
  - mean,
  - standard deviation,
  - and size of each treatment.
- Determine $t = f(\alpha, \upsilon)$ based on desired confidence interval.
  - Typically, $\alpha = 0.05$
  - Lookup Tables or use Excel Function-TINV
- Compare t_0 to t
  - Reject or Accept hypothesis?

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

$$\nu = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(\dfrac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \dfrac{\left(\dfrac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

Example

- The diameter of a ball bearing is measured with two different kinds of calipers, 12x each.

Reading	Caliper 1	Caliper 2
1	0.265	0.264
2	0.265	0.265
3	0.266	0.264
4	0.267	0.266
5	0.267	0.267
6	0.265	0.268
7	0.267	0.264
8	0.267	0.265
9	0.265	0.265
10	0.268	0.267
11	0.268	0.268
12	0.265	0.269
Count, n	12	12
Mean	0.2663	0.2660
St.Dev	0.0012	0.0018
Variance	0.00000148	0.00000309
t_0	0.4052	
v	20	Compare
t	2.086	

- Result
  - Is $|t_0| > |t|$ --no
  - **Accept H_0**
    - Same

- F Tests
  - Unlike the t-test, this test can compare two or more treatments (k=number of treatments).
  - Just as in the t-test, the F-test is used to test the null hypothesis that population variances corresponding to the samples are equal.
    - **Again – Same until proven different!**
    - Compares variance among the treatments versus the variance of the individuals within each of the treatments.

$$F \cong \frac{n \cdot s_{\bar{y}}^2}{s^2_{pooled}} \approx \frac{Signal}{Noise}$$