# MENG353 - FLUID MECHANICS

SOURCE: FUNDAMENTALS OF FLUIDMECHANICS
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CHAPTER 1 FUNDAMENTALS

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# Learning Objectives

After completing this chapter, you should be able to:

- list the dimensions and units of physical quantities.
- identify the key fluid properties used in the analysis of fluid behavior.
- calculate values for common fluid properties given appropriate information.
- explain effects of fluid compressibility.
- use the concepts of viscosity, vapor pressure, and surface tension.

# 1.1 SOME CHARACTERISTICS OF FLUIDS

- A **fluid** is defined as:

  "A substance that continually deforms (flows) under an applied **shear stress** regardless of the magnitude of the applied stress".

- It is a subset of the phases of matter and includes liquids, gases, plasmas and, to some extent, plastic solids.

# Distinction between a Solid and a Fluid

| Solid | Fluid |
|---|---|
| - *Definite Shape* and *definite volume.*<br>- *Does not flow easily.*<br>- *Molecules* are closer.<br>- *Attractive forces* between the molecules are large enough to retain its shape.<br>- An **ideal Elastic Solid** deform under load and comes back to original position upon removal of load.<br>- **Plastic Solid** does not comes back to original position upon removal of load, means permanent deformation takes place. | - *Indefinite Shape* and *Indefinite volume & it assumes the shape* of the container which it occupies.<br>- *Flow Easily.*<br>- *Molecules* are far apart.<br>- *Attractive forces* between the molecules are smaller.<br>- *Intermolecular cohesive forces* in a fluid are not great enough to hold the various elements of fluid together. Hence Fluid will flow under the action of applied stress. The flow will be continuous as long as stress is applied. |

# Distinction between a Gas and Liquid

- The molecules of a **gas** are much farther apart than those of a liquid.
- Hence a gas is very compressible, and when all external pressure is removed, it tends to expand indefinitely.
- A gas is therefore in equilibrium only when it is completely enclosed.

- A **liquid** is relatively incompressible.
- If all pressure, except that of its own vapor pressure, is removed, the cohesion between molecules holds them together, so that the liquid does not expand indefinitely.
- Therefore a liquid may have a free surface.

## 1.2 Dimensions, Dimensional Homogeneity, and Units

FLUID CHARACTERISTICS CAN BE DESCRIBED QUALITATIVELY IN TERMS OF CERTAIN BASIC QUANTITIES SUCH AS LENGTH, TIME, AND MASS.

# FUNDAMENTAL UNITS

- Fundamental units are the units which are not dependent with any other units like length, mass, time, electric current, thermodynamic temperature, amount of substance, or luminous intensity in the International System of Units, consisting respectively of the meter, kilogram, second, ampere, kelvin, mole etc are called fundamental units. Some examples are as follows :-
- length is fundamental unit,meter is it's SI unit, m is it's symbol
- mass is fundamental unit,kilo gram is it's SI unit, kg is it's symbol
- time is fundamental unit,second is it's SI unit, s is it's symbol
- electric current is fundamental unit,ampere is it's SI unit, A is it's symbol
- temperature is fundamental unit,kelvin is it's SI unit, K is it's symbol
- amount of substance is fundamental unit,mole is it's SI unit, mol is it's symbol
- luminous intensity is fundamental unit,candela is it's SI unit, cd is it's symbol

# The 7 Fundamental SI Units

| | Physical quantity | unit | abbreviation |
|---|---|---|---|
| 1 | Mass | kilogram | [kg] |
| 2 | Length | meter | [m] |
| 3 | Time | second | [s] |
| 4 | Temperature | Kelvin | [K] |
| 5 | Amount of substance | mole | [mol] |
| 6 | Electric current | ampere | [A] |
| 7 | Luminous intensity | Candela | [cd] |

**Table 1.1**

**Dimensions Associated with Common Physical Quantities**

| | *FLT* System | *MLT* System |
|---|---|---|
| Acceleration | $LT^{-2}$ | $LT^{-2}$ |
| Angle | $F^0L^0T^0$ | $M^0L^0T^0$ |
| Angular acceleration | $T^{-2}$ | $T^{-2}$ |
| Angular velocity | $T^{-1}$ | $T^{-1}$ |
| Area | $L^2$ | $L^2$ |
| Density | $FL^{-4}T^2$ | $ML^{-3}$ |
| Energy | $FL$ | $ML^2T^{-2}$ |
| Force | $F$ | $MLT^{-2}$ |
| Frequency | $T^{-1}$ | $T^{-1}$ |
| Heat | $FL$ | $ML^2T^{-2}$ |
| Length | $L$ | $L$ |
| Mass | $FL^{-1}T^2$ | $M$ |
| Modulus of elasticity | $FL^{-2}$ | $ML^{-1}T^{-2}$ |
| Moment of a force | $FL$ | $ML^2T^{-2}$ |
| Moment of inertia (area) | $L^4$ | $L^4$ |
| Moment of inertia (mass) | $FLT^2$ | $ML^2$ |
| Momentum | $FT$ | $MLT^{-1}$ |

| | *FLT* System | *MLT* System |
|---|---|---|
| Power | $FLT^{-1}$ | $ML^2T^{-3}$ |
| Pressure | $FL^{-2}$ | $ML^{-1}T^{-2}$ |
| Specific heat | $L^2T^{-2}\Theta^{-1}$ | $L^2T^{-2}\Theta^{-1}$ |
| Specific weight | $FL^{-3}$ | $ML^{-2}T^{-2}$ |
| Strain | $F^0L^0T^0$ | $M^0L^0T^0$ |
| Stress | $FL^{-2}$ | $ML^{-1}T^{-2}$ |
| Surface tension | $FL^{-1}$ | $MT^{-2}$ |
| Temperature | $\Theta$ | $\Theta$ |
| Time | $T$ | $T$ |
| Torque | $FL$ | $ML^2T^{-2}$ |
| Velocity | $LT^{-1}$ | $LT^{-1}$ |
| Viscosity (dynamic) | $FL^{-2}T$ | $ML^{-1}T^{-1}$ |
| Viscosity (kinematic) | $L^2T^{-1}$ | $L^2T^{-1}$ |
| Volume | $L^3$ | $L^3$ |
| Work | $FL$ | $ML^2T^{-2}$ |

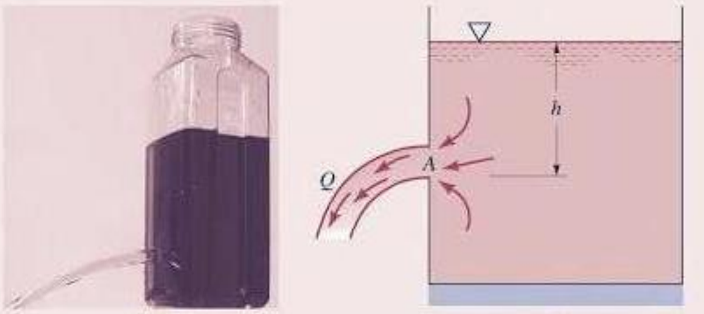
## EXAMPLE 1.1 Restricted and General Homogeneous Equations

**GIVEN** A liquid flows through an orifice located in the side of a tank as shown in Fig. E1.1. A commonly used equation for determining the volume rate of flow, $Q$, through the orifice is

$$Q = 0.61\, A\sqrt{2gh}$$

where $A$ is the area of the orifice, $g$ is the acceleration of gravity, and $h$ is the height of the liquid above the orifice.

**FIND** Investigate the dimensional homogeneity of this formula.




(a) (b)

Figure E1.1

### SOLUTION

The dimensions of the various terms in the equation are $Q =$ volume/time $\doteq L^3T^{-1}$, $A =$ area $\doteq L^2$, $g =$ acceleration of gravity $\doteq LT^{-2}$, and $h =$ height $\doteq L$.

These terms, when substituted into the equation, yield the dimensional form:

$$(L^3T^{-1}) \doteq (0.61)(L^2)(\sqrt{2})(LT^{-2})^{1/2}(L)^{1/2}$$

or

$$(L^3T^{-1}) \doteq [0.61\sqrt{2}](L^3T^{-1})$$

It is clear from this result that the equation is dimensionally homogeneous (both sides of the formula have the same dimensions of $L^3T^{-1}$), and the number $0.61\sqrt{2}$ is dimensionless.

If we were going to use this relationship repeatedly, we might be tempted to simplify it by replacing $g$ with its standard value of 32.2 ft/s$^2$ and rewriting the formula as

$$Q = 4.90\, A\sqrt{h} \qquad (1)$$

A quick check of the dimensions reveals that

$$L^3T^{-1} \doteq (4.90)(L^{5/2})$$

and, therefore, the equation expressed as Eq. 1 can only be dimensionally correct if the number 4.90 has the dimensions of $L^{1/2}T^{-1}$. Whenever a number appearing in an equation or formula has dimensions, it means that the specific value of the number will depend on the system of units used. Thus, for the case being considered with feet and seconds used as units, the number 4.90 has units of ft$^{1/2}$/s. Equation 1 will only give the correct value for $Q$ (in ft$^3$/s) when $A$ is expressed in square feet and $h$ in feet. Thus, Eq. 1 is a *restricted* homogeneous equation, whereas the original equation is a *general* homogeneous equation that would be valid for any consistent system of units.

**COMMENT** A quick check of the dimensions of the various terms in an equation is a useful practice and will often be helpful in eliminating errors—that is, as noted previously, all physically meaningful equations must be dimensionally homogeneous. We have briefly alluded to units in this example, and this important topic will be considered in more detail in the next section.

# SI Units

| Quantity | Basic Definition | Standard SI Units | Other Units Often Used |
|---|---|---|---|
| Length | — | meter (m) | millimeter (mm); kilometer (km) |
| Time | — | second (s) | hour (h); minute (min) |
| Mass | Quantity of a substance | kilogram (kg) | $N \cdot s^2/m$ |
| Force or weight | Push or pull on an object | newton (N) | $kg \cdot m/s^2$ |
| Pressure | Force/area | $N/m^2$ or pascal (Pa) | kilopascals (kPa); bar |
| Energy | Force times distance | $N \cdot m$ or Joule (J) | $kg \cdot m^2/s^2$ |
| Power | Energy/time | $N \cdot m/s$ or J/s | watt (W); kW |
| Volume | $(\text{Length})^3$ | $m^3$ | liter (L) |
| Area | $(\text{Length})^2$ | $m^2$ | $mm^2$ |
| Volume flow rate | Volume/time | $m^3/s$ | L/s; L/min; $m^3/h$ |
| Weight flow rate | Weight/time | N/s | kN/s; kN/min |
| Mass flow rate | Mass/time | kg/s | kg/h |
| Specific weight | Weight/volume | $N/m^3$ | $kg/m^2 \cdot s^2$ |
| Density | Mass/volume | $kg/m^3$ | $N \cdot s^2/m^4$ |

# FPS Units

| Quantity | Basic Definition | Standard U.S. Units | Other Units Often Used |
|---|---|---|---|
| Length | — | feet (ft) | inches (in); miles (mi) |
| Time | — | second (s) | hour (h); minute (min) |
| Mass | Quantity of a substance | slugs | $lb \cdot s^2/ft$ |
| Force or weight | Push or pull on an object | pound (lb) | kip (1000 lb) |
| Pressure | Force/area | $lb/ft^2$ or psf | $lb/in^2$ or psi; $kip/in^2$ or ksi |
| Energy | Force times distance | $lb \cdot ft$ | $lb \cdot in$ |
| Power | Energy/time | $lb \cdot ft/s$ | horsepower (hp) |
| Volume | $(Length)^2$ | $ft^3$ | gallon (gal) |
| Area | $(Length)^3$ | $ft^2$ | $in^2$ |
| Volume flow rate | Volume/time | $ft^3/s$ or cfs | gal/min (gpm); $ft^3/min$ (cfm) |
| Weight flow rate | Weight/time | lb/s | lb/min; lb/h |
| Mass flow rate | Mass/time | slugs/s | slugs/min; slugs/h |
| Specific weight | Weight/volume | $lb/ft^3$ | |
| Density | Mass/volume | $slugs/ft^3$ | |

# 1.3 ANALYSIS OF FLUID BEHAVIOR

The study of fluid mechanics involves the same fundamental laws you have encountered in physics and other mechanics courses. These laws include Newton's laws of motion, conservation of mass, and the first and second laws of thermodynamics. Thus, there are strong similarities between the general approach to fluid mechanics and to rigid-body and deformable-body solid mechanics. This is indeed helpful since many of the concepts and techniques of analysis used in fluid mechanics will be ones you have encountered before in other courses.

The broad subject of fluid mechanics can be generally subdivided into *fluid statics,* in which the fluid is at rest, and *fluid dynamics*, in which the fluid is moving. In the following chapters we will consider both of these areas in detail. Before we can proceed, however, it will be necessary to define and discuss certain fluid *properties* that are intimately related to fluid behavior. It is obvious that different fluids can have grossly different characteristics. For example, gases are light and compressible, whereas liquids are heavy (by comparison) and relatively incompressible. A syrup flows slowly from a container, but water flows rapidly when poured from the same container. To quantify these differences, certain fluid properties are used. In the following several sections, properties that play an important role in the analysis of fluid behavior are considered.

# 1.4 MEASURES OF FLUID MASS AND WEIGHT

### 1.4.1 Density

The ***density*** of a fluid, designated by the Greek symbol $\rho$ (rho), is defined as its mass per unit volume. Density is typically used to characterize the mass of a fluid system. In the BG system, $\rho$ has units of $slugs/ft^3$ and in SI the units are $kg/m^3$.

The *specific volume, $v$*, is the *volume* per unit mass and is therefore the reciprocal of the density—that is,

$$v = \frac{1}{\rho} \tag{1.5}$$

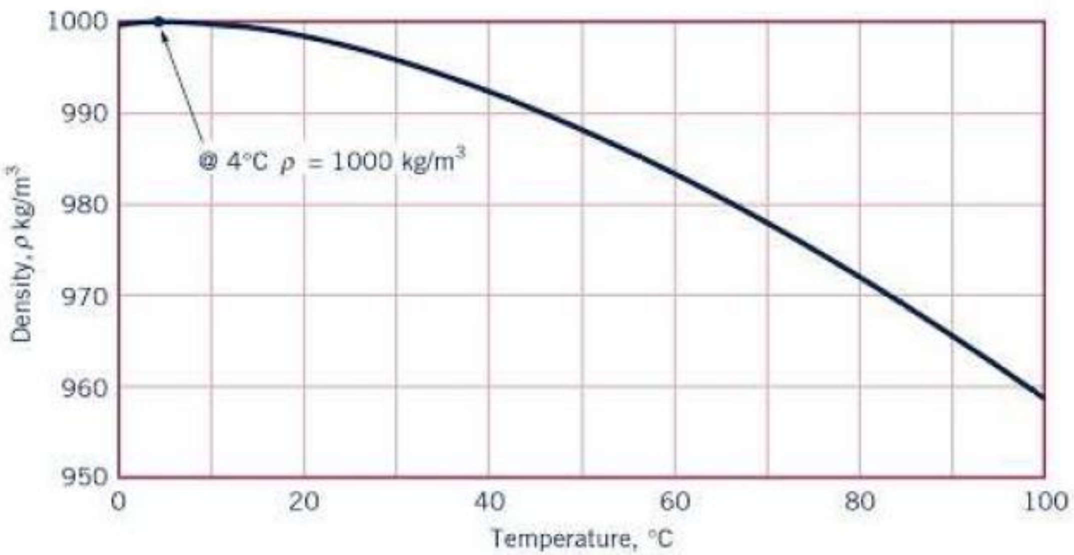



**Figure 1.3** **Density of water as a function of temperature.**

### 1.4.2 Specific Weight

The ***specific weight*** of a fluid, designated by the Greek symbol $\gamma$ (gamma), is defined as its *weight* per unit volume. Thus, specific weight is related to density through the equation

$$\gamma = \rho g \tag{1.6}$$

### 1.4.3 Specific Gravity

The ***specific gravity*** of a fluid, designated as $SG$, is defined as the ratio of the density of the fluid to the density of water at some specified temperature. Usually the specified temperature is taken as 4 °C (39.2 °F), and at this temperature the density of water is 1.94 slugs/ft$^3$ or 1000 kg/m$^3$. In equation form, specific gravity is expressed as

$$SG = \frac{\rho}{\rho_{H_2O@4\,°C}} \tag{1.7}$$

$$\rho_{Hg} = (13.55)(1.94\ \text{slugs/ft}^3) = 26.3\ \text{slugs/ft}^3$$

$$\rho_{Hg} = (13.55)(1000\ \text{kg/m}^3) = 13.6 \times 10^3\ \text{kg/m}^3$$

## 1.5 Ideal Gas Law

Gases are highly compressible in comparison to liquids, with changes in gas density directly related to changes in pressure and temperature through the equation

$$\rho = \frac{p}{RT}$$

or, in the more standard form,

$$p = \rho RT, \qquad (1.8)$$

Pressure in a fluid at rest is defined as the normal force per unit area exerted on a plane surface (real or imaginary) immersed in a fluid and is created by the bombardment of the surface with the fluid molecules. From the definition, pressure has the dimension of $FL^{-2}$ and in BG units is expressed as $lb/ft^2$ (psf) or $lb/in.^2$ (psi) and in SI units as $N/m^2$. In SI, 1 $N/m^2$ defined as a *pascal*, abbreviated as Pa, and pressures are commonly specified in pascals. The pressure in the ideal gas law must be expressed as an ***absolute pressure,*** denoted (abs), which means that it is measured relative to absolute zero pressure (a pressure that would only occur in a perfect vacuum). Standard sea-level atmospheric pressure (by international agreement) is 14.696 psi (abs) or 101.33 kPa (abs). For most calculations these pressures can be rounded to 14.7 psi and 101 kPa, respectively. In engineering it is common practice to measure pressure relative to the local atmospheric pressure, and when measured in this fashion it is called ***gage pressure.*** Thus, the absolute pressure can be obtained from the gage pressure by adding the value of the atmospheric pressure.

## EXAMPLE 1.3 Ideal Gas Law

**GIVEN** The compressed air tank shown in Fig. E1.3*a* has a volume of 0.84 ft$^3$. The temperature is 70 °F and the atmospheric pressure is 14.7 psi (abs).

**FIND** When the tank is filled with air at a gage pressure of 50 psi, determine the density of the air and the weight of air in the tank.

### SOLUTION

The air density can be obtained from the ideal gas law (Eq. 1.8)

$$\rho = \frac{p}{RT}$$

so that

$$\rho = \frac{(50\ \text{lb/in.}^2 + 14.7\ \text{lb/in.}^2)(144\ \text{in.}^2/\text{ft}^2)}{(1716\ \text{ft}\cdot\text{lb/slug}\cdot{}^\circ\text{R})[(70+460)\,{}^\circ\text{R}]}$$

$$= 0.0102\ \text{slugs/ft}^3 \qquad \text{(Ans)}$$

Note that both the pressure and temperature were changed to absolute values.

The weight, $\mathcal{W}$, of the air is equal to

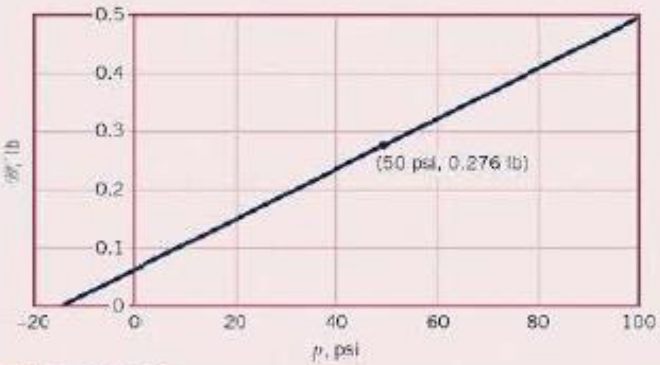
$$\mathcal{W} = \rho g \times (\text{volume})$$
$$= (0.0102\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(0.84\ \text{ft}^3)$$
$$= 0.276\ \text{slug}\cdot\text{ft/s}^2$$


■ Figure E1.3*a* **(Photograph courtesy of Jenny Products, Inc.)**

so that since 1 lb = 1 slug · ft/s$^2$

$$\mathcal{W} = 0.276\ \text{lb} \qquad \text{(Ans)}$$

**COMMENT** By repeating the calculations for various values of the pressure, $p$, the results shown in Fig. E1.3*b* are obtained. Note that doubling the gage pressure does not double the amount of air in the tank, but doubling the absolute pressure does. Thus, a scuba diving tank at a gage pressure of 100 psi does not contain twice the amount of air as when the gage reads 50 psi.




■ Figure E1.3*b*

## 1.6 Viscosity

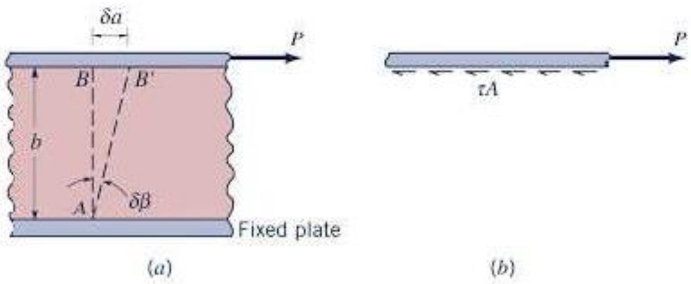



**Figure 1.4** (*a*) Deformation of material placed between two parallel plates. (*b*) Forces acting on upper plate.

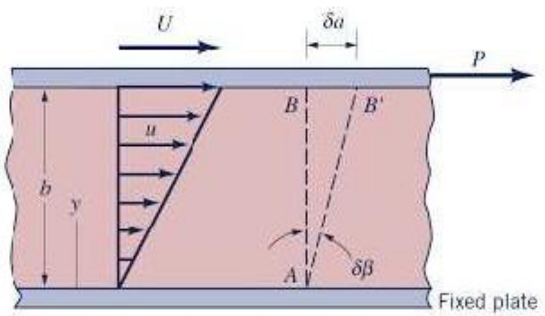
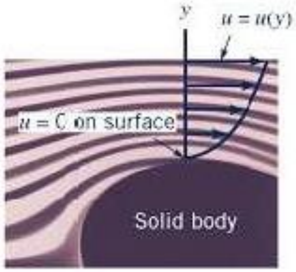



**Figure 1.5** Behavior of a fluid placed between two parallel plates.

The fluid between the two plates moves with velocity $u = u(y)$ that would be found to vary linearly, $u = Uy/b$, as illustrated in Fig. 1.5. Thus, a *velocity gradient, $du/dy$*, is developed in the fluid between the plates. In this particular case the velocity gradient is a constant since $du/dy = U/b$, but in more complex flow situations, such as that shown by the photograph in the margin, this is not true. The experimental observation that the fluid "sticks" to the solid boundaries is a very important one in fluid mechanics and is usually referred to as the ***no-slip condition***. All fluids, both liquids and gases, satisfy this condition for typical flows.




In a small time increment, $\delta t$, an imaginary vertical line $AB$ in the fluid would rotate through an angle, $\delta\beta$, so that

$$\delta\beta \approx \tan\delta\beta = \frac{\delta a}{b}$$

Since $\delta a = U\,\delta t$, it follows that

$$\delta\beta = \frac{U\,\delta t}{b}$$

We note that in this case, $\delta\beta$ is a function not only of the force $P$ (which governs $U$) but also of time. Thus, it is not reasonable to attempt to relate the shearing stress, $\tau$, to $\delta\beta$ as is done for solids. Rather, we consider the *rate* at which $\delta\beta$ is changing and define the ***rate of shearing strain***, $\dot{\gamma}$, as

$$\dot{\gamma} = \lim_{\delta t \to 0} \frac{\delta\beta}{\delta t}$$

which in this instance is equal to

$$\dot{\gamma} = \frac{U}{b} = \frac{du}{dy}$$

A continuation of this experiment would reveal that as the shearing stress, $\tau$, is increased by increasing $P$ (recall that $\tau = P/A$), the rate of shearing strain is increased in direct proportion—that is,

$$\tau \propto \dot{\gamma}$$

or

$$\tau \propto \frac{du}{dy}$$

This result indicates that for common fluids such as water, oil, gasoline, and air the shearing stress and rate of shearing strain (velocity gradient) can be related with a relationship of the form

$$\tau = \mu \frac{du}{dy} \tag{1.9}$$

where the constant of proportionality is designated by the Greek symbol $\mu$ (mu) and is called the ***absolute viscosity***, *dynamic viscosity*, or simply the *viscosity* of the fluid. In accordance with Eq. 1.9, plots of $\tau$ versus $du/dy$ should be linear with the slope equal to the viscosity as illustrated in Fig. 1.6. The actual value of the viscosity depends on the particular fluid, and for a particular fluid the viscosity is also highly dependent on temperature as illustrated in Fig. 1.6 with the two curves for water. Fluids for which the shearing stress is *linearly* related to the rate of shearing strain (also referred to as the rate of angular deformation) are designated as ***Newtonian fluids*** after Isaac Newton (1642–1727). Fortunately, most common fluids, both liquids and gases, are Newtonian.

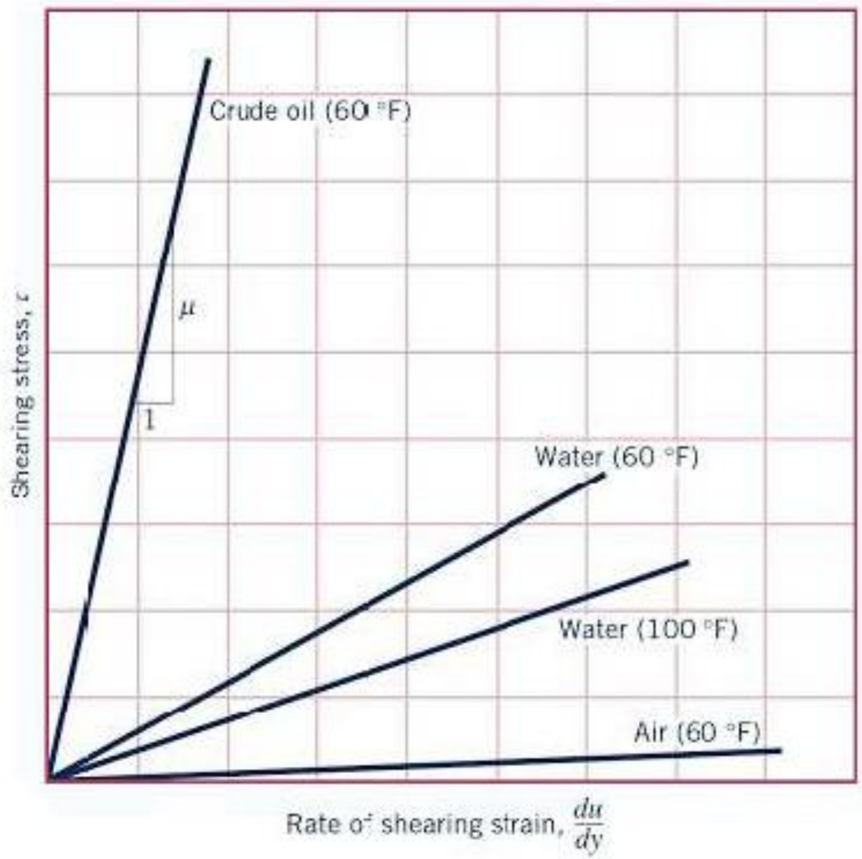



**Figure 1.6** **Linear variation of shearing stress with rate of shearing strain for common fluids.**

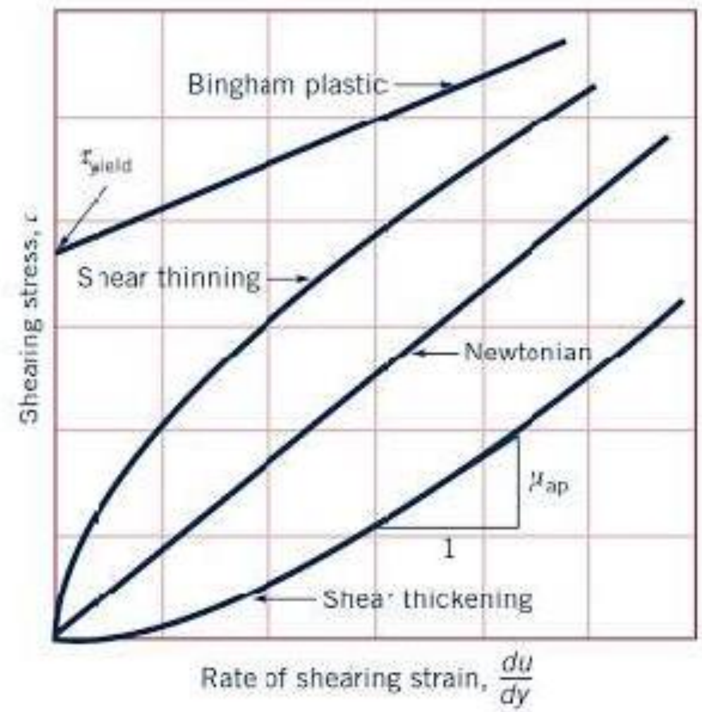



**Figure 1.7** Variation of shearing stress with rate of shearing strain for several types of fluids, including common non-Newtonian fluids.

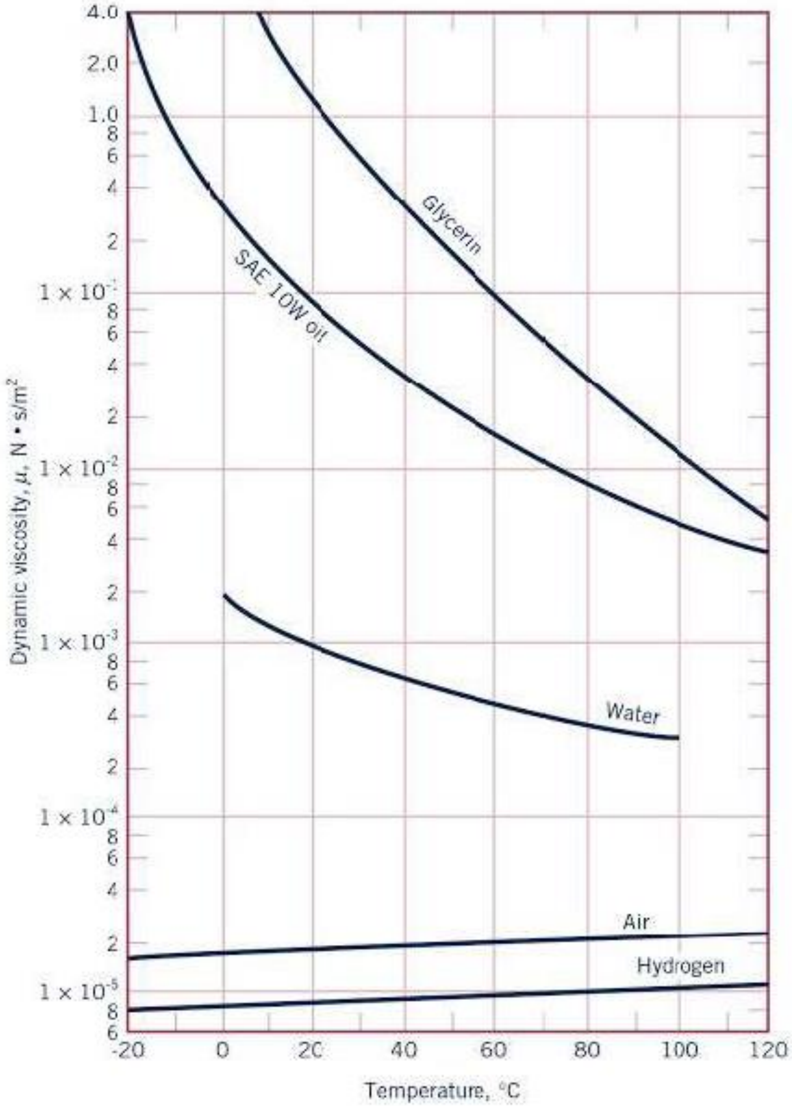



**Figure 1.8** **Dynamic (absolute) viscosity of some common fluids as a function of temperature.**

The effect of temperature on viscosity can be closely approximated using two empirical formulas. For gases the *Sutherland equation* can be expressed as

$$\mu = \frac{CT^{3/2}}{T + S} \tag{1.10}$$

where $C$ and $S$ are empirical constants, and $T$ is absolute temperature. Thus, if the viscosity is known at two temperatures, $C$ and $S$ can be determined. Or, if more than two viscosities are known, the data can be correlated with Eq. 1.10 by using some type of curve-fitting scheme.

For liquids an empirical equation that has been used is

$$\mu = De^{B/T} \tag{1.11}$$

where $D$ and $B$ are constants and $T$ is absolute temperature. This equation is often referred to as *Andrade's equation*. As was the case for gases, the viscosity must be known at least for two temperatures so the two constants can be determined. A more detailed discussion of the effect of temperature on fluids can be found in Ref. 1.

## EXAMPLE 1.4 Viscosity and Dimensionless Quantities

**GIVEN** A dimensionless combination of variables that is important in the study of viscous flow through pipes is called the *Reynolds number*, Re, defined as $\rho VD/\mu$ where, as indicated in Fig. E1.4, $\rho$ is the fluid density, $V$ the mean fluid velocity, $D$ the pipe diameter, and $\mu$ the fluid viscosity. A Newtonian fluid having a viscosity of $0.38\ \mathrm{N \cdot s/m^2}$ and a specific gravity of 0.91 flows through a 25-mm-diameter pipe with a velocity of 2.6 m/s.

**FIND** Determine the value of the Reynolds number using (a) SI units and (b) BG units.

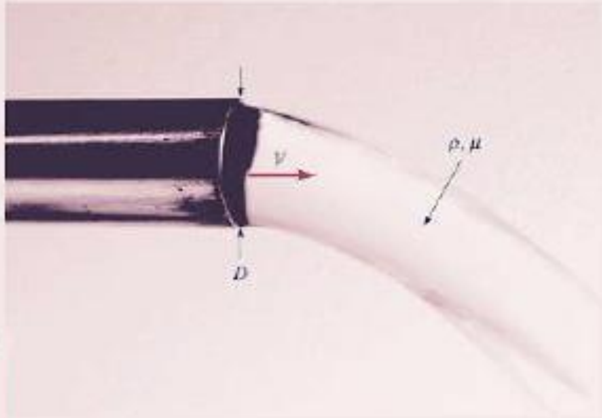



Figure E1.4

### SOLUTION

**(a)** The fluid density is calculated from the specific gravity as

$$\rho = SG\,\rho_{H_2O@4^\circ C} = 0.91\,(1000\ \mathrm{kg/m^3}) = 910\ \mathrm{kg/m^3}$$

and from the definition of the Reynolds number

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{(910\ \mathrm{kg/m^3})(2.6\ \mathrm{m/s})(25\ \mathrm{mm})(10^{-3}\ \mathrm{m/mm})}{0.38\ \mathrm{N\cdot s/m^2}}$$
$$= 156\ (\mathrm{kg\cdot m/s^2})/\mathrm{N}$$

However, since $1\ \mathrm{N} = 1\ \mathrm{kg\cdot m/s^2}$ it follows that the Reynolds number is unitless—that is,

$$\mathrm{Re} = 156 \qquad \text{(Ans)}$$

The value of any dimensionless quantity does not depend on the system of units used if all variables that make up the quantity are expressed in a consistent set of units. To check this, we will calculate the Reynolds number using BG units.

**(b)** We first convert all the SI values of the variables appearing in the Reynolds number to BG values. Thus,

$$\rho = (910\ \mathrm{kg/m^3})(1.940\times10^{-3}) = 1.77\ \mathrm{slugs/ft^3}$$
$$V = (2.6\ \mathrm{m/s})(3.281) = 8.53\ \mathrm{ft/s}$$
$$D = (0.025\ \mathrm{m})(3.281) = 8.20\times10^{-2}\ \mathrm{ft}$$
$$\mu = (0.38\ \mathrm{N\cdot s/m^2})(2.089\times10^{-2}) = 7.94\times10^{-3}\ \mathrm{lb\cdot s/ft^2}$$

and the value of the Reynolds number is

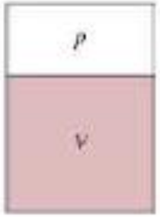
$$\mathrm{Re} = \frac{(1.77\ \mathrm{slugs/ft^3})(8.53\ \mathrm{ft/s})(8.20\times10^{-2}\ \mathrm{ft})}{7.94\times10^{-3}\ \mathrm{lb\cdot s/ft^2}}$$
$$= 156\ (\mathrm{slug\cdot ft/s^2})/\mathrm{lb} = 156 \qquad \text{(Ans)}$$

since $1\ \mathrm{lb} = 1\ \mathrm{slug\cdot ft/s^2}$.

**COMMENTS** The values from part (a) and part (b) are the same, as expected. Dimensionless quantities play an important role in fluid mechanics, and the significance of the Reynolds number as well as other important dimensionless combinations will be discussed in detail in Chapter 7. It should be noted that in the Reynolds number it is actually the ratio $\mu/\rho$ that is important, and this is the property that is defined as the kinematic viscosity.

## 1.7 Compressibility of Fluids

### 1.7.1 Bulk Modulus





An important question to answer when considering the behavior of a particular fluid is how easily can the volume (and thus the density) of a given mass of the fluid be changed when there is a change in pressure? That is, how compressible is the fluid? A property that is commonly used to characterize compressibility is the ***bulk modulus***, $E_v$, defined as

$$E_v = -\frac{dp}{dV/V} \tag{1.12}$$

where $dp$ is the differential change in pressure needed to create a differential change in volume, $dV$, of a volume $V$. This is illustrated by the figure in the margin. The negative sign is included since an increase in pressure will cause a decrease in volume. Since a decrease in volume of a given mass, $m = \rho V$, will result in an increase in density, Eq. 1.12 can also be expressed as

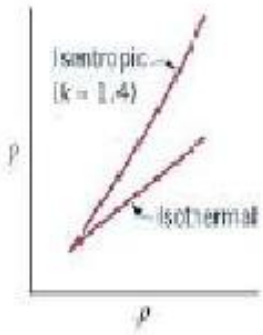
$$E_v = \frac{dp}{d\rho/\rho} \tag{1.13}$$

VIDEO *V1.7 Water balloon*

The bulk modulus (also referred to as the *bulk modulus of elasticity*) has dimensions of pressure, $FL^{-2}$. In BG units, values for $E_v$ are usually given as lb/in.$^2$ (psi) and in SI units as N/m$^2$ (Pa). Large values for the bulk modulus indicate that the fluid is relatively incompressible—that is, it takes a large pressure change to create a small change in volume. As expected, values of $E_v$ for common liquids are large (see Tables 1.5 and 1.6). For example, at atmospheric pressure and a temperature of 60 °F it would require a pressure of 3120 psi to compress a unit volume of water 1%. This result is representative of the compressibility of liquids. Since such large pressures are required to effect a change in volume, we conclude that liquids can be considered as *incompressible* for most practical engineering applications. As liquids are compressed the bulk modulus increases, but the bulk modulus near atmospheric pressure is usually the one of interest. The use of bulk modulus as a property describing compressibility is most prevalent when dealing with liquids, although the bulk modulus can also be determined for gases.

### 1.7.2 Compression and Expansion of Gases

When gases are compressed (or expanded), the relationship between pressure and density depends on the nature of the process. If the compression or expansion takes place under constant temperature conditions (*isothermal process*), then from Eq. 1.8




$$\frac{p}{\rho} = \text{constant} \tag{1.14}$$

If the compression or expansion is frictionless and no heat is exchanged with the surroundings (*isentropic process*), then

$$\frac{p}{\rho^k} = \text{constant} \tag{1.15}$$

where $k$ is the ratio of the specific heat at constant pressure, $c_p$, to the specific heat at constant volume, $c_v$ (i.e., $k = c_p/c_v$). The two specific heats are related to the gas constant, $R$, through the equation $R = c_p - c_v$. As was the case for the ideal gas law, the pressure in both Eqs. 1.14 and 1.15 must be expressed as an absolute pressure.

## EXAMPLE 1.6 Isentropic Compression of a Gas

**GIVEN** A cubic foot of air at an absolute pressure of 14.7 psi is compressed isentropically to $\frac{1}{2}$ ft$^3$ by the tire pump shown in Fig. E1.6*a*.

**FIND** What is the final pressure?

### SOLUTION

For an isentropic compression

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$

where the subscripts $i$ and $f$ refer to initial and final states, respectively. Since we are interested in the final pressure, $p_f$, it follows that

$$p_f = \left(\frac{\rho_f}{\rho_i}\right)^k p_i$$

Figure E1.6*a*

As the volume, $V$, is reduced by one-half, the density must double, since the mass, $m = \rho V$, of the gas remains constant. Thus, with $k = 1.40$ for air

$$p_f = (2)^{1.40}(14.7 \text{ psi}) = 38.8 \text{ psi (abs)} \qquad \textbf{(Ans)}$$

**COMMENT** By repeating the calculations for various values of the ratio of the final volume to the initial volume, $V_f/V_i$, the results shown in Fig. E1.6*b* are obtained. Note that even though air is often considered to be easily compressed (at least compared to liquids), it takes considerable pressure to significantly reduce a given volume of air as is done in an automobile engine where the compression ratio is on the order of $V_f/V_i = 1/8 = 0.125$.

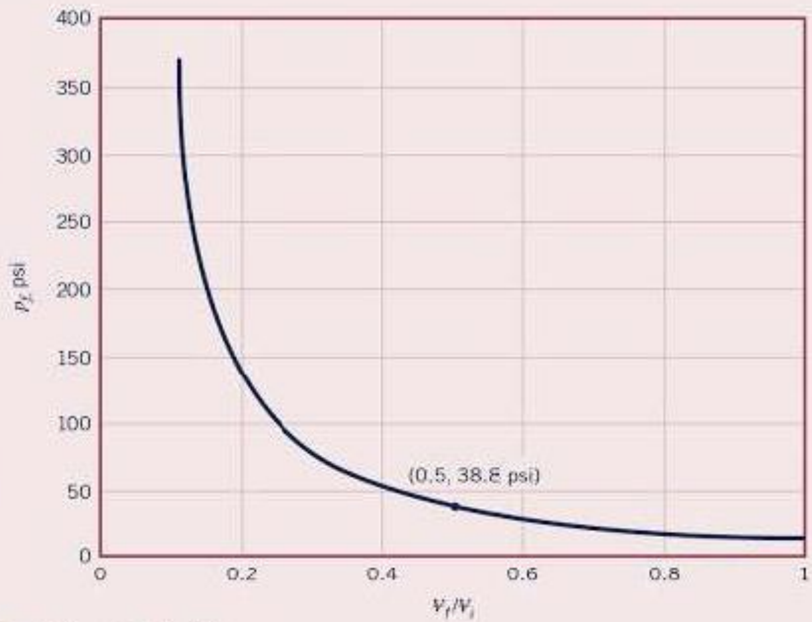



**Figure E1.6*b***

### 1.7.3 Speed of Sound

Another important consequence of the compressibility of fluids is that disturbances introduced at some point in the fluid propagate at a finite velocity. For example, if a fluid is flowing in a pipe and a valve at the outlet is suddenly closed (thereby creating a localized disturbance), the effect of the valve closure is not felt instantaneously upstream. It takes a finite time for the increased pressure created by the valve closure to propagate to an upstream location. Similarly, a loudspeaker diaphragm causes a localized disturbance as it vibrates, and the small change in pressure created by the motion of the diaphragm is propagated through the air with a finite velocity. The velocity at which these small disturbances propagate is called the *acoustic velocity* or the ***speed of sound, c***. It will be shown in Chapter 11 that the speed of sound is related to changes in pressure and density of the fluid medium through the equation

$$c = \sqrt{\frac{dp}{d\rho}} \tag{1.18}$$

or in terms of the bulk modulus defined by Eq. 1.13

$$c = \sqrt{\frac{E_v}{\rho}} \tag{1.19}$$

Since the disturbance is small, there is negligible heat transfer and the process is assumed to be isentropic. Thus, the pressure–density relationship used in Eq. 1.18 is that for an isentropic process.

For gases undergoing an isentropic process, $E_v = kp$ (Eq. 1.17) so that

$$c = \sqrt{\frac{kp}{\rho}}$$

and making use of the ideal gas law, it follows that

$$c = \sqrt{kRT} \tag{1.20}$$

Thus, for ideal gases the speed of sound is proportional to the square root of the absolute temperature. For example, for air at 60 °F with $k = 1.40$ and $R = 1716 \text{ ft} \cdot \text{lb/slug} \cdot {}^\circ\text{R}$, it follows that $c = 1117 \text{ ft/s}$. The speed of sound in air at various temperatures can be found in Appendix B

## EXAMPLE 1.7 Speed of Sound and Mach Number

**GIVEN** A jet aircraft flies at a speed of 550 mph at an altitude of 35,000 ft, where the temperature is −66 °F and the specific heat ratio is $k = 1.4$.

**FIND** Determine the ratio of the speed of the aircraft, $V$, to that of the speed of sound, $c$, at the specified altitude.

### SOLUTION

From Eq. 1.20 the speed of sound can be calculated as

$$c = \sqrt{kRT} = \sqrt{(1.40)(1716 \text{ ft lb/slug °R})(-66 + 460)\text{°R}} = 973 \text{ ft/s}$$

Since the air speed is

$$V = \frac{(550 \text{ mi/hr})(5280 \text{ ft/mi})}{(3600 \text{ s/hr})} = 807 \text{ ft/s}$$

the ratio is

$$\frac{V}{c} = \frac{807 \text{ ft/s}}{973 \text{ ft/s}} = 0.829 \qquad \textbf{(Ans)}$$

**COMMENT** This ratio is called the *Mach number*, Ma. If Ma < 1.0 the aircraft is flying at *subsonic* speeds, whereas for Ma > 1.0 it is flying at *supersonic* speeds. The Mach number is an important dimensionless parameter used in the study of the flow of gases at high speeds and will be further discussed in Chapters 7 and 11.

By repeating the calculations for different temperatures, the results shown in Fig. E1.7 are obtained. Because the speed of sound increases with increasing temperature, for a constant airplane speed, the Mach number decreases as the temperature increases.

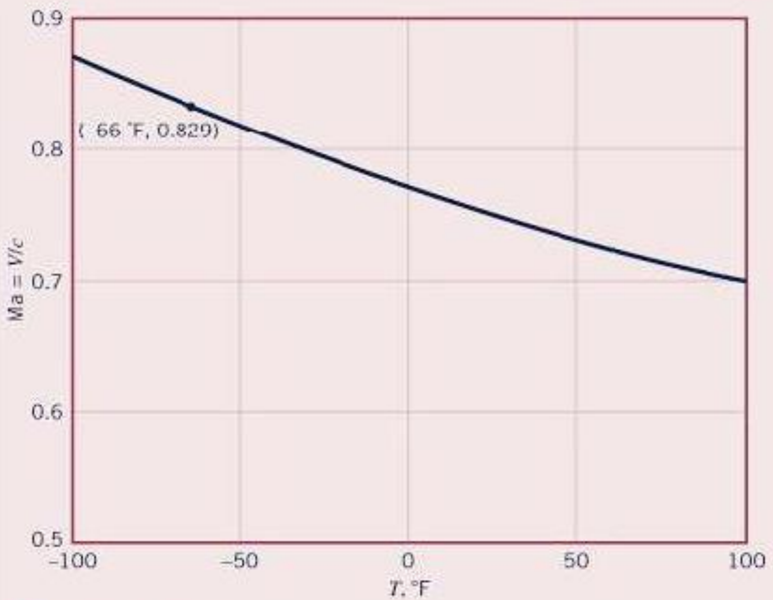
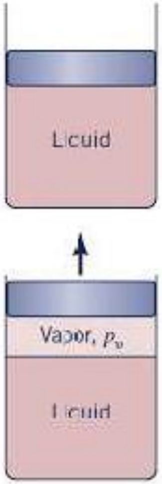



**Figure E1.7**

## 1.8 Vapor Pressure




*A liquid boils when the pressure is reduced to the vapor pressure.*

It is a common observation that liquids such as water and gasoline will evaporate if they are simply placed in a container open to the atmosphere. Evaporation takes place because some liquid molecules at the surface have sufficient momentum to overcome the intermolecular cohesive forces and escape into the atmosphere. If the container is closed with a small air space left above the surface, and this space evacuated to form a vacuum, a pressure will develop in the space as a result of the vapor that is formed by the escaping molecules. When an equilibrium condition is reached so that the number of molecules leaving the surface is equal to the number entering, the vapor is said to be saturated and the pressure that the vapor exerts on the liquid surface is termed the ***vapor pressure***, $p_v$. Similarly, if the end of a completely liquid-filled container is moved as shown in the figure in the margin without letting any air into the container, the space between the liquid and the end becomes filled with vapor at a pressure equal to the vapor pressure.

Since the development of a vapor pressure is closely associated with molecular activity, the value of vapor pressure for a particular liquid depends on temperature. Values of vapor pressure for water at various temperatures can be found in Appendix B (Tables B.1 and B.2), and the values of vapor pressure for several common liquids at room temperatures are given in Tables 1.5 and 1.6.

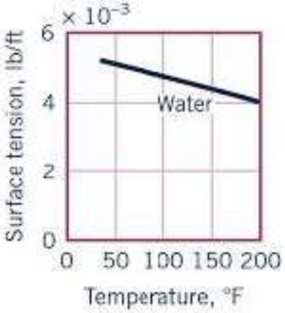
*Boiling*, which is the formation of vapor bubbles within a fluid mass, is initiated when the absolute pressure in the fluid reaches the vapor pressure. As commonly observed in the kitchen, water at standard atmospheric pressure will boil when the temperature reaches 212 °F (100 °C)—that is, the vapor pressure of water at 212 °F is 14.7 psi (abs). However, if we attempt to boil water at a higher elevation, say 30,000 ft above sea level (the approximate elevation of Mt. Everest), where the atmospheric pressure is 4.37 psi (abs), we find that boiling will start when the temperature is about 157 °F. At this temperature the vapor pressure of water is 4.37 psi (abs).

## 1.9 Surface Tension

V1.9 Floating razor blade

At the interface between a liquid and a gas, or between two immiscible liquids, forces develop in the liquid surface that cause the surface to behave as if it were a "skin" or "membrane" stretched over the fluid mass. Although such a skin is not actually present, this conceptual analogy allows us to explain several commonly observed phenomena. For example, a steel needle or a razor blade will float on water if placed gently on the surface because the tension developed in the hypothetical skin supports it. Small droplets of mercury will form into spheres when placed on a smooth surface because the cohesive forces in the surface of the mercury tend to hold all the molecules together in a compact shape. Similarly, discrete bubbles will form in a liquid. (See the photograph at the beginning of Chapter 1.)

These various types of surface phenomena are due to the unbalanced cohesive forces acting on the liquid molecules at the fluid surface. Molecules in the interior of the fluid mass are surrounded by molecules that are attracted to each other equally. However, molecules along the surface are subjected to a net force toward the interior. The apparent physical consequence of this unbalanced force along the surface is to create the hypothetical skin or membrane. A tensile force may be considered to be acting in the plane of the surface along any line in the surface. The intensity of the molecular attraction per unit length along any line in the surface is called the ***surface tension*** and is designated by the Greek symbol $\sigma$ (sigma). For a given liquid the surface tension depends on temperature as well as the other fluid it is in contact with at the interface. The dimensions of surface tension are $FL^{-1}$ with BG units of lb/ft and SI units of N/m. Values of surface tension for some common liquids (in contact with air) are given in Appendix B (Tables B.1 and B.2) for water at various temperatures. As indicated by the figure in the margin, the value of the surface tension decreases as the temperature increases.



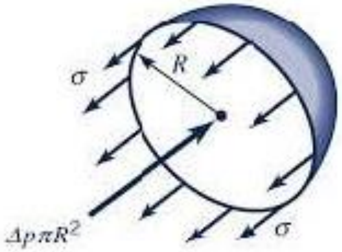



**Figure 1.9** **Forces acting on one-half of a liquid drop.**

The pressure inside a drop of fluid can be calculated using the free-body diagram in Fig. 1.9. If the spherical drop is cut in half (as shown), the force developed around the edge due to surface tension is $2\pi R\sigma$. This force must be balanced by the pressure difference, $\Delta p$, between the internal pressure, $p_i$, and the external pressure, $p_e$, acting over the circular area, $\pi R^2$. Thus,

$$2\pi R\sigma = \Delta p\ \pi R^2$$

or

$$\Delta p = p_i - p_e = \frac{2\sigma}{R} \tag{1.21}$$

It is apparent from this result that the pressure inside the drop is greater than the pressure surrounding the drop. (Would the pressure on the inside of a bubble of water be the same as that on the inside of a drop of water of the same diameter and at the same temperature?)

Among common phenomena associated with surface tension is the rise (or fall) of a liquid in a capillary tube. If a small open tube is inserted into water, the water level in the tube will rise above the water level outside the tube, as is illustrated in Fig. 1.10*a*. In this situation we have a liquid–gas–solid interface. For the case illustrated there is an attraction (adhesion) between the wall of the tube and liquid molecules which is strong enough to overcome the mutual attraction (cohesion) of the molecules and pull them up the wall. Hence, the liquid is said to *wet* the solid surface.

The height, $h$, is governed by the value of the surface tension, $\sigma$, the tube radius, $R$, the specific weight of the liquid, $\gamma$, and the *angle of contact*, $\theta$, between the fluid and tube. From the free-body diagram of Fig. 1.10*b* we see that the vertical force due to the surface tension is equal to $2\pi R\sigma \cos\theta$ and the weight is $\gamma\pi R^2 h$, and these two forces must balance for equilibrium. Thus,

$$\gamma\pi R^2 h = 2\pi R\sigma \cos\theta$$

so that the height is given by the relationship

$$h = \frac{2\sigma\cos\theta}{\gamma R} \tag{1.22}$$

The angle of contact is a function of both the liquid and the surface. For water in contact with clean glass $\theta \approx 0°$. It is clear from Eq. 1.22 that the height is inversely proportional to the tube radius, and therefore, as indicated by the figure in the margin, the rise of a liquid in a tube as a result of capillary action becomes increasingly pronounced as the tube radius is decreased.

If adhesion of molecules to the solid surface is weak compared to the cohesion between molecules, the liquid will not wet the surface and the level in a tube placed in a nonwetting liquid will actually be depressed, as shown in Fig. 1.10*c*. Mercury is a good example of a nonwetting liquid when it is in contact with a glass tube. For nonwetting liquids the angle of contact is greater than 90°, and for mercury in contact with clean glass $\theta \approx 130°$.

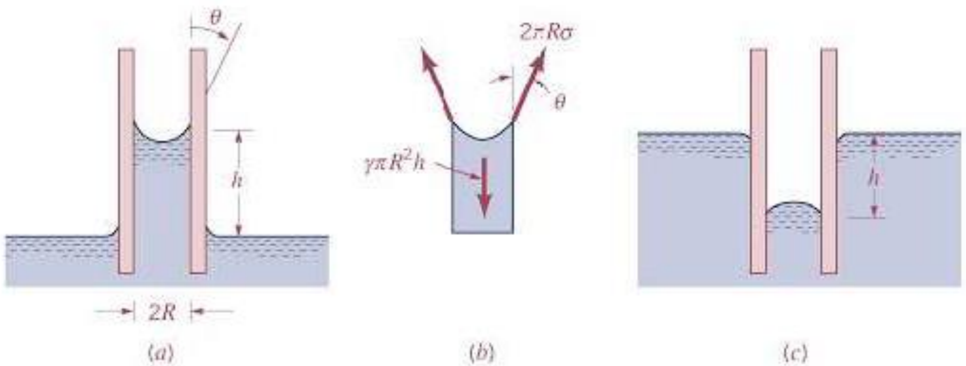



**Figure 1.10** **Effect of capillary action in small tubes. (*a*) Rise of column for a liquid that wets the tube. (*b*) Free-body diagram for calculating column height. (*c*) Depression of column for a nonwetting liquid.**

## EXAMPLE 1.8 Capillary Rise in a Tube

**GIVEN** Pressures are sometimes determined by measuring the height of a column of liquid in a vertical tube.

**FIND** What diameter of clean glass tubing is required so that the rise of water at 20 °C in a tube due to capillary action (as opposed to pressure in the tube) is less than $h = 1.0$ mm?

### SOLUTION

From Eq. 1.22

$$h = \frac{2\sigma \cos\theta}{\gamma R}$$

so that

$$R = \frac{2\sigma \cos\theta}{\gamma h}$$

For water at 20 °C (from Table B.2), $\sigma = 0.0728$ N/m and $\gamma = 9.789$ kN/m$^3$. Since $\theta \approx 0°$ it follows that for $h = 1.0$ mm,

$$R = \frac{2(0.0728 \text{ N/m})(1)}{(9.789 \times 10^3 \text{ N/m}^3)(1.0 \text{ mm})(10^{-3} \text{ m/mm})} = 0.0149 \text{ m}$$

and the minimum required tube diameter, $D$, is

$$D = 2R = 0.0298 \text{ m} = 29.8 \text{ mm} \qquad \textbf{(Ans)}$$

**COMMENT** By repeating the calculations for various values of the capillary rise, $h$, the results shown in Fig. E1.8 are obtained. Note that as the allowable capillary rise is decreased, the diameter of the tube must be significantly increased. There is always some capillarity effect, but it can be minimized by using a large enough diameter tube.

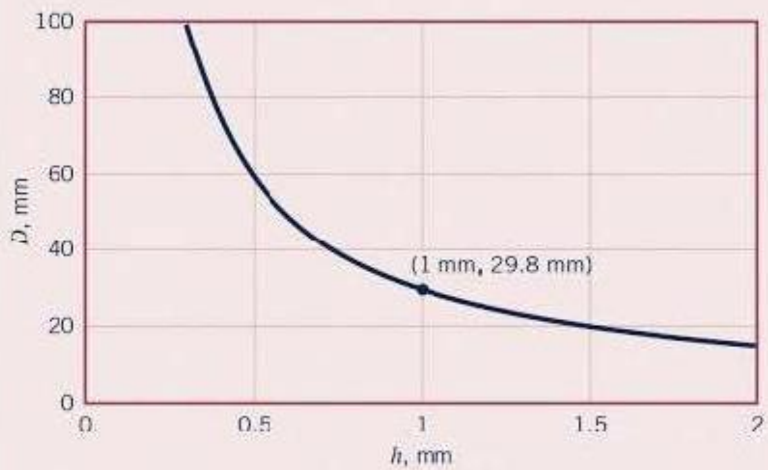



Figure E1.8

At the beginning of the twentieth century, both the fields of theoretical hydrodynamics and experimental hydraulics were highly developed, and attempts were being made to unify the two. In 1904 a classic paper was presented by a German professor, Ludwig Prandtl (1875–1953), who introduced the concept of a "fluid boundary layer," which laid the foundation for the unification of the theoretical and experimental aspects of fluid mechanics. Prandtl's idea was that for flow next to a

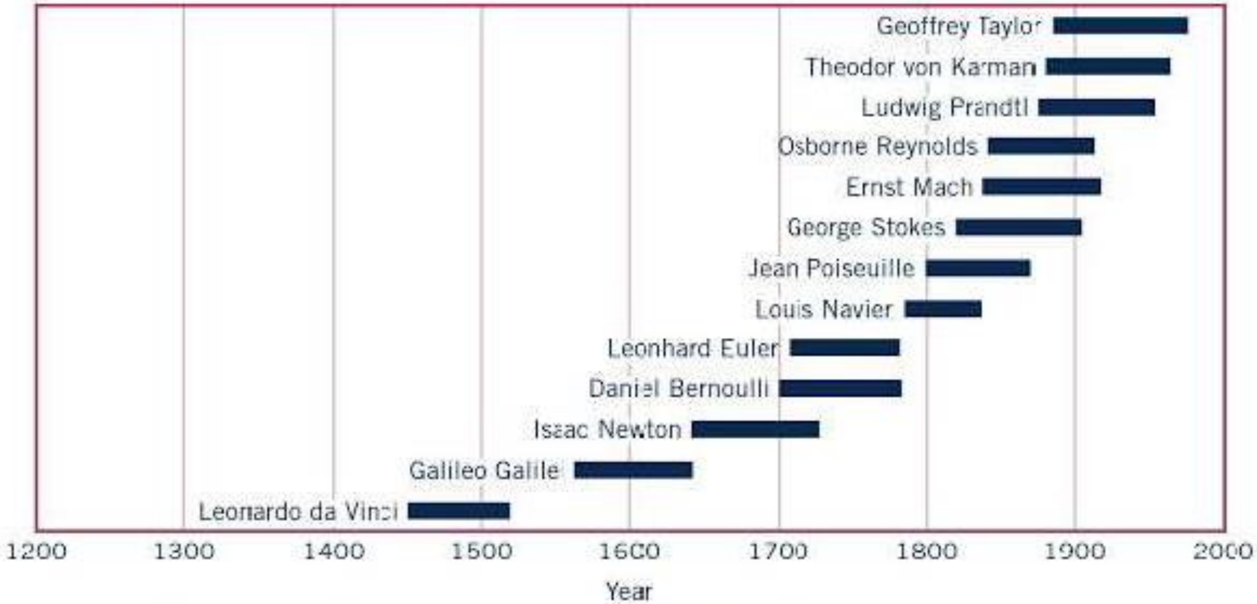



Figure 1.11 **Time line of some contributors to the science of fluid mechanics.**


Leonardo da Vinci


Isaac Newton


Daniel Bernoulli


Ernst Mach

**Table 1.9**

**Chronological Listing of Some Contributors to the Science of Fluid Mechanics Noted in the Text**[a]

**ARCHIMEDES** (287–212 B.C.)
Established elementary principles of buoyancy and flotation.

**SEXTUS JULIUS FRONTINUS** (A.D. 40–103)
Wrote treatise on Roman methods of water distribution.

**LEONARDO da VINCI** (1452–1519)
Expressed elementary principle of continuity; observed and sketched many basic flow phenomena; suggested designs for hydraulic machinery.

**GALILEO GALILEI** (1564–1642)
Indirectly stimulated experimental hydraulics; revised Aristotelian concept of vacuum.

**EVANGELISTA TORRICELLI** (1608–1647)
Related barometric height to weight of atmosphere, and form of liquid jet to trajectory of free fall.

**BLAISE PASCAL** (1623–1662)
Finally clarified principles of barometer, hydraulic press, and pressure transmissibility.

**ISAAC NEWTON** (1642–1727)
Explored various aspects of fluid resistance—inertial, viscous, and wave; discovered jet contraction.

**HENRI de PITOT** (1695–1771)
Constructed double-tube device to indicate water velocity through differential head.

**DANIEL BERNOULLI** (1700–1782)
Experimented and wrote on many phases of fluid motion, coining name "hydrodynamics"; devised manometry technique and adapted primitive energy principle to explain velocity-head indication; proposed jet propulsion.

**LEONHARD EULER** (1707–1783)
First explained role of pressure in fluid flow; formulated basic equations of motion and so-called Bernoulli theorem; introduced concept of cavitation and principle of centrifugal machinery.

**JEAN le ROND d'ALEMBERT** (1717–1783)
Originated notion of velocity and acceleration components, differential expression of continuity, and paradox of zero resistance to steady nonuniform motion.

**ANTOINE CHEZY** (1718–1798)
Formulated similarity parameter for predicting flow characteristics of one channel from measurements on another.

**GIOVANNI BATTISTA VENTURI** (1746–1822)
Performed tests on various forms of mouthpieces—in particular, conical contractions and expansions.

**LOUIS MARIE HENRI NAVIER** (1785–1836)
Extended equations of motion to include "molecular" forces.

**AUGUSTIN LOUIS de CAUCHY** (1789–1857)
Contributed to the general field of theoretical hydrodynamics and to the study of wave motion.

**GOTTHILF HEINRICH LUDWIG HAGEN** (1797–1884)
Conducted original studies of resistance in and transition between laminar and turbulent flow.

**JEAN LOUIS POISEUILLE** (1799–1869)
Performed meticulous tests on resistance of flow through capillary tubes.

**HENRI PHILIBERT GASPARD DARCY** (1803–1858)
Performed extensive tests on filtration and pipe resistance; initiated open-channel studies carried out by Bazin.

**JULIUS WEISBACH** (1806–1871)
Incorporated hydraulics in treatise on engineering mechanics, based on original experiments; noteworthy for flow patterns, nondimensional coefficients, weir, and resistance equations.

**WILLIAM FROUDE** (1810–1879)
Developed many towing-tank techniques, in particular the conversion of wave and boundary layer resistance from model to prototype scale.

**ROBERT MANNING** (1816–1897)
Proposed several formulas for open-channel resistance.

**GEORGE GABRIEL STOKES** (1819–1903)
Derived analytically various flow relationships ranging from wave mechanics to viscous resistance—particularly that for the settling of spheres.

**ERNST MACH** (1838–1916)
One of the pioneers in the field of supersonic aerodynamics.

## 1.11 Chapter Summary and Study Guide

This introductory chapter discussed several fundamental aspects of fluid mechanics. Methods for describing fluid characteristics both quantitatively and qualitatively are considered. For a quantitative description, units are required. The concept of dimensions is introduced in which basic dimensions such as length, $L$, time, $T$, and mass, $M$, are used to provide a description of various quantities of interest. The use of dimensions is helpful in checking the generality of equations, as well as serving as the basis for the powerful tool of dimensional analysis discussed in detail in Chapter 7.

Various important fluid properties are defined, including fluid density, specific weight, specific gravity, viscosity, bulk modulus, speed of sound, vapor pressure, and surface tension. The ideal gas law is introduced to relate pressure, temperature, and density in common gases, along with a brief discussion of the compression and expansion of gases. The distinction between absolute and gage pressure is introduced. This important idea is explored more fully in Chapter 2.

The following checklist provides a study guide for this chapter. When your study of the entire chapter and end-of-chapter exercises has been completed you should be able to

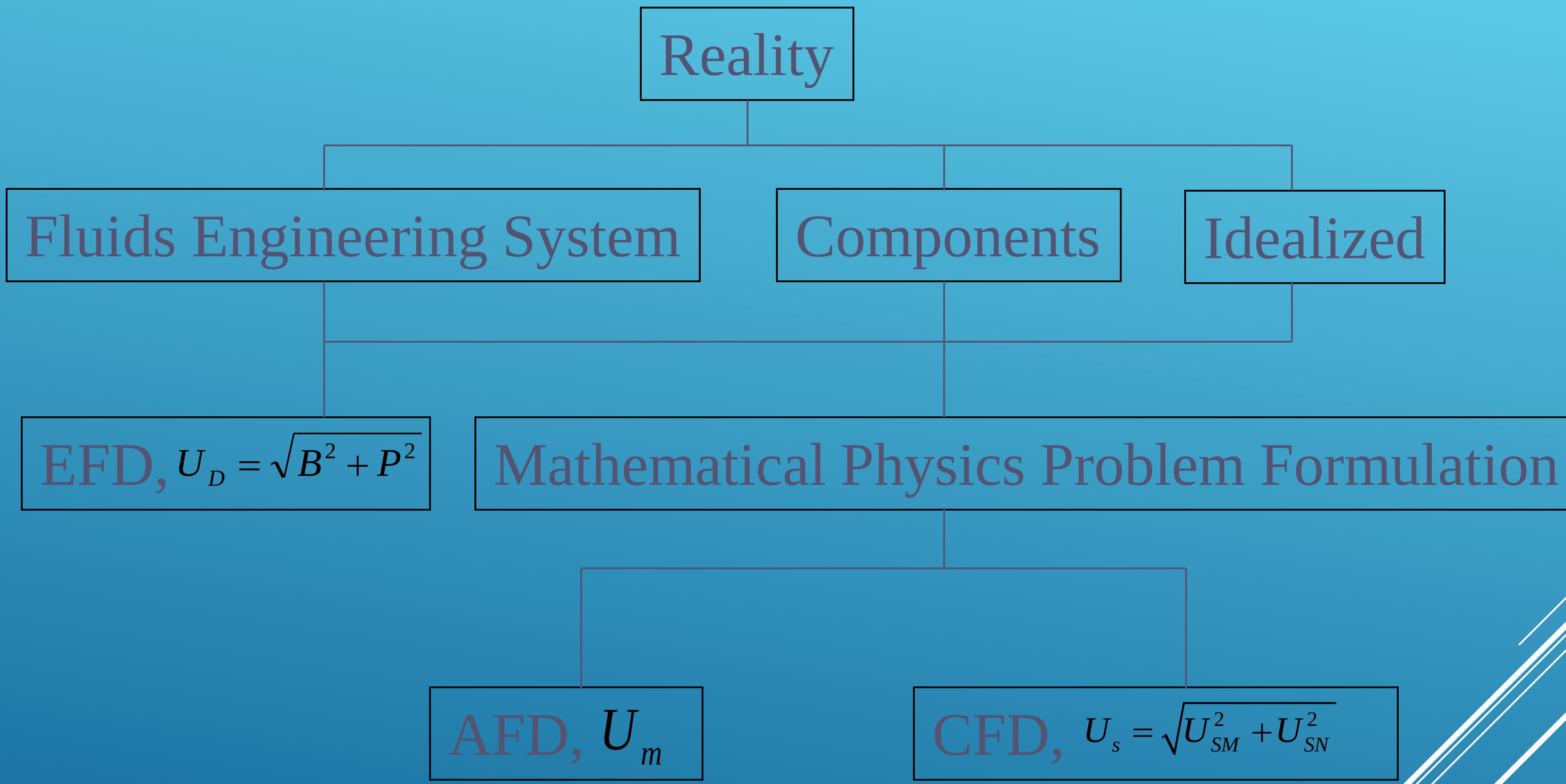
- write out meanings of the terms listed here in the margin and understand each of the related concepts. These terms are particularly important and are set in ***italic, bold, and color*** type in the text.
- determine the dimensions of common physical quantities.
- determine whether an equation is a general or restricted homogeneous equation.

- correctly use units and systems of units in your analyses and calculations.
- calculate the density, specific weight, or specific gravity of a fluid from a knowledge of any two of the three.
- calculate the density, pressure, or temperature of an ideal gas (with a given gas constant) from a knowledge of any two of the three.
- relate the pressure and density of a gas as it is compressed or expanded using Eqs. 1.14 and 1.15.
- use the concept of viscosity to calculate the shearing stress in simple fluid flows.
- calculate the speed of sound in fluids using Eq. 1.19 for liquids and Eq. 1.20 for gases.
- determine whether boiling or cavitation will occur in a liquid using the concept of vapor pressure.
- use the concept of surface tension to solve simple problems involving liquid–gas or liquid–solid–gas interfaces.

Some of the important equations in this chapter are:

| | | |
|---|---|---|
| Specific weight | $\gamma = \rho g$ | (1.6) |
| Specific gravity | $SG = \dfrac{\rho}{\rho_{H_2O@4\,°C}}$ | (1.7) |
| Ideal gas law | $p = \rho R T$ | (1.8) |
| Newtonian fluid shear stress | $\tau = \mu \dfrac{du}{dy}$ | (1.9) |
| Bulk modulus | $E_v = -\dfrac{dp}{dV/V}$ | (1.12) |
| Speed of sound in an ideal gas | $c = \sqrt{kRT}$ | (1.20) |
| Capillary rise in a tube | $h = \dfrac{2\sigma \cos\theta}{\gamma R}$ | (1.22) |

# Fluids Engineering





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# Analytical Fluid Dynamics

- The theory of mathematical physics problem formulation
- Control volume & differential analysis
- Exact solutions only exist for simple geometry and conditions
- Approximate solutions for practical applications
  - Linear
  - Empirical relations using EFD data



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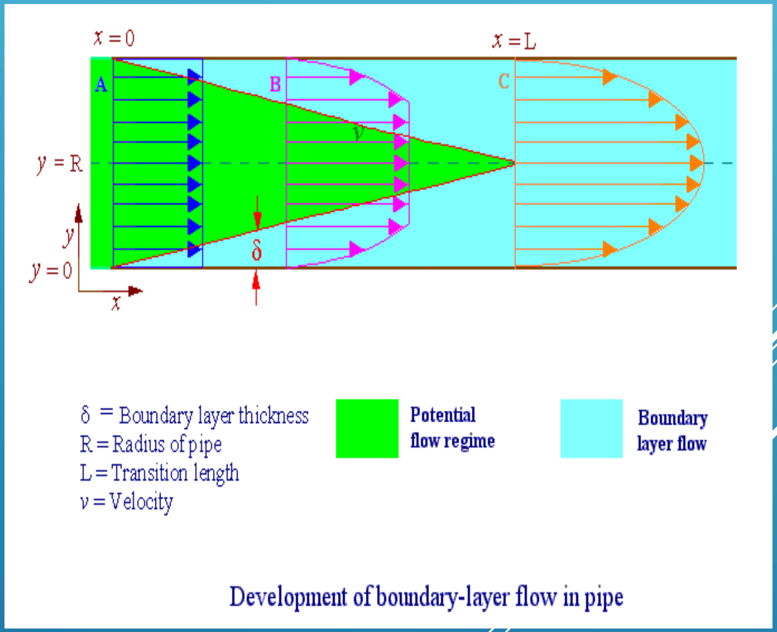
# Analytical Fluid Dynamics

- Example: laminar pipe flow

Assumptions: Fully developed, Low $\mathrm{Re} = \dfrac{\rho U D}{\mu} < 2000$

Approach: Simplify momentum equation, integrate, apply boundary conditions to determine integration constants and use energy equation to calculate head loss

Schematic



Development of boundary-layer flow in pipe

$$\cancelto{0}{\frac{Du}{Dt}} = -\frac{\partial p}{\partial x} + \mu\left[\cancelto{0}{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right] + \cancelto{0}{g_x}$$

Exact solution :

$$u(r) = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)$$

Friction factor: $f = \dfrac{8\tau_w}{\rho \bar{V}^2} = \dfrac{8\mu \left.\dfrac{du}{dy}\right|_w}{\rho \bar{V}^2} = \dfrac{64}{\mathrm{Re}}$

Head loss: $\dfrac{p_1}{\gamma} + z_1 = \dfrac{p_2}{\gamma} + z_2 + h_f \qquad h_f = f\dfrac{L}{D}\dfrac{V^2}{2g} = \dfrac{32\mu L V}{\gamma D^2}$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# Experimental Fluid Dynamics (EFD)

**Definition:**

Use of experimental methodology and procedures for solving fluids engineering systems, including full and model scales, large and table top facilities, measurement systems (instrumentation, data acquisition and data reduction), uncertainty analysis, and dimensional analysis and similarity.

**EFD philosophy:**

- Decisions on conducting experiments are governed by the ability of the expected test outcome, to achieve the test objectives within allowable uncertainties.
- Integration of UA into all test phases should be a key part of entire experimental program
    - test design
    - determination of error sources
    - estimation of uncertainty
    - documentation of the results

# Purpose

- Science & Technology: understand and investigate a phenomenon/process, substantiate and validate a theory (hypothesis)
- Research & Development: document a process/system, provide benchmark data (standard procedures, validations), calibrate instruments, equipment, and facilities
- Industry: design optimization and analysis, provide data for direct use, product liability, and acceptance
- Teaching: instruction/demonstration



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# Full and model scale

- Scales: model, and full-scale
- Selection of the model scale: governed by dimensional analysis and similarity



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# Computational Fluid Dynamics

- CFD is use of computational methods for solving fluid engineering systems, including modeling (mathematical & Physics) and numerical methods (solvers, finite differences, and grid generations, etc.).
- Rapid growth in CFD technology since advent of computer

ENIAC 1, 1946

IBM WorkStation



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# Purpose

- The objective of CFD is to model the continuous fluids with Partial Differential Equations (PDEs) and discretize PDEs into an algebra problem, solve it, validate it and achieve simulation based design instead of "build & test"

- Simulation of physical fluid phenomena that are difficult to be measured by experiments: scale simulations (full-scale ships, airplanes), hazards (explosions,radiations,pollution), physics (weather prediction, planetary boundary layer, stellar evolution).



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# Modeling

- Mathematical physics problem formulation of fluid engineering system
- Governing equations: Navier-Stokes equations (momentum), continuity equation, pressure Poisson equation, energy equation, ideal gas law, combustions (chemical reaction equation), multi-phase flows(e.g. Rayleigh equation), and turbulent models (RANS, LES, DES).
- Coordinates: Cartesian, cylindrical and spherical coordinates result in different form of governing equations
- Initial conditions(initial guess of the solution) and Boundary Conditions (no-slip wall, free-surface, zero-gradient, symmetry, velocity/pressure inlet/outlet)
- Flow conditions: Geometry approximation, domain, Reynolds Number, and Mach Number, etc.



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## MATHEMATICAL MODELING OF ENGINEERING PROBLEMS

### Experimental vs. Analytical Analysis

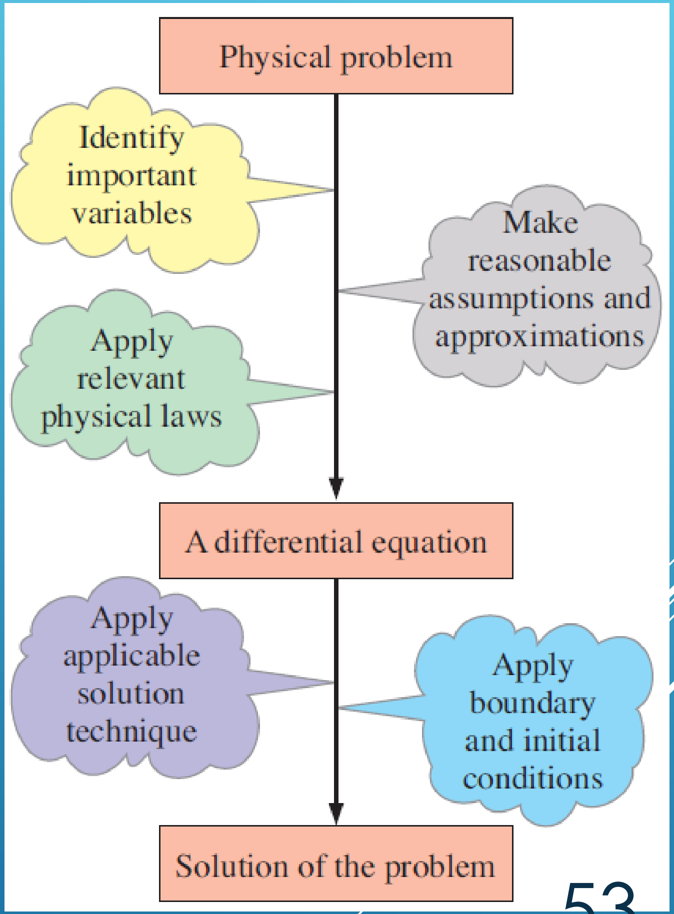
An engineering device or process can be studied either *experimentally* (testing and taking measurements) or *analytically* (by analysis or calculations).

- The experimental approach has the advantage that we deal with the actual physical system, and the desired quantity is determined by measurement, within the limits of experimental error. However, this approach is expensive, time-consuming, and often impractical.
- The analytical approach (including the numerical approach) has the advantage that it is fast and inexpensive, but the results obtained are subject to the accuracy of the assumptions, approximations, and idealizations made in the analysis.

# MODELING IN ENGINEERING

- **Why do we need differential equations?** The descriptions of most scientific problems involve equations that relate the changes in some key variables to each other.
- In the limiting case of infinitesimal or differential changes in variables, we obtain *differential equations* that provide precise mathematical formulations for the physical principles and laws by representing the rates of change as *derivatives*.
- Therefore, differential equations are used to investigate a wide variety of problems in sciences and engineering.
- **Do we always need differential equations?** Many problems encountered in practice can be solved without resorting to differential equations and the complications associated with them.

## MATHEMATICAL MODELING OF PHYSICAL PROBLEMS.



## PROBLEM-SOLVING TECHNIQUE

- Step 1: Problem Statement
- Step 2: Schematic
- Step 3: Assumptions and Approximations
- Step 4: Physical Laws
- Step 5: Properties
- Step 6: Calculations
- Step 7: Reasoning, Verification, and Discussion