MENG353 - FLUID MECHANICS

SOURCE: FUNDAMENTALS OF FLUIDMECHANICS
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CHAPTER 5 FINITE CONTROL VOLUME ANALYSIS

FALL 2017 - 18

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Learning Objectives

After completing this chapter, you should be able to:

- select an appropriate finite control volume to solve a fluid mechanics problem.
- apply conservation of mass and energy and Newton's second law of motion to the contents of a finite control volume to get important answers.
- know how velocity changes and energy transfers in fluid flows are related to forces and torques.
- understand why designing for minimum loss of energy in fluid flows is so important.

5.1 Conservation of Mass—The Continuity Equation

5.1.1 Derivation of the Continuity Equation

A system is defined as a collection of unchanging contents, so the *conservation of mass* principle for a system is simply stated as

time rate of change of the system mass = 0

or

$$\frac{DM_{\text{sys}}}{Dt} = 0 \tag{5.1}$$

where the system mass, M_{sys}, is more generally expressed as

$$M_{\text{sys}} = \int_{\text{sys}} \rho \, d\forall \tag{5.2}$$

and the integration is over the volume of the system. In words, Eq. 5.2 states that the system mass is equal to the sum of all the density-volume element products for the contents of the system.

For a system and a fixed, nondeforming control volume that are coincident at an instant of time, as illustrated in Fig. 5.1, the Reynolds transport theorem (Eq. 4.19) with B = mass and $b = 1$ allows us to state that

$$\frac{D}{Dt}\int_{\text{sys}} \rho \, d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \tag{5.3}$$

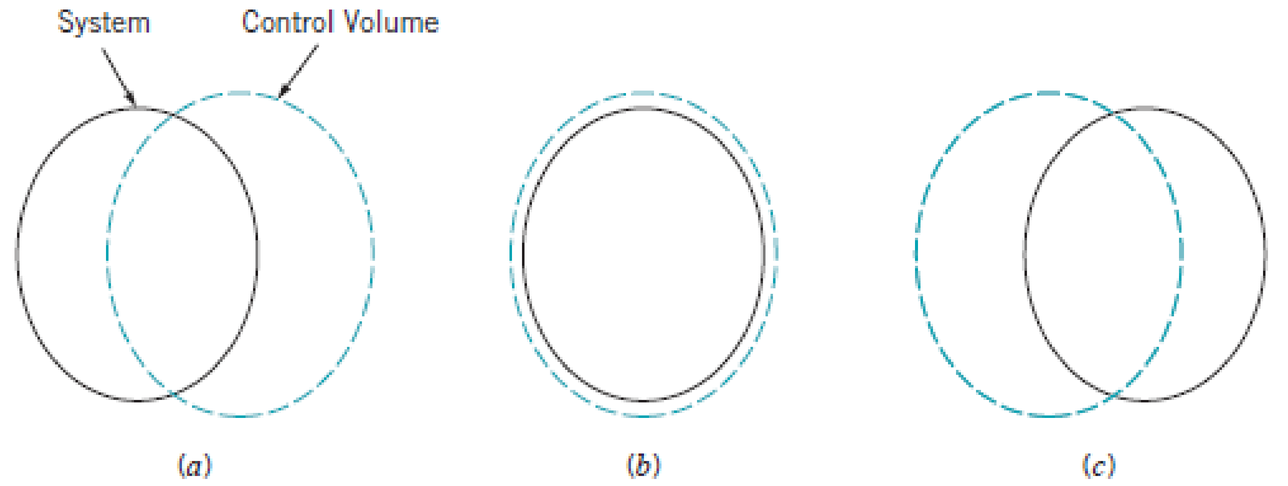
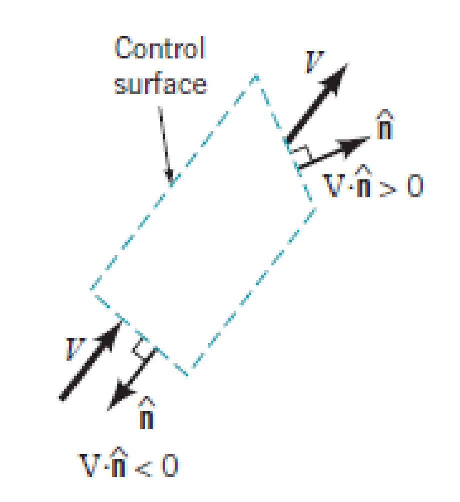


FIGURE 5.1 System and control volume at three different instances of time. (*a*) System and control volume at time $t - \delta t$. (*b*) System and control volume at time t, coincident condition. (*c*) System and control volume at time $t + \delta t$.



or

$$\begin{array}{ccccc}\text{time rate of change} & & \text{time rate of change} & & \text{net rate of flow} \\ \text{of the mass of the} & = & \text{of the mass of the} & + & \text{of mass through} \\ \text{coincident system} & & \text{contents of the coin-} & & \text{the control} \\ & & \text{cident control volume} & & \text{surface}\end{array}$$

In Eq. 5.3, we express the time rate of change of the system mass as the sum of two control volume quantities, the time rate of change of the mass of the contents of the control volume,

$$\frac{\partial}{\partial t}\int_{\mathrm{cv}} \rho\, d\maltese$$

and the net rate of mass flow through the control surface,

$$\int_{\mathrm{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

When a flow is steady, all field properties (i.e., properties at any specified point) including density remain constant with time and the time rate of change of the mass of the contents of the control volume is zero. That is,

$$\frac{\partial}{\partial t}\int_{\mathrm{cv}} \rho\, d\maltese = 0$$

the result is the net mass flowrate through the control surface, or

$$\int_{\mathrm{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \sum \dot{m}_{\mathrm{out}} - \sum \dot{m}_{\mathrm{in}} \qquad \textbf{(5.4)}$$

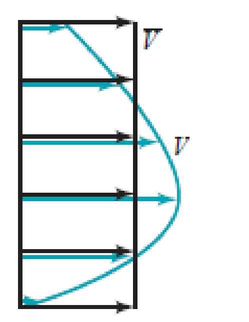
The control volume expression for conservation of mass, which is commonly called the *continuity equation*, for a fixed, nondeforming control volume is obtained by combining Eqs. 5.1, 5.2, and 5.3 to obtain

$$\boxed{\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0} \qquad (5.5)$$

An often-used expression for *mass flowrate*, $\dot{m}$, through a section of control surface having area A is

$$\boxed{\dot{m} = \rho Q = \rho A V} \qquad (5.6)$$

$$\dot{m} = \int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$



$$\boxed{\overline{V} = \frac{\int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA}{\rho A}} \qquad (5.7)$$

5.1.2 Fixed, Nondeforming Control Volume

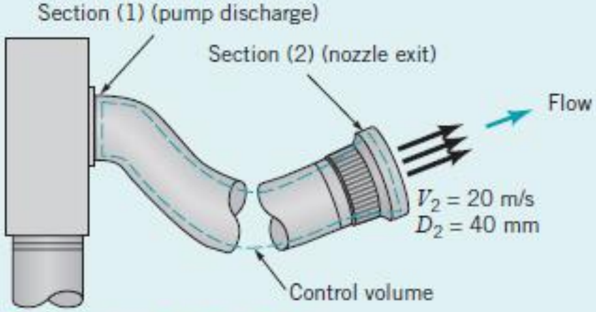
In many applications of fluid mechanics, an appropriate control volume to use is fixed and nondeforming. Several example problems that involve the continuity equation for fixed, nondeforming control volumes (Eq. 5.5) follow.

EXAMPLE 5.1 Conservation of Mass—Steady, Incompressible Flow

GIVEN Water flows steadily through a nozzle at the end of a fire hose as illustrated in Fig. E5.1*a*. According to local regulations, the nozzle exit velocity must be at least 20 m/s as shown in Fig. E5.1*b*.

FIND Determine the minimum pumping capacity, Q, required in m^3/s.

■ FIGURE E5.1*a*



■ FIGURE E5.1*b*

Solution

The pumping capacity sought is the volume flowrate delivered by the fire pump to the hose and nozzle. Since we desire knowledge about the pump discharge flowrate and we have information about the nozzle exit flowrate, we link these two flowrates with the control volume designated with the dashed line in Fig. E5.1*b*. This control volume contains, at any instant, water that is within the hose and nozzle from the pump discharge to the nozzle exit plane.

Equation 5.5 is applied to the contents of this control volume to give

$$\cancel{\frac{\partial}{\partial t}\int_{\text{cv}}}^{\,0\text{ (flow is steady)}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0 \qquad (1)$$

The time rate of change of the mass of the contents of this control volume is zero because the flow is steady. Because there is only one inflow [the pump discharge, section (1)] and one outflow [the nozzle exit, section (2)], Eq. (1) becomes

$$\rho_2 A_2 V_2 - \rho_1 A_1 V_1 = 0$$

so that with $\dot{m} = \rho A V$

$$\dot{m}_1 = \dot{m}_2 \qquad (2)$$

Because the mass flowrate is equal to the product of fluid density, ρ, and volume flowrate, Q (see Eq. 5.6), we obtain from Eq. 2

$$\rho_2 Q_2 = \rho_1 Q_1 \qquad (3)$$

Liquid flow at low speeds, as in this example, may be considered incompressible. Therefore

$$\rho_2 = \rho_1 \qquad (4)$$

and from Eqs. 3 and 4

$$Q_2 = Q_1 \qquad (5)$$

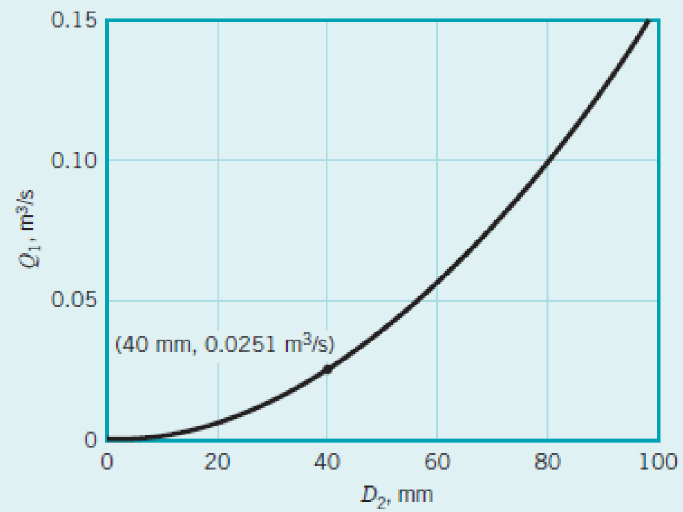
The pumping capacity is equal to the volume flowrate at the nozzle exit. If, for simplicity, the velocity distribution at the nozzle exit plane, section (2), is considered uniform (one-dimensional), then from Eq. 5

$$Q_1 = Q_2 = V_2 A_2$$

$$= V_2 \frac{\pi}{4} D_2^2 = (20 \text{ m/s}) \frac{\pi}{4} \left(\frac{40 \text{ mm}}{1000 \text{ mm/m}} \right)^2$$

$$= 0.0251 \text{ m}^3\text{/s} \qquad \textbf{(Ans)}$$

COMMENT By repeating the calculations for various values of the nozzle exit diameter, D_2, the results shown in Fig. E5.1*c* are obtained. The flowrate is proportional to the exit area, which varies as the diameter squared. Hence, if the diameter were doubled, the flowrate would increase by a factor of four, provided the exit velocity remained the same.

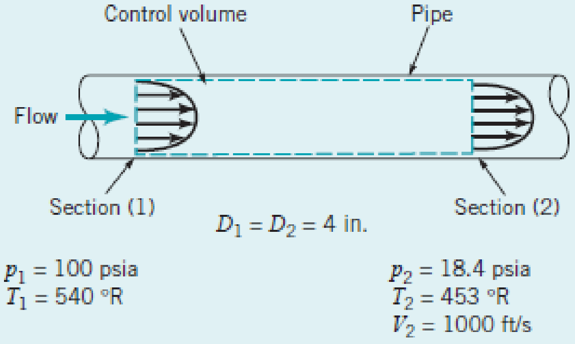


■ FIGURE E5.1c

EXAMPLE 5.2 Conservation of Mass—Steady, Compressible Flow

GIVEN Air flows steadily between two sections in a long, straight portion of 4-in. inside diameter pipe as indicated in Fig. E5.2. The uniformly distributed temperature and pressure at each section are given. The average air velocity (nonuniform velocity distribution) at section (2) is 1000 ft/s.

FIND Calculate the average air velocity at section (1).



■ FIGURE E5.2

SOLUTION

The average fluid velocity at any section is that velocity which yields the section mass flowrate when multiplied by the section average fluid density and section area (Eq. 5.7). We relate the flows at sections (1) and (2) with the control volume designated with a dashed line in Fig. E5.2.

Equation 5.5 is applied to the contents of this control volume to obtain

$$\overbrace{\frac{\partial}{\partial t}\int_{cv} \rho \, d\mathcal{V}}^{0 \text{ (flow is steady)}} + \int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0$$

The time rate of change of the mass of the contents of this control volume is zero because the flow is steady. The control surface integral involves mass flowrates at sections (1) and (2) so that from Eq. 5.4 we get

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \dot{m}_2 - \dot{m}_1 = 0$$

or

$$\dot{m}_1 = \dot{m}_2 \qquad \textbf{(1)}$$

and from Eqs. 1, 5.6, and 5.7 we obtain

$$\rho_1 A_1 \overline{V}_1 = \rho_2 A_2 \overline{V}_2 \qquad \textbf{(2)}$$

or since $A_1 = A_2$

$$\overline{V}_1 = \frac{\rho_2}{\rho_1} \overline{V}_2 \qquad \textbf{(3)}$$

Air at the pressures and temperatures involved in this example problem behaves like an ideal gas. The ideal gas equation of state (Eq. 1.8) is

$$\rho = \frac{p}{RT} \qquad \textbf{(4)}$$

Thus, combining Eqs. 3 and 4 we obtain

$$\overline{V}_1 = \frac{p_2 T_1 \overline{V}_2}{p_1 T_2}$$

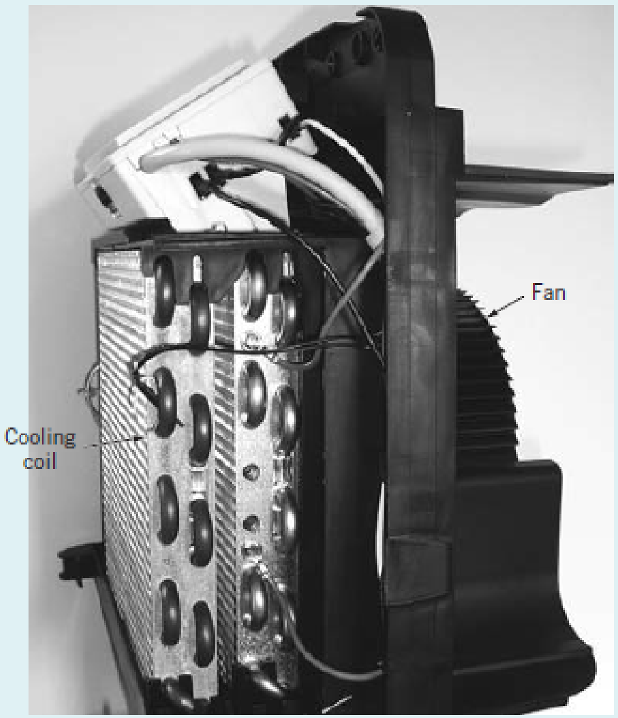
$$= \frac{(18.4 \text{ psia})(540\ ^\circ\text{R})(1000 \text{ ft/s})}{(100 \text{ psia})(453\ ^\circ\text{R})} = 219 \text{ ft/s} \qquad \textbf{(Ans)}$$

COMMENT We learn from this example that the continuity equation (Eq. 5.5) is valid for compressible as well as incompressible flows. Also, nonuniform velocity distributions can be handled with the average velocity concept. Significant average velocity changes can occur in pipe flow if the fluid is compressible.

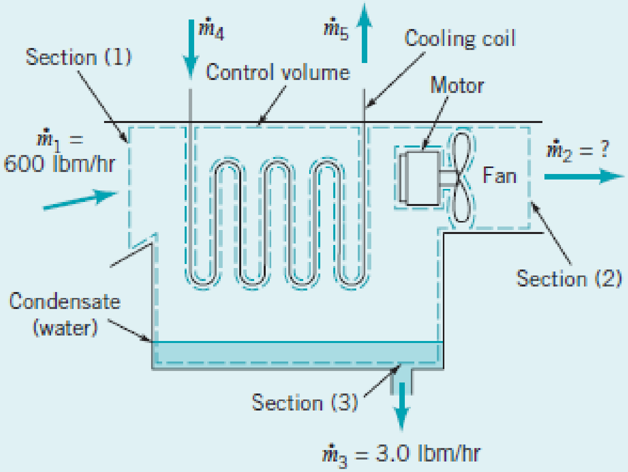
EXAMPLE 5.3 Conservation of Mass—Two Fluids

GIVEN The inner workings of a dehumidifier are shown in Fig. E5.3*a*. Moist air (a mixture of dry air and water vapor) enters the dehumidifier at the rate of 600 lbm/hr. Liquid water drains out of the dehumidifier at a rate of 3.0 lbm/hr. A simplified sketch of the process is provided in Fig. E5.3*b*.

FIND Determine the mass flowrate of the dry air and the water vapor leaving the dehumidifier.



■ FIGURE E5.3*a*



■ FIGURE E5.3*b*

Solution

The unknown mass flowrate at section (2) is linked with the known flowrates at sections (1) and (3) with the control volume designated with a dashed line in Fig. E5.3*b*. The contents of the control volume are the air and water vapor mixture and the condensate (liquid water) in the dehumidifier at any instant.

Not included in the control volume are the fan and its motor, and the condenser coils and refrigerant. Even though the flow in the vicinity of the fan blade is unsteady, it is unsteady in a cyclical way. Thus, the flowrates at sections (1), (2), and (3) appear steady and the time rate of change of the mass of the contents of the control volume may be considered equal to zero on a time-average basis. The application of Eqs. 5.4 and 5.5 to the control volume contents results in

$$\int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = -\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = 0$$

or

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 600 \text{ lbm/hr} - 3.0 \text{ lbm/hr}$$
$$= 597 \text{ lbm/hr} \qquad \textbf{(Ans)}$$

COMMENT Note that the continuity equation (Eq. 5.5) can be used when there is more than one stream of fluid flowing through the control volume.

The answer is the same with a control volume which includes the cooling coils to be within the control volume. The continuity equation becomes

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 + \dot{m}_4 - \dot{m}_5 \qquad \textbf{(1)}$$

where $\dot{m}_4$ is the mass flowrate of the cooling fluid flowing into the control volume, and $\dot{m}_5$ is the flowrate out of the control volume through the cooling coil. Since the flow through the coils is steady, it follows that $\dot{m}_4 = \dot{m}_5$. Hence, Eq. 1 gives the same answer as obtained with the original control volume.

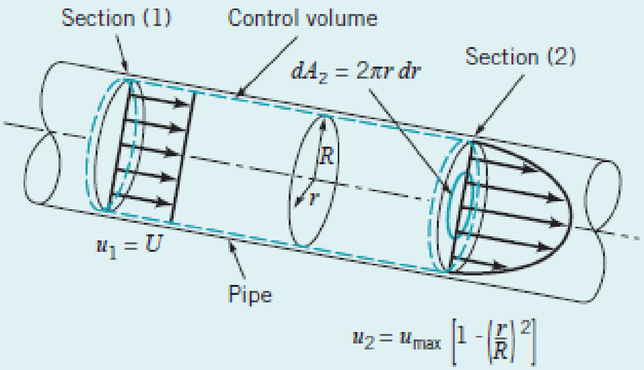
EXAMPLE 5.4 Conservation of Mass—Nonuniform Velocity Profile

GIVEN Incompressible, laminar water flow develops in a straight pipe having radius R as indicated in Fig. E5.4*a*. At section (1), the velocity profile is uniform; the velocity is equal to a constant value U and is parallel to the pipe axis everywhere. At section (2), the velocity profile is axisymmetric and parabolic, with zero velocity at the pipe wall and a maximum value of u_{max} at the centerline.

FIND

(a) How are U and u_{max} related?

(b) How are the average velocity at section (2), $\overline{V}_2$, and u_{max} related?



■ FIGURE E5.4*a*

SOLUTION

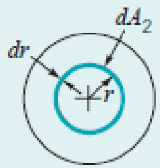
(a) An appropriate control volume is sketched (dashed lines) in Fig. E5.4*a*. The application of Eq. 5.5 to the contents of this control volume yields

$$\cancelto{0 \text{ (flow is steady)}}{\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, dV} + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0 \qquad \textbf{(1)}$$

At the inlet, section (1), the velocity is uniform with $V_1 = U$ so that

$$\int_{(1)} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = -\rho_1 A_1 U \qquad \textbf{(2)}$$

At the outlet, section (2), the velocity is not uniform. However, the net flowrate through this section is the sum of flows through numerous small washer-shaped areas of size $dA_2 = 2\pi r \, dr$ as shown by the shaded area element in Fig. E5.4*b*. On each of



■ FIGURE E5.4*b*

these infinitesimal areas the fluid velocity is denoted as u_2. Thus, in the limit of infinitesimal area elements, the summation is replaced by an integration and the outflow through section (2) is given by

$$\int_{(2)} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \rho_2 \int_0^R u_2 2\pi r \, dr \qquad \textbf{(3)}$$

By combining Eqs. 1, 2, and 3 we get

$$\rho_2 \int_0^R u_2 2\pi r \, dr - \rho_1 A_1 U = 0 \qquad \textbf{(4)}$$

Since the flow is considered incompressible, $\rho_1 = \rho_2$. The parabolic velocity relationship for flow through section (2) is used in Eq. 4 to yield

$$2\pi u_{\max} \int_0^R \left[1 - \left(\frac{r}{R}\right)^2\right] r \, dr - A_1 U = 0 \qquad \textbf{(5)}$$

Integrating, we get from Eq. 5

$$2\pi u_{\max} \left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)_0^R - \pi R^2 U = 0$$

or

$$u_{\max} = 2U \qquad \textbf{(Ans)}$$

(b) Since this flow is incompressible, we conclude from Eq. 5.7 that U is the average velocity at all sections of the control volume. Thus, the average velocity at section (2), $\overline{V}_2$, is one-half the maximum velocity, $u_{\max}$, there or

$$\overline{V}_2 = \frac{u_{\max}}{2} \qquad \textbf{(Ans)}$$

COMMENT The relationship between the maximum velocity at section (2) and the average velocity is a function of the "shape" of the velocity profile. For the parabolic profile assumed in this example, the average velocity, $u_{\max}/2$, is the actual "average" of the maximum velocity at section (2), $u_2 = u_{\max}$, and the minimum velocity at that section, $u_2 = 0$. However, as shown in Fig. E5.4*c*, if the velocity profile is a different shape (non-parabolic), the average velocity is not necessarily one half of the maximum velocity.

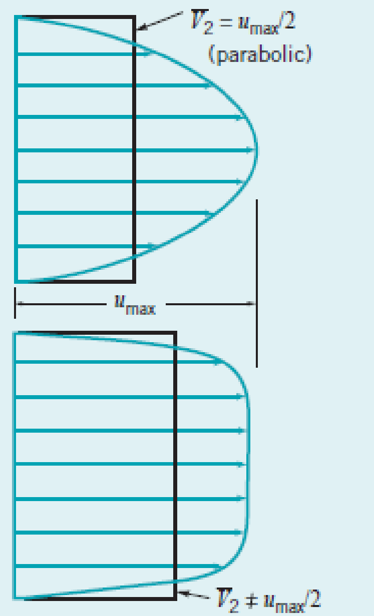
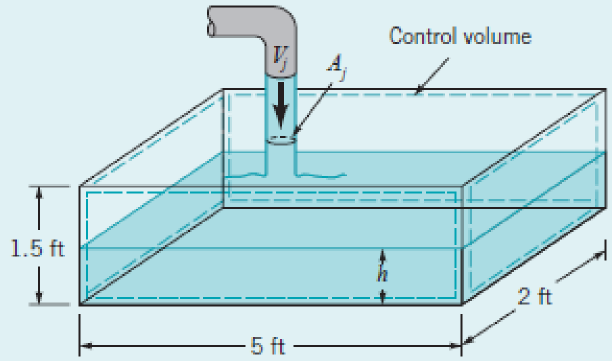


FIGURE E5.4*c*

EXAMPLE 5.5 Conservation of Mass—Unsteady Flow

GIVEN A bathtub is being filled with water from a faucet. The rate of flow from the faucet is steady at 9 gal/min. The tub volume is approximated by a rectangular space as indicated in Fig. E5.5*a*.

FIND Estimate the time rate of change of the depth of water in the tub, $\partial h/\partial t$, in inches per minute at any instant.



■ FIGURE E5.5*a*

SOLUTION

We use the fixed, nondeforming control volume outlined with a dashed line in Fig. E5.5*a*. This control volume includes in it, at any instant, the water accumulated in the tub, some of the water flowing from the faucet into the tub, and some air. Application of Eqs. 5.4 and 5.5 to these contents of the control volume results in

$$\frac{\partial}{\partial t}\int_{\substack{\text{air}\\\text{volume}}} \rho_{\text{air}}\, d\tilde{V}_{\text{air}} + \frac{\partial}{\partial t}\int_{\substack{\text{water}\\\text{volume}}} \rho_{\text{water}}\, d\tilde{V}_{\text{water}} - \dot{m}_{\text{water}} + \dot{m}_{\text{air}} = 0 \qquad (1)$$

Recall that the mass, *dm*, of fluid contained in a small volume $d\tilde{V}$ is $dm = \rho\, d\tilde{V}$. Hence, the two *integrals* in Eq. 1 represent the total amount of air and water in the control volume, and the sum of the first two *terms* is the time rate of change of mass within the control volume.

for air, and

$$\frac{\partial}{\partial t}\int_{\substack{\text{water}\\\text{volume}}} \rho_{\text{water}}\, d\tilde{V}_{\text{water}} = \dot{m}_{\text{water}} \qquad (2)$$

for water. The volume of water in the control volume is given by

$$\int_{\substack{\text{water}\\\text{volume}}} \rho_{\text{water}}\, d\tilde{V}_{\text{water}} = \rho_{\text{water}}\left[h(2\text{ ft})(5\text{ ft}) + (1.5\text{ ft} - h)A_j\right] \qquad (3)$$

Note that the time rate of change of air mass and water mass are each not zero. Recognizing, however, that the air mass must be conserved, we know that the time rate of change of the mass of air in the control volume must be equal to the rate of air mass flow out of the control volume. For simplicity, we disregard any water evaporation that occurs. Thus, applying Eqs. 5.4 and 5.5 to the air only and to the water only, we obtain

$$\frac{\partial}{\partial t}\int_{\substack{\text{air}\\\text{volume}}} \rho_{\text{air}}\, d\forall_{\text{air}} + \dot{m}_{\text{air}} = 0$$

where A_j is the cross-sectional area of the water flowing from the faucet into the tub. Combining Eqs. 2 and 3, we obtain

$$\rho_{\text{water}}\,(10\text{ ft}^2 - A_j)\frac{\partial h}{\partial t} = \dot{m}_{\text{water}}$$

and, thus, since $\dot{m} = \rho Q$,

$$\frac{\partial h}{\partial t} = \frac{Q_{\text{water}}}{(10\text{ ft}^2 - A_j)}$$

For $A_j \ll 10\text{ ft}^2$ we can conclude that

$$\frac{\partial h}{\partial t} = \frac{Q_{\text{water}}}{(10\text{ ft}^2)}$$

or

$$\frac{\partial h}{\partial t} = \frac{(9\text{ gal/min})(12\text{ in./ft})}{(7.48\text{ gal/ft}^3)(10\text{ ft}^2)} = 1.44\text{ in./min} \qquad \textbf{(Ans)}$$

COMMENT By repeating the calculations for the same flowrate but with various water jet diameters, D_j, the results shown in Fig. E5.5*b* are obtained. With the flowrate held constant, the value of $\partial h/\partial t$ is nearly independent of the jet diameter for values of the diameter less than about 10 in.

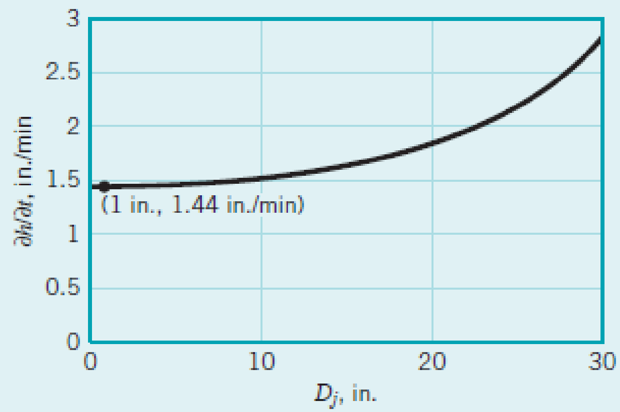


FIGURE E5.5*b*

The preceding example problems illustrate some important results of applying the conservation of mass principle to the contents of a fixed, nondeforming control volume. The dot product $\mathbf{V} \cdot \hat{\mathbf{n}}$ is "+" for flow out of the control volume and "−" for flow into the control volume. Thus, mass flowrate out of the control volume is "+" and mass flowrate in is "−." When the flow is steady, the time rate of change of the mass of the contents of the control volume

$$\frac{\partial}{\partial t} \int_{\text{cv}} \rho \, d\forall$$

is zero and the net amount of mass flowrate, $\dot{m}$, through the control surface is therefore also zero

$$\sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} = 0 \qquad \textbf{(5.9)}$$

If the steady flow is also incompressible, the net amount of volume flowrate, Q, through the control surface is also zero:

$$\sum Q_{\text{out}} - \sum Q_{\text{in}} = 0 \qquad \textbf{(5.10)}$$

An unsteady, but cyclical flow can be considered steady on a time-average basis. When the flow is unsteady, the instantaneous time rate of change of the mass of the contents of the control volume is not necessarily zero and can be an important variable. When the value of

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall$$

is "+," the mass of the contents of the control volume is increasing. When it is "−," the mass of the contents of the control volume is decreasing.

When the flow is uniformly distributed over the opening in the control surface (one-dimensional flow),

$$\dot{m} = \rho A V$$

where V is the uniform value of the velocity component normal to the section area A. When the velocity is nonuniformly distributed over the opening in the control surface,

$$\dot{m} = \rho A \overline{V} \tag{5.11}$$

where $\overline{V}$ is the average value of the component of velocity normal to the section area A as defined by Eq. 5.7.

For steady flow involving only one stream of a specific fluid flowing through the control volume at sections (1) and (2),

$$\dot{m} = \rho_1 A_1 \overline{V}_1 = \rho_2 A_2 \overline{V}_2 \tag{5.12}$$

and for incompressible flow,

$$Q = A_1 \overline{V}_1 = A_2 \overline{V}_2 \tag{5.13}$$

5.1.3 Moving, Nondeforming Control Volume

It is sometimes necessary to use a nondeforming control volume attached to a moving reference frame. Examples include control volumes containing a gas turbine engine on an aircraft in flight, the exhaust stack of a ship at sea, and the gasoline tank of an automobile passing by.

As discussed in Section 4.4.6, when a moving control volume is used, the fluid velocity relative to the moving control volume (relative velocity) is an important flow field variable. The relative velocity, $\mathbf{W}$, is the fluid velocity seen by an observer moving with the control volume. The control volume velocity, $\mathbf{V}_{cv}$, is the velocity of the control volume as seen from a fixed coordinate system. The absolute velocity, $\mathbf{V}$, is the fluid velocity seen by a stationary observer in a fixed coordinate system. These velocities are related to each other by the vector equation

$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{cv} \tag{5.14}$$

as illustrated by the figure in the margin. This is the same as Eq. 4.22, introduced earlier.

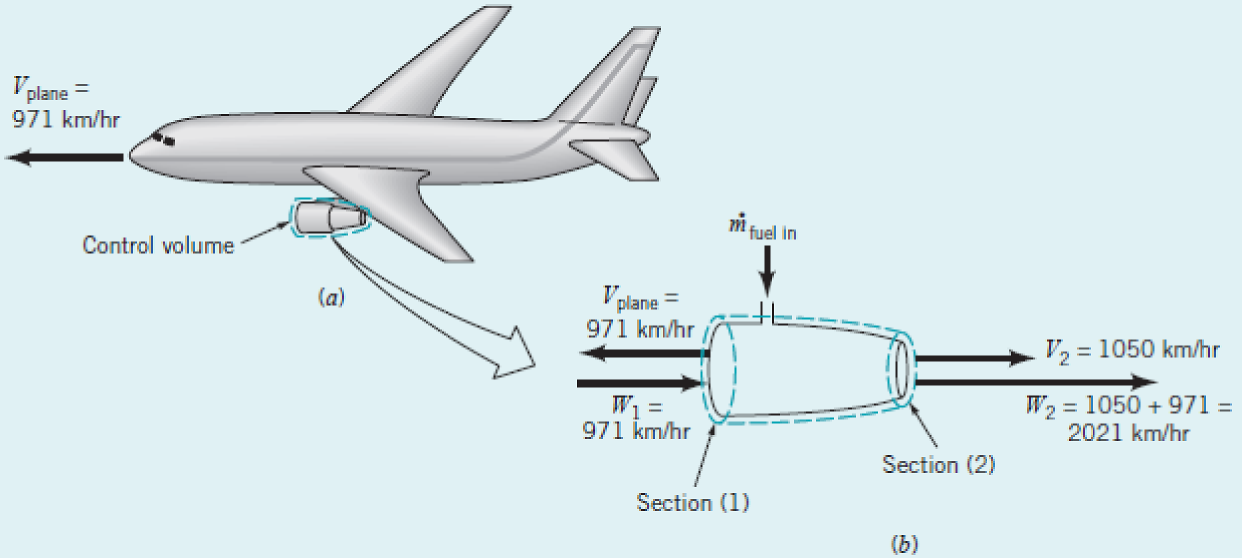
For a system and a moving, nondeforming control volume that are coincident at an instant of time, the Reynolds transport theorem (Eq. 4.23) for a moving control volume leads to

$$\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA \tag{5.15}$$

Example 5.6 Conservation of Mass—Compressible Flow with a Moving Control Volume

GIVEN An airplane moves forward at a speed of 971 km/hr as shown in Fig. E5.6a. The frontal intake area of the jet engine is 0.80 m^2 and the entering air density is 0.736 kg/m^3. A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is 0.558 m^2, and the exhaust gas density is 0.515 kg/m^3.

FIND Estimate the mass flowrate of fuel into the engine in kg/hr.



■ FIGURE E5.6

SOLUTION

The control volume, which moves with the airplane (see Fig. E5.6b), surrounds the engine and its contents and includes all fluids involved at an instant. The application of Eq. 5.16 to these contents of the control volume yields

$$\overset{0\ \text{(flow relative to moving control volume is considered steady on a time-average basis)}}{\cancel{\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\Psi}} + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0 \qquad (1)$$

Assuming one-dimensional flow, we evaluate the surface integral in Eq. 1 and get

$$-\dot{m}_{\substack{\text{fuel}\\\text{in}}} - \rho_1 A_1 W_1 + \rho_2 A_2 W_2 = 0$$

or

$$\dot{m}_{\substack{\text{fuel}\\\text{in}}} = \rho_2 A_2 W_2 - \rho_1 A_1 W_1 \qquad (2)$$

We consider the intake velocity, W_1, relative to the moving control volume, as being equal in magnitude to the speed of the airplane, 971 km/hr. The exhaust velocity, W_2, also needs to be measured relative to the moving control volume. Since a fixed observer noted that the exhaust gases were moving away from the engine at a speed of 1050 km/hr, the speed of the exhaust gases relative to the moving control volume, W_2, is determined as follows by using Eq. 5.14

$$V_2 = W_2 + V_{\text{plane}}$$

or

$$W_2 = V_2 - V_{\text{plane}} = 1050 \text{ km/hr} - (-971 \text{ km/hr})$$
$$= 2021 \text{ km/hr}$$

and is shown in Fig. E5.6*b*.

From Eq. 2,

$$\dot{m}_{\substack{\text{fuel}\\\text{in}}} = (0.515 \text{ kg/m}^3)(0.558 \text{ m}^2)(2021 \text{ km/hr})(1000 \text{ m/km})$$
$$- (0.736 \text{ kg/m}^3)(0.80 \text{ m}^2)(971 \text{ km/hr})(1000 \text{ m/km})$$
$$= (580{,}800 - 571{,}700) \text{ kg/hr}$$
$$\dot{m}_{\substack{\text{fuel}\\\text{in}}} = 9100 \text{ kg/hr} \qquad \textbf{(Ans)}$$

COMMENT Note that the fuel flowrate was obtained as the difference of two large, nearly equal numbers. Precise values of W_2 and W_1 are needed to obtain a modestly accurate value of $\dot{m}_{\substack{\text{fuel}\\\text{in}}}$.

EXAMPLE 5.7 Conservation of Mass—Relative Velocity

GIVEN Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in Fig. E5.7. The exit area of each of the two nozzles is 30 mm^2.

FIND Determine the average speed of the water leaving the nozzle, relative to the nozzle, if

(a) the rotary sprinkler head is stationary,

(b) the sprinkler head rotates at 600 rpm, and

(c) the sprinkler head accelerates from 0 to 600 rpm.

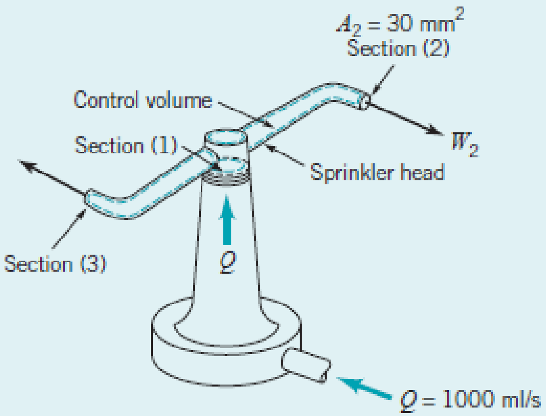


FIGURE E5.7

SOLUTION

(a) We specify a control volume that contains the water in the rotary sprinkler head at any instant. This control volume is nondeforming, but it moves (rotates) with the sprinkler head.
The application of Eq. 5.16 to the contents of this control volume for situation (a), (b), or (c) of the problem results in the same expression, namely

0 flow is steady or the control volume is filled with an incompressible fluid

$$\cancel{\frac{\partial}{\partial t}\int_{\mathrm{cv}} \rho \, d\Psi} + \int_{\mathrm{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0$$

or

$$\sum \rho_{\mathrm{out}} A_{\mathrm{out}} W_{\mathrm{out}} - \sum \rho_{\mathrm{in}} A_{\mathrm{in}} W_{\mathrm{in}} = 0 \qquad (1)$$

The time rate of change of the mass of water in the control volume is zero because the flow is steady and the control volume is filled with water.

Because there is only one inflow [at the base of the rotating arm, section (1)] and two outflows [the two nozzles at the tips of the arm, sections (2) and (3), each have the same area and fluid velocity], Eq. 1 becomes

$$\rho_2 A_2 W_2 + \rho_3 A_3 W_3 - \rho_1 A_1 W_1 = 0 \qquad (2)$$

Hence, for incompressible flow with $\rho_1 = \rho_2 = \rho_3$, Eq. 2 becomes

$$A_2 W_2 + A_3 W_3 - A_1 W_1 = 0$$

With $Q = A_1 W_1$, $A_2 = A_3$, and $W_2 = W_3$ it follows that

$$W_2 = \frac{Q}{2A_2}$$

or

$$W_2 = \frac{(1000\ \mathrm{ml/s})(0.001\ \mathrm{m^3/liter})(10^6\ \mathrm{mm^2/m^2})}{(1000\ \mathrm{ml/liter})(2)(30\ \mathrm{mm^2})}$$
$$= 16.7\ \mathrm{m/s} \qquad \textbf{(Ans)}$$

(b), (c) The value of W_2 is independent of the speed of rotation of the sprinkler head and represents the average velocity of the water exiting from each nozzle with respect to the nozzle for cases (a), (b), and (c).

COMMENT The velocity of water discharging from each nozzle, when viewed from a stationary reference (i.e., V_2), will vary as the rotation speed of the sprinkler head varies since from Eq. 5.14,

$$V_2 = W_2 - U$$

where $U = \omega R$ is the speed of the nozzle and ω and R are the angular velocity and radius of the sprinkler head, respectively.

5.1.4 Deforming Control Volume

Occasionally, a deforming control volume can simplify the solution of a problem. A deforming control volume involves changing volume size and control surface movement. Thus, the Reynolds transport theorem for a moving control volume can be used for this case, and Eqs. 4.23 and 5.1 lead to

$$\boxed{\frac{DM_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0} \qquad \textbf{(5.17)}$$

The time rate of change term in Eq. 5.17,

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall$$

is usually nonzero and must be carefully evaluated because the extent of the control volume varies with time. The mass flowrate term in Eq. 5.17,

$$\int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA$$

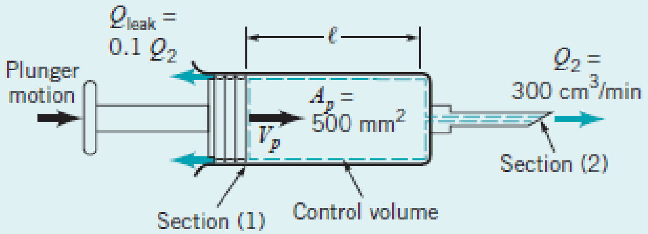
Since the control volume is deforming, the control surface velocity is not necessarily uniform and identical to the control volume velocity, $\mathbf{V}_{\text{cv}}$, as was true for moving, nondeforming control volumes. For the deforming control volume,

$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{\text{cs}} \qquad \textbf{(5.18)}$$

EXAMPLE 5.8 Conservation of Mass—Deforming Control Volume

GIVEN A syringe (Fig. E5.8) is used to inoculate a cow. The plunger has a face area of 500 mm^2. The liquid in the syringe is to be injected steadily at a rate of 300 cm^3/min. The leakage rate past the plunger is 0.10 times the volume flowrate out of the needle.

FIND With what speed should the plunger be advanced?



■ FIGURE E5.8

Solution

The control volume selected for solving this problem is the deforming one illustrated in Fig. E5.8. Section (1) of the control surface moves with the plunger. The surface area of section (1), A_1, is considered equal to the circular area of the face of the plunger, A_p, although this is not strictly true, since leakage occurs. The difference is small, however. Thus,

$$A_1 = A_p \tag{1}$$

Liquid also leaves the needle through section (2), which involves fixed area A_2. The application of Eq. 5.17 to the contents of this control volume gives

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\text{V} + \dot{m}_2 + \rho Q_{\text{leak}} = 0 \tag{2}$$

Even though Q_{leak} and the flow through section area A_2 are steady, the time rate of change of the mass of liquid in the shrinking control volume is not zero because the control volume is getting smaller. To evaluate the first term of Eq. 2, we note that

$$\int_{\text{cv}} \rho \, d\text{V} = \rho(\ell A_1 + \text{V}_{\text{needle}}) \tag{3}$$

where ℓ is the changing length of the control volume (see Fig. E5.8) and V_{needle} is the volume of the needle. From Eq. 3, we obtain

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\text{V} = \rho A_1 \frac{\partial \ell}{\partial t} \tag{4}$$

Note that

$$-\frac{\partial \ell}{\partial t} = V_p \tag{5}$$

where V_p is the speed of the plunger sought in the problem statement. Combining Eqs. 2, 4, and 5 we obtain

$$-\rho A_1 V_p + \dot{m}_2 + \rho Q_{\text{leak}} = 0 \tag{6}$$

However, from Eq. 5.6, we see that

$$\dot{m}_2 = \rho Q_2 \tag{7}$$

and Eq. 6 becomes

$$-\rho A_1 V_p + \rho Q_2 + \rho Q_{\text{leak}} = 0 \tag{8}$$

Solving Eq. 8 for V_p yields

$$V_p = \frac{Q_2 + Q_{\text{leak}}}{A_1} \tag{9}$$

Since $Q_{\text{leak}} = 0.1Q_2$, Eq. 9 becomes

$$V_p = \frac{Q_2 + 0.1Q_2}{A_1} = \frac{1.1Q_2}{A_1}$$

and

$$V_p = \frac{(1.1)(300 \text{ cm}^3/\text{min})}{(500 \text{ mm}^2)}\left(\frac{1000 \text{ mm}^3}{\text{cm}^3}\right)$$
$$= 660 \text{ mm/min} \qquad \textbf{(Ans)}$$

Example 5.9 Conservation of Mass—Deforming Control Volume

GIVEN Consider Example 5.5.

FIND Solve the problem of Example 5.5 using a deforming control volume that includes only the water accumulating in the bathtub.

Solution

For this deforming control volume, Eq. 5.17 leads to

$$\frac{\partial}{\partial t}\int_{\substack{\text{water}\\ \text{volume}}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0 \tag{1}$$

The first term of Eq. 1 can be evaluated as

$$\frac{\partial}{\partial t}\int_{\substack{\text{water}\\ \text{volume}}} \rho \, d\forall = \frac{\partial}{\partial t}[\rho h(2\text{ ft})(5\text{ ft})]$$

$$= \rho\,(10\text{ ft}^2)\frac{\partial h}{\partial t} \tag{2}$$

The second term of Eq. 1 can be evaluated as

$$\int_{\text{cs}} \rho \, \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = -\rho\left(V_j + \frac{\partial h}{\partial t}\right)A_j \tag{3}$$

where A_j and V_j are the cross-sectional area and velocity of the water flowing from the faucet into the tube. Thus, from Eqs. 1, 2, and 3 we obtain

$$\frac{\partial h}{\partial t} = \frac{V_j A_j}{(10\text{ ft}^2 - A_j)} = \frac{Q_{\text{water}}}{(10\text{ ft}^2 - A_j)}$$

or for $A_j \ll 10\text{ ft}^2$

$$\frac{\partial h}{\partial t} = \frac{9(\text{gal/min})(12\text{ in./ft})}{(7.48\text{ gal/ft}^3)(10\text{ ft}^2)} = 1.44\text{ in./min} \qquad \textbf{(Ans)}$$

COMMENT Note that these results using a deforming control volume are the same as that obtained in Example 5.5 with a fixed control volume.

5.2 Newton's Second Law—The Linear Momentum and Moment-of-Momentum Equations

5.2.1 Derivation of the Linear Momentum Equation

Newton's second law of motion for a system is

time rate of change of the linear momentum of the system	=	sum of external forces acting on the system

Since momentum is mass times velocity, the momentum of a small particle of mass $\rho d\forall$ is $\mathbf{V}\rho d\forall$. Thus, the momentum of the entire system is $\int_{\text{sys}} \mathbf{V}\rho d\forall$ and Newton's law becomes

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho \, d\forall = \sum \mathbf{F}_{\text{sys}} \quad (5.19)$$

$$\sum \mathbf{F}_{\text{sys}} = \sum \mathbf{F}_{\substack{\text{contents of the} \\ \text{coincident control volume}}} \quad (5.20)$$

Furthermore, for a system and the contents of a coincident control volume that is fixed and nondeforming, the Reynolds transport theorem [Eq. 4.19 with b set equal to the velocity (i.e., momentum per unit mass), and B_{sys} being the system momentum] allows us to conclude that

$$\frac{D}{Dt}\int_{sys} \mathbf{V}\rho\, d\forall = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho\, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA \tag{5.21}$$

or

time rate of change of the linear momentum of the system	=	time rate of change of the linear momentum of the contents of the control volume	+	net rate of flow of linear momentum through the control surface

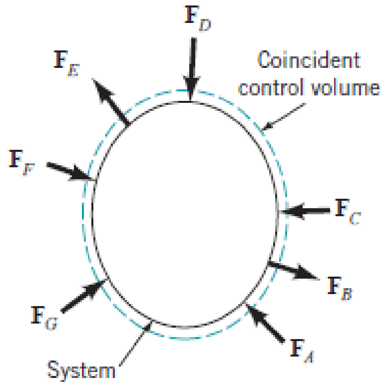
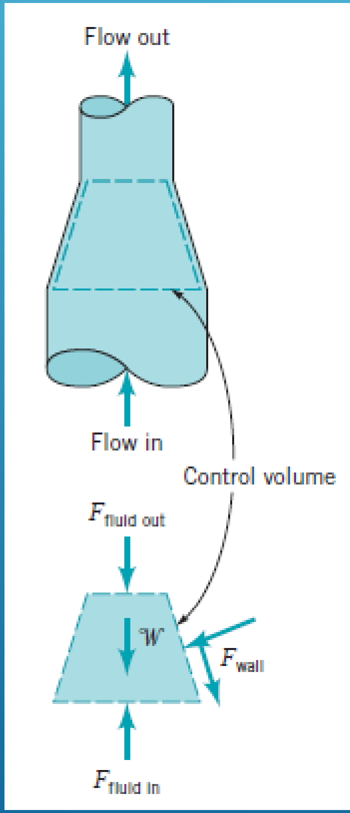


FIGURE 5.2 External forces acting on system and coincident control volume.

For a control volume that is fixed (and thus inertial) and nondeforming, Eqs. 5.19, 5.20, and 5.21 provide an appropriate mathematical statement of Newton's second law of motion as

$$\boxed{\frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho \, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}} \qquad (5.22)$$

We call Eq. 5.22 the ***linear momentum equation***.



5.2.2 Application of the Linear Momentum Equation

The linear momentum equation for an inertial control volume is a vector equation (Eq. 5.22). In engineering applications, components of this vector equation resolved along orthogonal coordinates, for example, x, y, and z (rectangular coordinate system) or r, θ, and x (cylindrical coordinate system), will normally be used. A simple example involving steady, incompressible flow is considered first.

EXAMPLE 5.10 Linear Momentum—Change in Flow Direction

GIVEN As shown in Fig. E5.10*a*, a horizontal jet of water exits a nozzle with a uniform speed of $V_1 = 10$ ft/s, strikes a vane, and is turned through an angle θ.

FIND Determine the anchoring force needed to hold the vane stationary if gravity and viscous effects are negligible.

Solution

We select a control volume that includes the vane and a portion of the water (see Figs. E5.10*b*, *c*) and apply the linear momentum equation to this fixed control volume. The only portions of the control surface across which fluid flows are section (1) (the entrance) and section (2) (the exit). Hence, the x and z components of Eq. 5.22 become

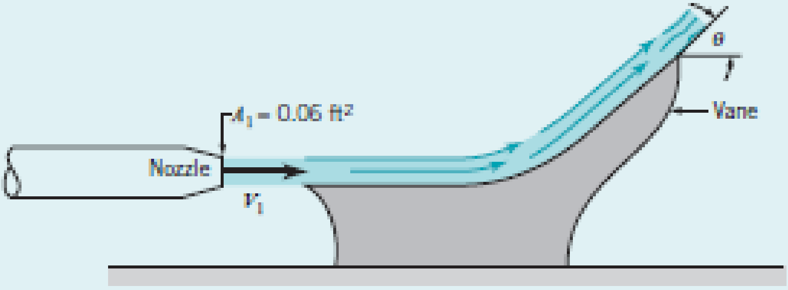
$$\frac{\partial}{\partial t}\int_{\text{cv}} \overbrace{\cancel{u}\,\rho\, d\Psi}^{0 \text{ (flow is steady)}} + \int_{\text{cs}} u\,\rho\,\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \sum F_x$$

and

$$\frac{\partial}{\partial t}\int_{\text{cv}} \overbrace{\cancel{w}\,\rho\, d\Psi}^{0 \text{ (flow is steady)}} + \int_{\text{cs}} w\,\rho\,\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \sum F_z$$

or

$$u_2\rho A_2 V_2 - u_1\rho A_1 V_1 = \sum F_x \qquad \textbf{(1)}$$



(*a*)

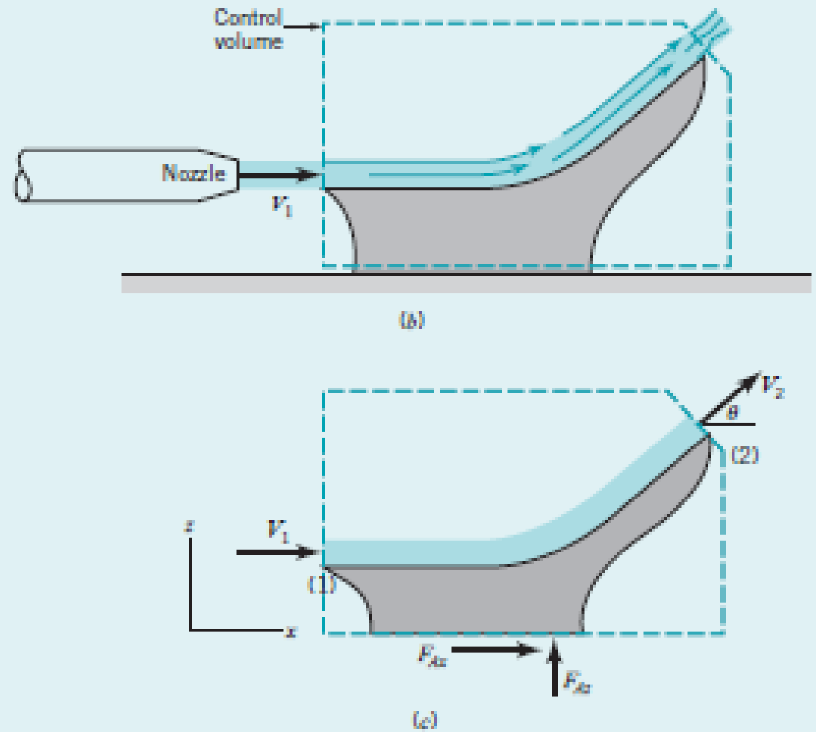


FIGURE E5.10

and

$$w_2 \rho A_2 V_2 - w_1 \rho A_1 V_1 = \sum F_z \qquad (2)$$

where $\mathbf{V} = u\hat{\mathbf{i}} + w\hat{\mathbf{k}}$, and $\sum F_x$ and $\sum F_z$ are the net x and z components of force acting on the contents of the control volume. Depending on the particular flow situation being considered and the coordinate system chosen, the x and z components of velocity, u and w, can be positive, negative, or zero. In this example the flow is in the positive directions at both the inlet and the outlet.

With negligible gravity and viscous effects, and since $p_1 = p_2$, the speed of the fluid remains constant so that $V_1 = V_2 = 10$ ft/s (see the Bernoulli equation, Eq. 3.7). Hence, at section (1), $u_1 = V_1$, $w_1 = 0$, and at section (2), $u_2 = V_1 \cos\theta$, $w_2 = V_1 \sin\theta$.

By using this information, Eqs. 1 and 2 can be written as

$$V_1 \cos\theta\, \rho A_2 V_1 - V_1\, \rho A_1 V_1 = F_{Ax} \tag{3}$$

and

$$V_1 \sin\theta\, \rho A_2 V_1 - 0\, \rho A_1 V_1 = F_{Az} \tag{4}$$

Equations 3 and 4 can be simplified by using conservation of mass, which states that for this incompressible flow $A_1V_1 = A_2V_2$, or $A_1 = A_2$ since $V_1 = V_2$. Thus

$$F_{Ax} = -\rho A_1 V_1^2 + \rho A_1 V_1^2 \cos\theta = -\rho A_1 V_1^2 (1 - \cos\theta) \tag{5}$$

and

$$F_{Az} = \rho A_1 V_1^2 \sin\theta \tag{6}$$

With the given data we obtain

$$\begin{aligned} F_{Ax} &= -(1.94 \text{ slugs/ft}^3)(0.06 \text{ ft}^2)(10 \text{ ft/s})^2(1 - \cos\theta) \\ &= -11.64(1 - \cos\theta) \text{ slugs} \cdot \text{ft/s}^2 \\ &= -11.64(1 - \cos\theta) \text{ lb} \qquad \textbf{(Ans)} \end{aligned}$$

and

$$\begin{aligned} F_{Az} &= (1.94 \text{ slugs/ft}^3)(0.06 \text{ ft}^2)(10 \text{ ft/s})^2 \sin\theta \\ &= 11.64 \sin\theta \text{ lb} \qquad \textbf{(Ans)} \end{aligned}$$

EXAMPLE 5.11 Linear Momentum—Weight, Pressure, and Change in Speed

GIVEN As shown in Fig. E5.11*a*, water flows through a nozzle attached to the end of a laboratory sink faucet with a flowrate of 0.6 liters/s. The nozzle inlet and exit diameters are 16 and 5 mm, respectively, and the nozzle axis is vertical. The mass of the nozzle is 0.1 kg. The pressure at section (1) is 464 kPa.

FIND Determine the anchoring force required to hold the nozzle in place.

SOLUTION

The anchoring force sought is the reaction force between the faucet and nozzle threads. To evaluate this force we select a control volume that includes the entire nozzle and the water contained in the nozzle at an instant, as is indicated in Figs. E5.11*a* and E5.11*b*. All of the vertical forces acting on the contents of this control volume are identified in Fig. E5.11*b*. The action of atmospheric pressure cancels out in every direction and is not shown. Gage pressure forces do not cancel out in the vertical direction and are shown. Application of the vertical or z direction component of Eq. 5.22 to the contents of this control volume leads to

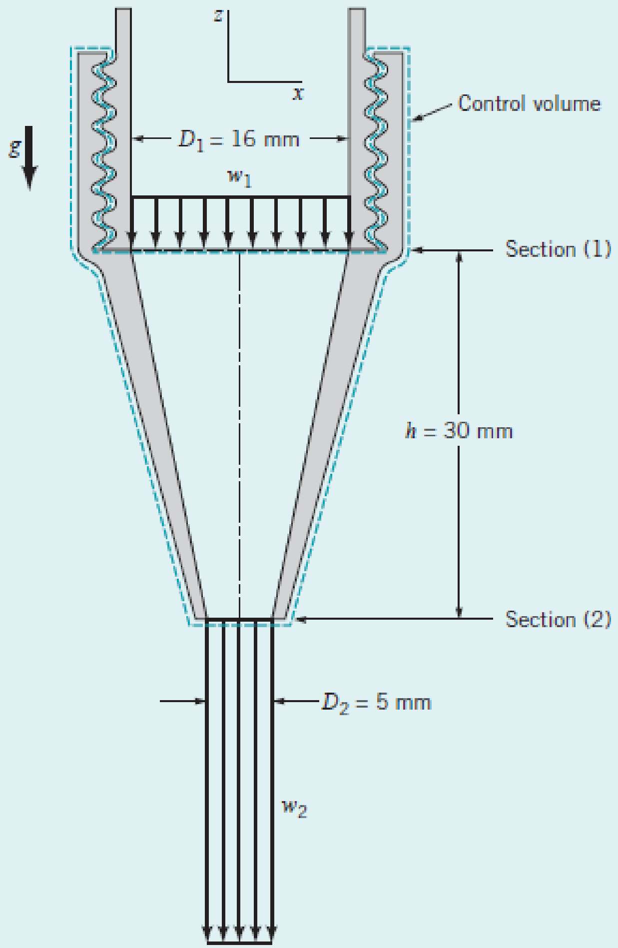
$$\overbrace{\cancel{\frac{\partial}{\partial t}\int_{\text{cv}} w\rho\, d\mathcal{V}}}^{0\ \text{(flow is steady)}} + \int_{\text{cs}} w\rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = F_A - \mathcal{W}_n - p_1A_1 - \mathcal{W}_w + p_2A_2 \qquad (1)$$

where w is the z direction component of fluid velocity, and the various parameters are identified in the figure.

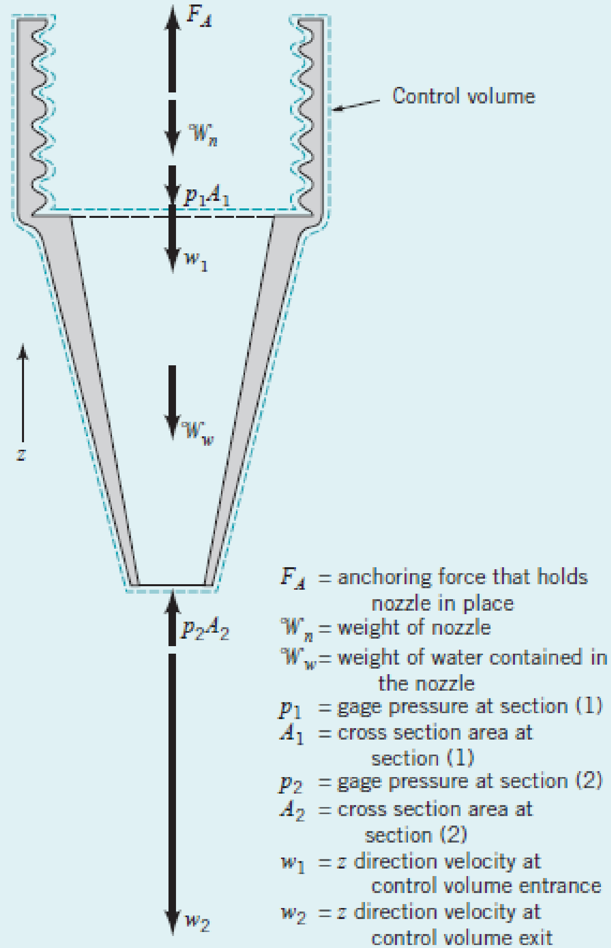
Note that the positive direction is considered "up" for the forces. We will use this same sign convention for the fluid velocity, w, in Eq. 1. In Eq. 1, the dot product, $\mathbf{V}\cdot\hat{\mathbf{n}}$, is "+" for flow out of the control volume and "−" for flow into the control volume. For this particular example

$$\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \pm|w|\, dA \qquad (2)$$

with the "+" used for flow out of the control volume and "−" used for flow in. To evaluate the control surface integral in Eq. 1, we need to assume a distribution for fluid velocity, w, and fluid density, ρ. For simplicity, we assume that w is uniformly distributed or constant, with magnitudes of w_1 and w_2 over cross-sectional areas A_1 and A_2. Also, this flow is incompressible so the



■ FIGURE E5.11a



■ FIGURE E5.11b

fluid density, ρ, is constant throughout. Proceeding further we obtain for Eq. 1

$$(-\dot{m}_1)(-w_1) + \dot{m}_2(-w_2) \\ = F_A - \mathcal{W}_n - p_1A_1 - \mathcal{W}_w + p_2A_2 \quad (3)$$

where $\dot{m} = \rho A V$ is the mass flowrate.

Note that $-w_1$ and $-w_2$ are used because both of these velocities are "down." Also, $-\dot{m}_1$ is used because it is associated with flow into the control volume. Similarly, $+\dot{m}_2$ is used because it is associated with flow out of the control volume. Solving Eq. 3 for the anchoring force, F_A, we obtain

$$F_A = \dot{m}_1 w_1 - \dot{m}_2 w_2 + \mathcal{W}_n + p_1A_1 + \mathcal{W}_w - p_2A_2 \quad (4)$$

From the conservation of mass equation, Eq. 5.12, we obtain

$$\dot{m}_1 = \dot{m}_2 = \dot{m} \quad (5)$$

which when combined with Eq. 4 gives

$$F_A = \dot{m}(w_1 - w_2) + \mathcal{W}_n + p_1A_1 + \mathcal{W}_w - p_2A_2 \quad (6)$$

It is instructive to note how the anchoring force is affected by the different actions involved. As expected, the nozzle weight, $\mathcal{W}_n$, the water weight, $\mathcal{W}_w$, and gage pressure force at section (1), p_1A_1, all increase the anchoring force, while the gage pressure force at section (2), p_2A_2, acts to decrease the anchoring force. The change in the vertical momentum flowrate, $\dot{m}(w_1 - w_2)$, will, in this instance, decrease the anchoring force because this change is negative $(w_2 > w_1)$.

To complete this example we use quantities given in the problem statement to quantify the terms on the right-hand side of Eq. 6.

From Eq. 5.6,

$$\dot{m} = \rho w_1 A_1 = \rho Q \\ = (999\ \text{kg/m}^3)(0.6\ \text{liter/s})(10^{-3}\ \text{m}^3/\text{liter}) \\ = 0.599\ \text{kg/s} \quad (7)$$

and

$$w_1 = \frac{Q}{A_1} = \frac{Q}{\pi(D_1^2/4)} \\ = \frac{(0.6\ \text{liter/s})(10^{-3}\ \text{m}^3/\text{liter})}{\pi(16\ \text{mm})^2/4(1000^2\ \text{mm}^2/\text{m}^2)} = 2.98\ \text{m/s} \quad (8)$$

Also from Eq. 5.6,

$$w_2 = \frac{Q}{A_2} = \frac{Q}{\pi(D_2^2/4)} \\ = \frac{(0.6\ \text{liter/s})(10^{-3}\ \text{m}^3/\text{liter})}{\pi(5\ \text{mm})^2/4(1000^2\ \text{mm}^2/\text{m}^2)} = 30.6\ \text{m/s} \quad (9)$$

The weight of the nozzle, $\mathcal{W}_n$, can be obtained from the nozzle mass, m_n, with

$$\mathcal{W}_n = m_n g = (0.1\ \text{kg})(9.81\ \text{m/s}^2) = 0.981\ \text{N} \quad (10)$$

The weight of the water in the control volume, $\mathcal{W}_w$, can be obtained from the water density, ρ, and the volume of water, $\mathcal{V}_w$, in the truncated cone of height h. That is,

$$\mathcal{W}_w = \rho \mathcal{V}_w g$$

where

$$\begin{aligned}\mathcal{V}_w &= \tfrac{1}{12}\pi h(D_1^2 + D_2^2 + D_1 D_2) \\ &= \frac{1}{12}\pi \frac{(30\text{ mm})}{(1000\text{ mm/m})} \\ &\quad \times \left[\frac{(16\text{ mm})^2 + (5\text{ mm})^2 + (16\text{ mm})(5\text{ mm})}{(1000^2\text{ mm}^2/\text{m}^2)}\right] \\ &= 2.84 \times 10^{-6}\text{ m}^3\end{aligned}$$

Thus,

$$\begin{aligned}\mathcal{W}_w &= (999\text{ kg/m}^3)(2.84 \times 10^{-6}\text{ m}^3)(9.81\text{ m/s}^2) \\ &= 0.0278\text{ N}\end{aligned} \qquad \textbf{(11)}$$

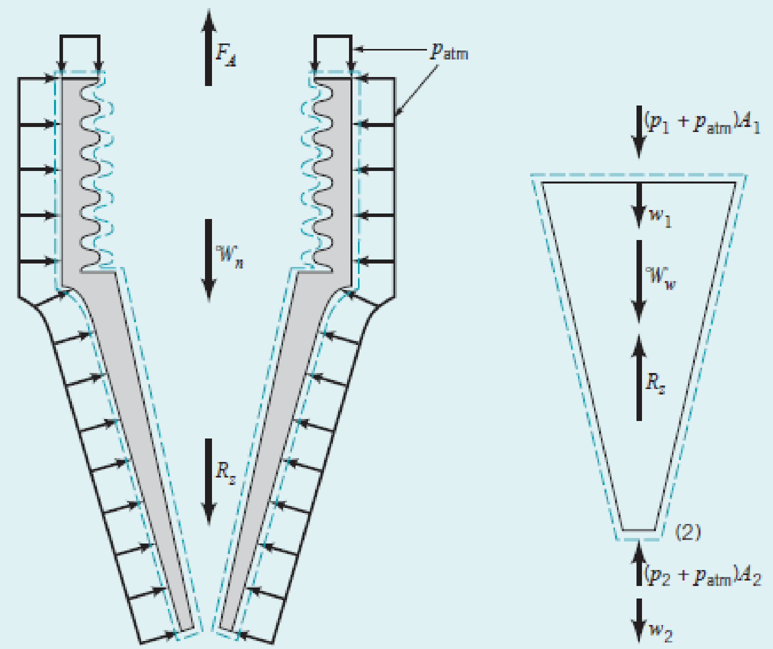
The gage pressure at section (2), p_2, is zero since, as discussed in Section 3.6.1, when a subsonic flow discharges to the atmosphere as in the present situation, the discharge pressure is essentially atmospheric. The anchoring force, F_A, can now be determined from Eqs. 6 through 11 with

$$\begin{aligned}F_A &= (0.599\text{ kg/s})(2.98\text{ m/s} - 30.6\text{ m/s}) + 0.981\text{ N} \\ &\quad + (464\text{ kPa})(1000\text{ Pa/kPa})\frac{\pi(16\text{ mm})^2}{4(1000^2\text{ mm}^2/\text{m}^2)} \\ &\quad + 0.0278\text{ N} - 0\end{aligned}$$

or

$$\begin{aligned}F_A &= -16.5\text{ N} + 0.981\text{ N} + 93.3\text{ N} + 0.0278\text{ N} \\ &= 77.8\text{ N}\end{aligned} \qquad \textbf{(Ans)}$$

Since the anchoring force, F_A, is positive, it acts upward in the z direction. The nozzle would be pushed off the pipe if it were not fastened securely.



EXAMPLE 5.14 Linear Momentum—Weight, Pressure, Friction, and Nonuniform Velocity Profile

GIVEN Consider the flow of Example 5.4 to be vertically upward.

FIND Develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

SOLUTION

A control volume (see dashed lines in Fig. E5.14) that includes only fluid from section (1) to section (2) is selected. The forces acting on the fluid in this control volume are identified in Fig. E5.14. The application of the axial component of Eq. 5.22 to the fluid in this control volume results in

$$\int_{\text{cs}} w\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = p_1 A_1 - R_z - \mathcal{W} - p_2 A_2 \tag{1}$$

where R_z is the resultant force of the wetted pipe wall on the fluid. Further, for uniform flow at section (1), and because the flow at section (2) is out of the control volume, Eq. 1 becomes

$$(+w_1)(-\dot{m}_1) + \int_{A_2} (+w_2)\rho(+w_2\, dA_2) = p_1 A_1 - R_z - \mathcal{W} - p_2 A_2 \tag{2}$$

The positive direction is considered up. The surface integral over the cross-sectional area at section (2), A_2, is evaluated by using the parabolic velocity profile obtained in Example 5.4, $w_2 = 2w_1[1 - (r/R)^2]$, as

$$\int_{A_2} w_2 \rho w_2\, dA_2 = \rho \int_0^R w_2^2\, 2\pi r\, dr$$

$$= 2\pi\rho \int_0^R (2w_1)^2 \left[1 - \left(\frac{r}{R}\right)^2\right]^2 r\, dr$$

or

$$\int_{A_2} w_2 \rho w_2\, dA_2 = 4\pi\rho w_1^2 \frac{R^2}{3} \tag{3}$$

Combining Eqs. 2 and 3 we obtain

$$-w_1^2 \rho\pi R^2 + \tfrac{4}{3} w_1^2 \rho\pi R^2 = p_1 A_1 - R_z - \mathcal{W} - p_2 A_2 \tag{4}$$

Solving Eq. 4 for the pressure drop from section (1) to section (2), $p_1 - p_2$, we obtain

$$p_1 - p_2 = \frac{\rho w_1^2}{3} + \frac{R_z}{A_1} + \frac{\mathcal{W}}{A_1} \qquad \textbf{(Ans)}$$

COMMENT We see that the drop in pressure from section (1) to section (2) occurs because of the following:

1. The change in momentum flow between the two sections associated with going from a uniform velocity profile to a parabolic velocity profile, $\rho w_1^2/3$
2. Pipe wall friction, R_z
3. The weight of the water column, $\mathcal{W}$; a hydrostatic pressure effect.

If the velocity profiles had been identically parabolic at sections (1) and (2), the momentum flowrate at each section would have

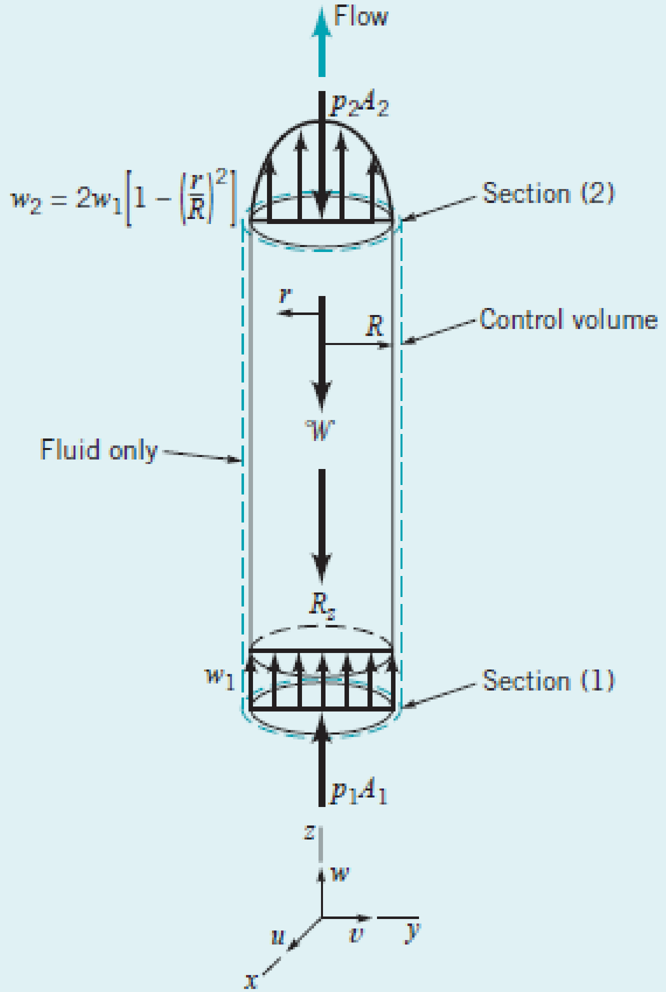
Note that although the average velocity is the same at section (1) as it is at section (2) ($\overline{V}_1 = \overline{V}_2 = w_1$), the momentum flux across section (1) is not the same as it is across section (2). If it were, the left-hand side of Eq. (4) would be zero. For this nonuniform flow the momentum flux can be written in terms of the average velocity, $\overline{V}$, and the *momentum coefficient*, β, as

$$\beta = \frac{\int w\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA}{\rho \overline{V}^2 A}$$

Hence the momentum flux can be written as

$$\int_{cs} w\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = -\beta_1 w_1^2 \rho \pi R^2 + \beta_2 w_1^2 \rho \pi R^2$$

where $\beta_1 = 1$ ($\beta = 1$ for uniform flow) and $\beta_2 = 4/3$ ($\beta > 1$ for any nonuniform flow).



All of the linear momentum examples considered thus far have involved stationary and nondeforming control volumes which are thus inertial because there is no acceleration. A nondeforming control volume translating in a straight line at constant speed is also inertial because there is no acceleration. For a system and an inertial, moving, nondeforming control volume that are both coincident at an instant of time, the Reynolds transport theorem (Eq. 4.23) leads to

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho\, d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA \tag{5.23}$$

When we combine Eq. 5.23 with Eqs. 5.19 and 5.20, we get

$$\frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}} \tag{5.24}$$

When the equation relating absolute, relative, and control volume velocities (Eq. 5.14) is used with Eq. 5.24, the result is

$$\frac{\partial}{\partial t}\int_{\text{cv}} (\mathbf{W}+\mathbf{V}_{\text{cv}})\rho\, d\forall + \int_{\text{cs}} (\mathbf{W}+\mathbf{V}_{\text{cv}})\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}} \tag{5.25}$$

For a constant control volume velocity, $\mathbf{V}_{cv}$, and steady flow in the control volume reference frame,

$$\frac{\partial}{\partial t}\int_{cv}(\mathbf{W}+\mathbf{V}_{cv})\rho\, d\forall = 0 \qquad (5.26)$$

Also, for this inertial, nondeforming control volume

$$\int_{cs}(\mathbf{W}+\mathbf{V}_{cv})\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \int_{cs}\mathbf{W}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA + \mathbf{V}_{cv}\int_{cs}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA \qquad (5.27)$$

For steady flow (on an instantaneous or time-average basis), Eq. 5.15 gives

$$\int_{cs}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = 0 \qquad (5.28)$$

Combining Eqs. 5.25, 5.26, 5.27, and 5.28, we conclude that the linear momentum equation for an inertial, moving, nondeforming control volume that involves steady (instantaneous or time-average) flow is

$$\boxed{\int_{cs}\mathbf{W}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}} \qquad (5.29)$$

5.2.3 Derivation of the Moment-of-Momentum Equation[2]

In many engineering problems, the moment of a force with respect to an axis, namely, *torque*, is important. Newton's second law of motion has already led to a useful relationship between forces and linear momentum flow. The linear momentum equation can also be used to solve problems involving torques. However, by forming the moment of the linear momentum and the resultant force associated with each particle of fluid with respect to a point in an inertial coordinate system, we will develop a ***moment-of-momentum equation*** that relates *torques* and *angular momentum flow* for the contents of a control volume. When torques are important, the moment-of-momentum equation is often more convenient to use than the linear momentum equation.

Application of Newton's second law of motion to a particle of fluid yields

$$\frac{D}{Dt}(\mathbf{V}\rho\,\delta\mathcal{V}) = \delta\mathbf{F}_{\text{particle}} \qquad \textbf{(5.30)}$$

$$\mathbf{r} \times \frac{D}{Dt}(\mathbf{V}\rho\, \delta \forall) = \mathbf{r} \times \delta \mathbf{F}_{\text{particle}} \tag{5.31}$$

where $\mathbf{r}$ is the position vector from the origin of the inertial coordinate system to the fluid particle (Fig. 5.3). We note that

$$\frac{D}{Dt}[(\mathbf{r} \times \mathbf{V})\rho\, \delta \forall] = \frac{D\mathbf{r}}{Dt} \times \mathbf{V}\rho\, \delta \forall + \mathbf{r} \times \frac{D(\mathbf{V}\rho\, \delta \forall)}{Dt} \tag{5.32}$$

and

$$\frac{D\mathbf{r}}{Dt} = \mathbf{V} \tag{5.33}$$

Thus, since

$$\mathbf{V} \times \mathbf{V} = 0 \tag{5.34}$$

by combining Eqs. 5.31, 5.32, 5.33, and 5.34, we obtain the expression

$$\frac{D}{Dt}[(\mathbf{r} \times \mathbf{V})\rho\, \delta \forall] = \mathbf{r} \times \delta \mathbf{F}_{\text{particle}} \tag{5.35}$$

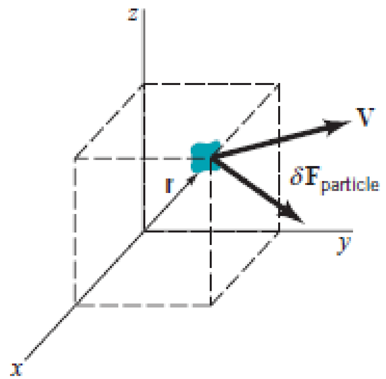


FIGURE 5.3 Inertial coordinate system.

Equation 5.35 is valid for every particle of a system. For a system (collection of fluid particles), we need to use the sum of both sides of Eq. 5.35 to obtain

$$\int_{\text{sys}} \frac{D}{Dt}[(\mathbf{r} \times \mathbf{V})\rho \, d\forall] = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} \tag{5.36}$$

where

$$\sum \mathbf{r} \times \delta \mathbf{F}_{\text{particle}} = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} \tag{5.37}$$

We note that

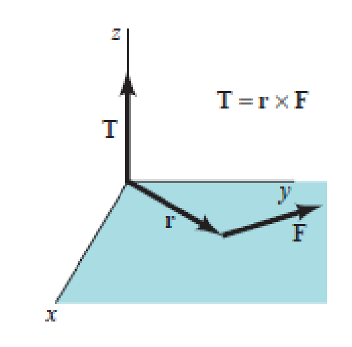
$$\frac{D}{Dt}\int_{\text{sys}} (\mathbf{r} \times \mathbf{V})\rho \, d\forall = \int_{\text{sys}} \frac{D}{Dt}[(\mathbf{r} \times \mathbf{V})\rho \, d\forall] \tag{5.38}$$

since the sequential order of differentiation and integration can be reversed without consequence. (Recall that the material derivative, $D(\ )/Dt$, denotes the time derivative following a given system; see Section 4.2.1.) Thus, from Eqs. 5.36 and 5.38 we get

$$\frac{D}{Dt}\int_{\text{sys}} (\mathbf{r} \times \mathbf{V})\rho \, d\forall = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} \tag{5.39}$$

or

the time rate of change of the moment-of-momentum of the system	=	sum of external torques acting on the system



$$\sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} = \sum (\mathbf{r} \times \mathbf{F})_{\text{cv}} \tag{5.40}$$

Further, for the system and the contents of the coincident control volume that is fixed and nondeforming, the Reynolds transport theorem (Eq. 4.19) leads to

$$\frac{D}{Dt} \int_{\text{sys}} (\mathbf{r} \times \mathbf{V}) \rho \, d\forall = \frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V}) \rho \, d\forall + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \tag{5.41}$$

or

time rate of change of the moment-of-momentum of the system	=	time rate of change of the moment-of-momentum of the contents of the control volume	+	net rate of flow of the moment-of-momentum through the control surface

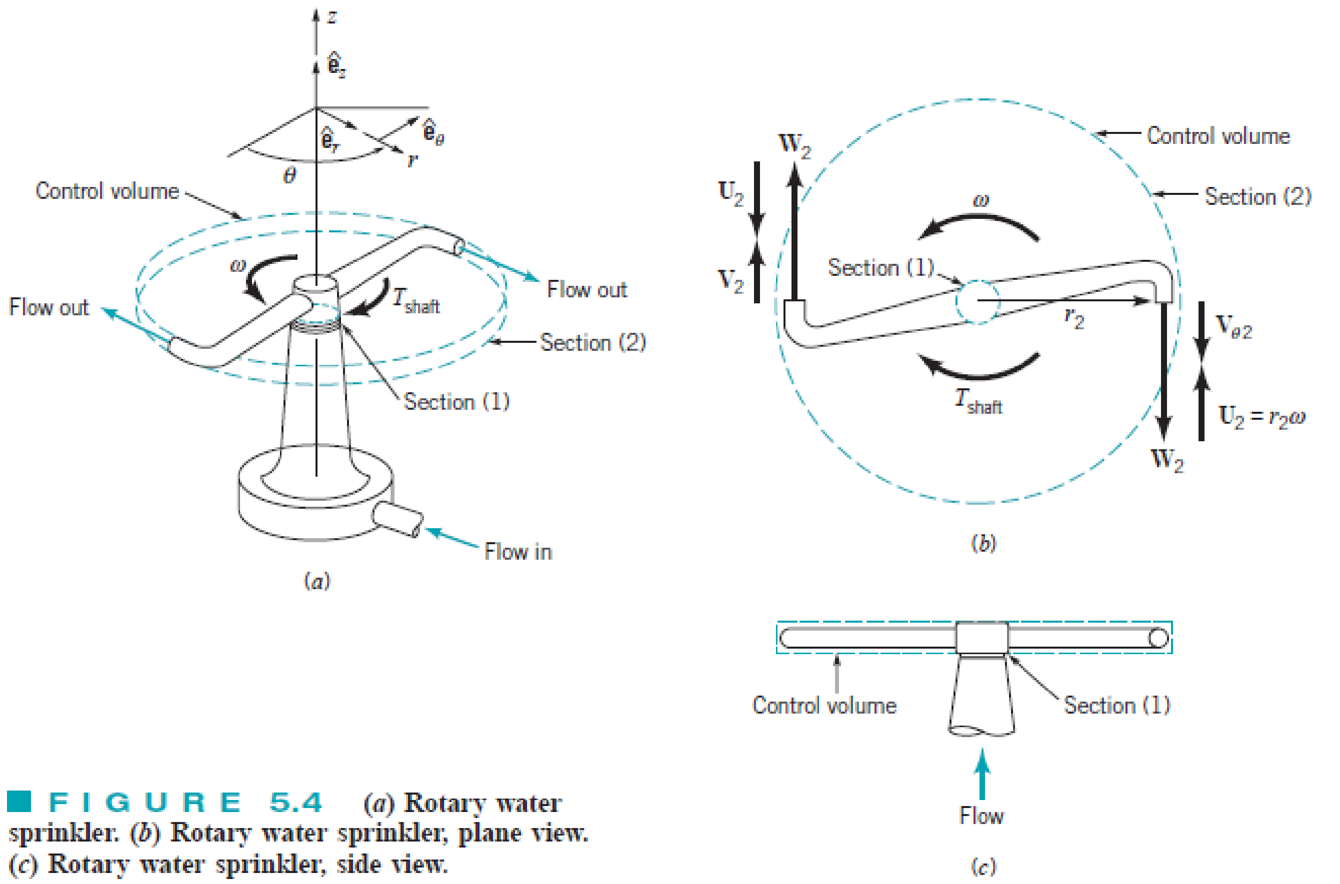
For a control volume that is fixed (and therefore inertial) and nondeforming, we combine Eqs. 5.39, 5.40, and 5.41 to obtain the moment-of-momentum equation:

$$\boxed{\frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V}) \rho \, d\forall + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \sum (\mathbf{r} \times \mathbf{F})_{\substack{\text{contents of the} \\ \text{control volume}}}} \tag{5.42}$$

5.2.4 Application of the Moment-of-Momentum Equation[3]

We simplify our use of Eq. 5.42 in several ways:

1. We assume that flows considered are one-dimensional (uniform distributions of average velocity at any section).



■ FIGURE 5.4 (*a*) Rotary water sprinkler. (*b*) Rotary water sprinkler, plane view. (*c*) Rotary water sprinkler, side view.

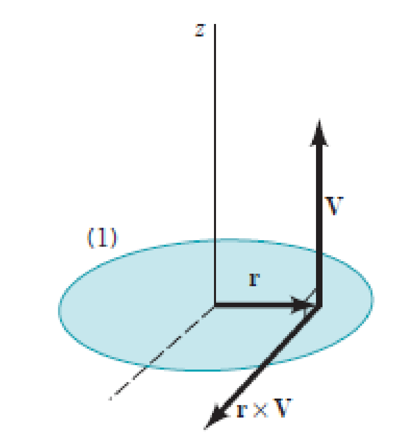
2. We confine ourselves to steady or steady-in-the-mean cyclical flows. Thus,

$$\frac{\partial}{\partial t}\int_{cv} (\mathbf{r} \times \mathbf{V})\rho \, d\forall = 0$$

at any instant of time for steady flows or on a time-average basis for cyclical unsteady flows.

3. We work only with the component of Eq. 5.42 resolved along the axis of rotation.

Change in moment of fluid velocity around an axis can result in torque and rotation around that same axis.



$$\mathbf{V} = \mathbf{W} + \mathbf{U} \tag{5.43}$$

where **U** is the velocity of the moving nozzle as measured relative to the fixed control surface.

The cross product and the dot product involved in the moment-of-momentum flow term of Eq. 5.42,

$$\int_{cs} (\mathbf{r} \times \mathbf{V})\rho\mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

$$\left[\int_{cs} (\mathbf{r} \times \mathbf{V})\rho\mathbf{V} \cdot \hat{\mathbf{n}}\, dA\right]_{\text{axial}} = (-r_2 V_{\theta 2})(+\dot{m}) \tag{5.44}$$

$$\sum \left[(\mathbf{r} \times \mathbf{F})_{\substack{\text{contents of the}\\ \text{control volume}}}\right]_{\text{axial}} = \mathbf{T}_{\text{shaft}} \tag{5.45}$$

$$-r_2 V_{\theta 2} \dot{m} = T_{\text{shaft}} \quad (5.46)$$

We interpret T_{shaft} being a negative quantity from Eq. 5.46 to mean that the shaft torque actually opposes the rotation of the sprinkler arms as shown in Fig. 5.4. The shaft torque, T_{shaft}, opposes rotation in all turbine devices.

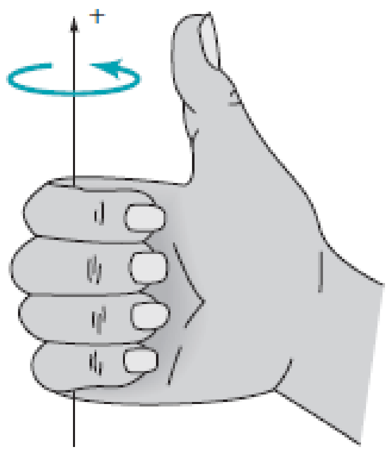


FIGURE 5.5 Right-hand rule convention.

We could evaluate the *shaft power*, $\dot{W}_{\text{shaft}}$, associated with *shaft torque*, T_{shaft}, by forming the product of T_{shaft} and the rotational speed of the shaft, ω. [We use the notation that W = work, $(\dot{\ }) = d(\ )/dt$, and thus $\dot{W}$ = power.] Thus, from Eq. 5.46 we get

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega = -r_2 V_{\theta 2}\dot{m}\,\omega \tag{5.47}$$

Since $r_2\omega$ is the speed of each sprinkler nozzle, U, we can also state Eq. 5.47 in the form

$$\dot{W}_{\text{shaft}} = -U_2 V_{\theta 2}\dot{m} \tag{5.48}$$

Shaft work per unit mass, w_{shaft}, is equal to $\dot{W}_{\text{shaft}}/\dot{m}$. Dividing Eq. 5.48 by the mass flowrate, $\dot{m}$, we obtain

$$w_{\text{shaft}} = -U_2 V_{\theta 2} \tag{5.49}$$

Negative shaft work as in Eqs. 5.47, 5.48, and 5.49 is work out of the control volume, that is, work done by the fluid on the rotor and thus its shaft.

The principles associated with this sprinkler example can be extended to handle most simplified turbomachine flows. The fundamental technique is not difficult. However, the geometry of some turbomachine flows is quite complicated.

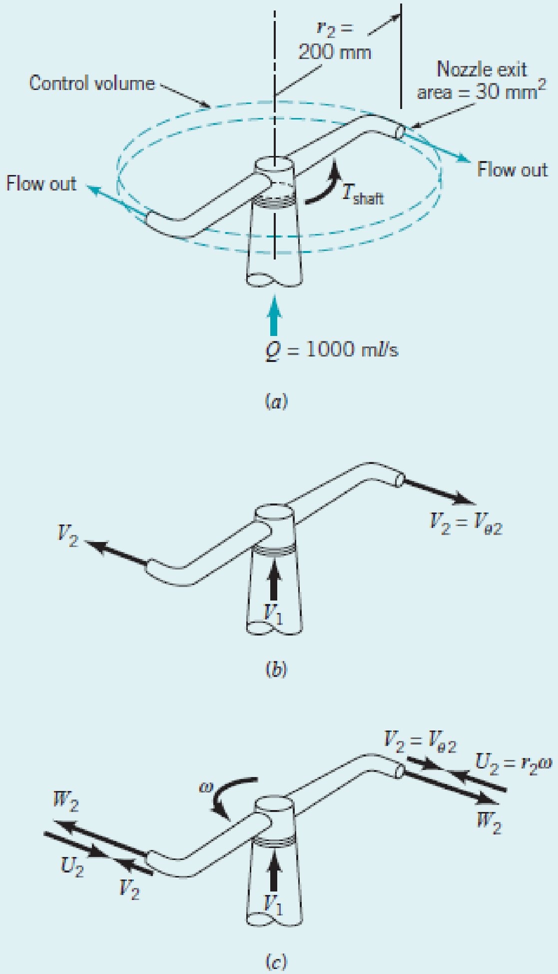
EXAMPLE 5.18 Moment-of-Momentum—Torque

GIVEN Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in Fig. E5.18*a*. The exit area of each of the two nozzles is 30 mm^2 and the flow leaving each nozzle is in the tangential direction. The radius from the axis of rotation to the centerline of each nozzle is 200 mm.

FIND **(a)** Determine the resisting torque required to hold the sprinkler head stationary.

(b) Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min.

(c) Determine the speed of the sprinkler if no resisting torque is applied.



■ FIGURE E5.18

SOLUTION

To solve parts (a), (b), and (c) of this example we can use the same fixed and nondeforming, disk-shaped control volume illustrated in Fig. 5.4. As indicated in Fig. E5.18*a*, the only axial torque considered is the one resisting motion, T_{shaft}.

(a) When the sprinkler head is held stationary as specified in part (a) of this example problem, the velocities of the fluid entering and leaving the control volume are shown in Fig. E5.18*b*. Equation 5.46 applies to the contents of this control volume. Thus,

$$T_{shaft} = -r_2 V_{\theta 2} \dot{m} \quad (1)$$

Since the control volume is fixed and nondeforming and the flow exiting from each nozzle is tangential,

$$V_{\theta 2} = V_2 \quad (2)$$

Equations 1 and 2 give

$$T_{shaft} = -r_2 V_2 \dot{m} \quad (3)$$

In Example 5.7, we ascertained that $V_2 = 16.7$ m/s. Thus, from Eq. 3 with

$$\dot{m} = Q\rho = \frac{(1000 \text{ ml/s})(10^{-3} \text{ m}^3/\text{liter})(999 \text{ kg/m}^3)}{(1000 \text{ ml/liter})}$$
$$= 0.999 \text{ kg/s}$$

we obtain

$$T_{\text{shaft}} = -\frac{(200 \text{ mm})(16.7 \text{ m/s})(0.999 \text{ kg/s})[1 (\text{N/kg})/(\text{m/s}^2)]}{(1000 \text{ mm/m})}$$

or

$$T_{\text{shaft}} = -3.34 \text{ N} \cdot \text{m} \qquad \textbf{(Ans)}$$

(b) When the sprinkler is rotating at a constant speed of 500 rpm, the flow field in the control volume is unsteady but cyclical. Thus, the flow field is steady in the mean. The velocities of the flow entering and leaving the control volume are as indicated in Fig. E5.18*c*. The absolute velocity of the fluid leaving each nozzle, V_2, is from Eq. 5.43,

$$V_2 = W_2 - U_2 \qquad \textbf{(4)}$$

where

$$W_2 = 16.7 \text{ m/s}$$

as determined in Example 5.7. The speed of the nozzle, U_2, is obtained from

$$U_2 = r_2\omega \qquad \textbf{(5)}$$

Application of the axial component of the moment-of-momentum equation (Eq. 5.46) leads again to Eq. 3. From Eqs. 4 and 5,

$$V_2 = 16.7 \text{ m/s} - r_2\omega$$
$$= 16.7 \text{ m/s} - \frac{(200 \text{ mm})(500 \text{ rev/min})(2\pi \text{ rad/rev})}{(1000 \text{ mm/m})(60 \text{ s/min})}$$

or

$$V_2 = 16.7 \text{ m/s} - 10.5 \text{ m/s} = 6.2 \text{ m/s}$$

Thus, using Eq. 3, with $\dot{m} = 0.999$ kg/s (as calculated previously), we get

$$T_{\text{shaft}} = -\frac{(200 \text{ mm})(6.2 \text{ m/s})\ 0.999 \text{ kg/s}\ [1 (\text{N/kg})/(\text{m/s}^2)]}{(1000 \text{ mm/m})}$$

or

$$T_{\text{shaft}} = -1.24 \text{ N} \cdot \text{m} \qquad \textbf{(Ans)}$$

(c) When no resisting torque is applied to the rotating sprinkler head, a maximum constant speed of rotation will occur as demonstrated below. Application of Eqs. 3, 4, and 5 to the contents of the control volume results in

$$T_{\text{shaft}} = -r_2(W_2 - r_2\omega)\dot{m} \tag{6}$$

For no resisting torque, Eq. 6 yields

$$0 = -r_2(W_2 - r_2\omega)\dot{m}$$

Thus,

$$\omega = \frac{W_2}{r_2} \tag{7}$$

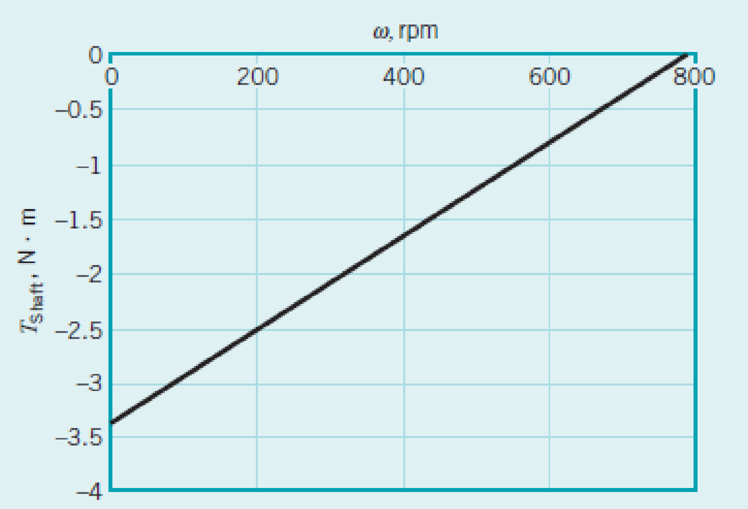
In Example 5.4, we learned that the relative velocity of the fluid leaving each nozzle, W_2, is the same regardless of the speed of rotation of the sprinkler head, ω, as long as the mass flowrate of the fluid, $\dot{m}$, remains constant. Thus, by using Eq. 7 we obtain

$$\omega = \frac{W_2}{r_2} = \frac{(16.7 \text{ m/s})(1000 \text{ mm/m})}{(200 \text{ mm})} = 83.5 \text{ rad/s}$$

or

$$\omega = \frac{(83.5 \text{ rad/s})(60 \text{ s/min})}{2\ \pi \text{ rad/rev}} = 797 \text{ rpm} \qquad \textbf{(Ans)}$$

For this condition ($T_{\text{shaft}} = 0$), the water both enters and leaves the control volume with zero angular momentum.



When the moment-of-momentum equation (Eq. 5.42) is applied to a more general, one-dimensional flow through a rotating machine, we obtain

$$\boxed{T_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm r_{\text{in}} V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm r_{\text{out}} V_{\theta\text{out}})} \quad \textbf{(5.50)}$$

The shaft power, $\dot{W}_{\text{shaft}}$, is related to shaft torque, T_{shaft}, by

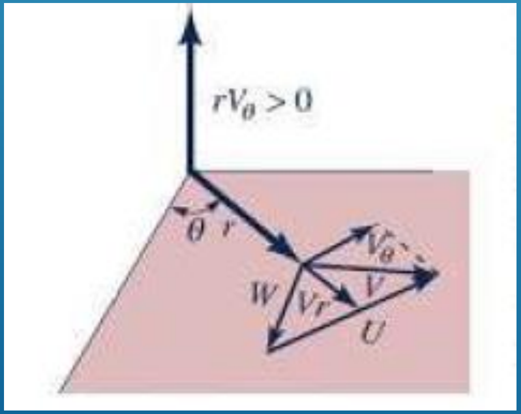
$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega \quad \textbf{(5.51)}$$

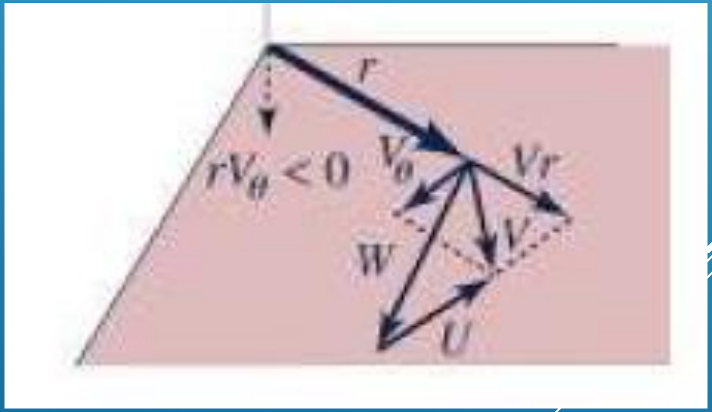
Thus, using Eqs. 5.50 and 5.51 with a "+" sign for T_{shaft} in Eq. 5.50, we obtain

$$\dot{W}_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm r_{\text{in}}\omega V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm r_{\text{out}}\omega V_{\theta\text{out}}) \quad \textbf{(5.52)}$$

or since $r\omega = U$

$$\boxed{\dot{W}_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm U_{\text{in}} V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm U_{\text{out}} V_{\theta\text{out}})} \quad \textbf{(5.53)}$$





From Eq. 5.53, we obtain

$$\boxed{w_{\text{shaft}} = -(\pm U_{\text{in}} V_{\theta \text{in}}) + (\pm U_{\text{out}} V_{\theta \text{out}})} \quad \textbf{(5.54)}$$

EXAMPLE 5.19 Moment-of-Momentum—Power

GIVEN An air fan has a bladed rotor of 12-in. outside diameter and 10-in. inside diameter as illustrated in Fig. E5.19*a*. The height of each rotor blade is constant at 1 in. from blade inlet to outlet. The flowrate is steady, on a time-average basis, at 230 ft^3/min and the absolute velocity of the air at blade inlet, $\mathbf{V}_1$, is radial. The blade discharge angle is 30° from the tangential direction. The rotor rotates at a constant speed of 1725 rpm.

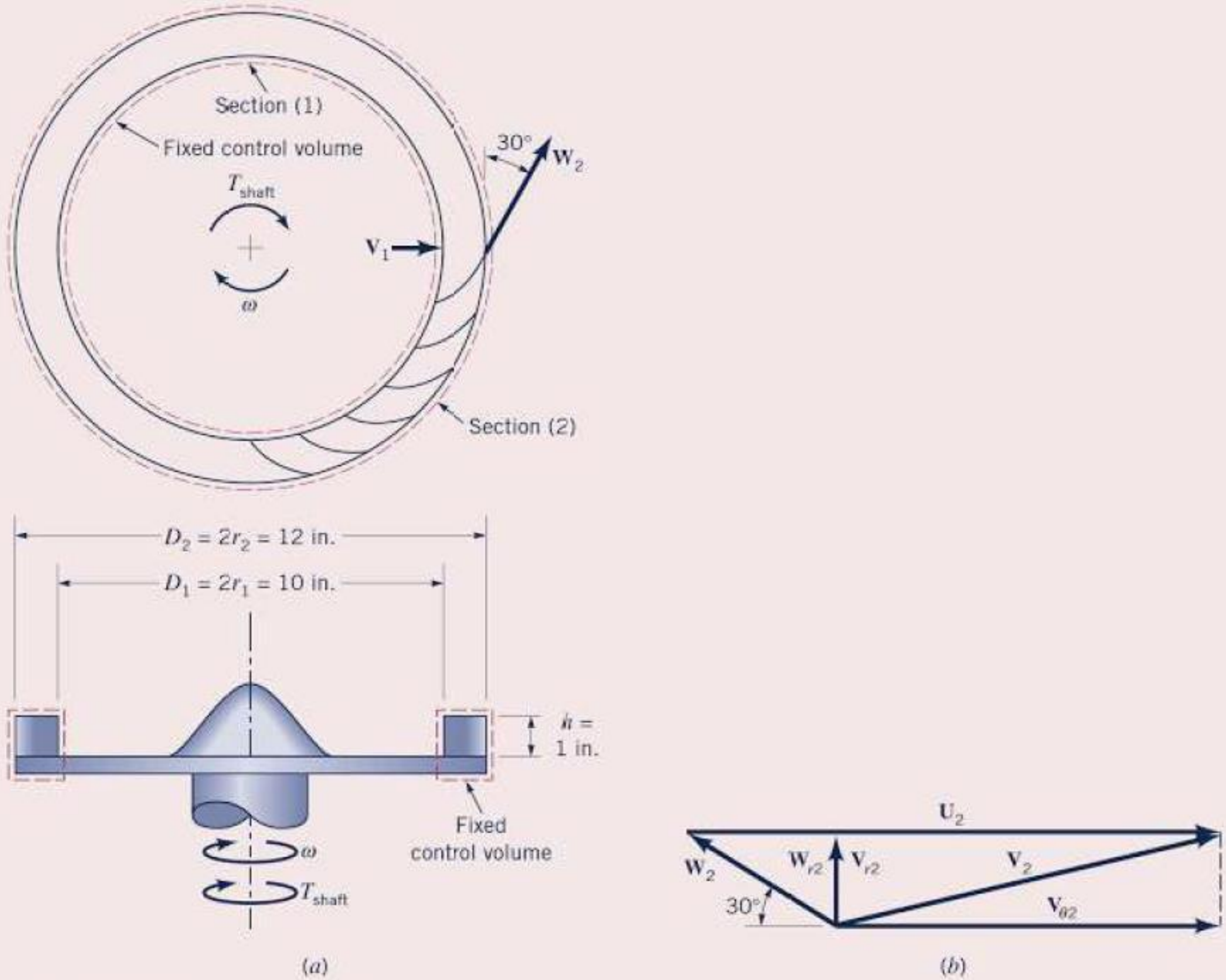
FIND Estimate the power required to run the fan.

SOLUTION

We select a fixed and nondeforming control volume that includes the rotating blades and the fluid within the blade row at an instant, as shown with a dashed line in Fig. E5.19*a*. The flow within this control volume is cyclical, but steady in the mean. The only torque we consider is the driving shaft torque, T_{shaft}. This torque is provided by a motor. We assume that the entering and leaving flows are each represented by uniformly distributed velocities and flow properties. Since shaft power is sought, Eq. 5.53 is appropriate. Application of Eq. 5.53 to the contents of the control volume in Fig. E5.19 gives

$$\dot{W}_{\text{shaft}} = -\dot{m}_1(\pm \underbrace{U_1 V_{\theta 1}}_{0\ (\mathbf{V}_1 \text{ is radial})}) + \dot{m}_2(\pm U_2 V_{\theta 2}) \quad \textbf{(1)}$$

From Eq. 1 we see that to calculate fan power, we need mass flowrate, $\dot{m}$, rotor exit blade velocity, U_2, and fluid tangential



Section (1)
Fixed control volume
T_{shaft}
ω
V_1
30°
W_2
Section (2)
$D_2 = 2r_2 = 12$ in.
$D_1 = 2r_1 = 10$ in.
$h =$ 1 in.
Fixed control volume
ω
T_{shaft}
(a)
U_2
W_2
W_{r2}
V_{r2}
V_2
$V_{\theta 2}$
30°
(b)

Figure E5.19

velocity at blade exit, $V_{\theta 2}$. The mass flowrate, $\dot{m}$, is easily obtained from Eq. 5.6 as

$$\dot{m} = \rho Q = \frac{(2.38 \times 10^{-3} \text{ slug/ft}^3)(230 \text{ ft}^3/\text{min})}{(60 \text{ s/min})}$$
$$= 0.00912 \text{ slug/s} \qquad \textbf{(2)}$$

Often, problems involving fans are solved using English Engineering units. Since 1 slug = 32.174 lbm, we could have used as the density of air $\rho_{\text{air}} = (2.38 \times 10^{-3} \text{ slug/ft}^3)(32.174 \text{ lbm/slug}) = 0.0766 \text{ lbm/ft}^3$.

Then

$$\dot{m} = \frac{(0.0766 \text{ lbm/ft}^3)(230 \text{ ft}^3/\text{min})}{(60 \text{ s/min})} = 0.294 \text{ lbm/s}$$

The rotor exit blade speed, U_2, is

$$U_2 = r_2\omega = \frac{(6 \text{ in.})(1725 \text{ rpm})(2\pi \text{ rad/rev})}{(12 \text{ in./ft})(60 \text{ s/min})}$$
$$= 90.3 \text{ ft/s} \qquad \textbf{(3)}$$

To determine the fluid tangential speed at the fan rotor exit, $V_{\theta 2}$, we use Eq. 5.43 to get

$$\mathbf{V}_2 = \mathbf{W}_2 + \mathbf{U}_2 \qquad \textbf{(4)}$$

The vector addition of Eq. 4 is shown in the form of a "velocity triangle" in Fig. E5.19*b*. From Fig. E5.19*b*, we can see that

$$V_{\theta 2} = U_2 - W_2 \cos 30^\circ \qquad \textbf{(5)}$$

To solve Eq. 5 for $V_{\theta 2}$ we need a value of W_2, in addition to the value of U_2 already determined (Eq. 3). To get W_2, we recognize that

$$W_2 \sin 30^\circ = V_{r2} \qquad \textbf{(6)}$$

where V_{r2} is the radial component of either $\mathbf{W}_2$ or $\mathbf{V}_2$. Also, using Eq. 5.6, we obtain

$$\dot{m} = \rho A_2 V_{r2} \qquad \textbf{(7)}$$

or since

$$A_2 = 2\pi r_2 h \qquad \textbf{(8)}$$

where h is the blade height, Eqs. 7 and 8 combine to form

$$\dot{m} = \rho 2\pi r_2 h V_{r2} \qquad \textbf{(9)}$$

Taking Eqs. 6 and 9 together we get

$$W_2 = \frac{\dot{m}}{\rho 2\pi r_2 h \sin 30^\circ} = \frac{\rho Q}{\rho 2\pi r_2 h \sin 30^\circ} \qquad \textbf{(10)}$$
$$= \frac{Q}{2\pi r_2 h \sin 30^\circ}$$

Substituting known values into Eq. 10, we obtain

$$W_2 = \frac{(230 \text{ ft}^3/\text{min})(12 \text{ in./ft})(12 \text{ in./ft})}{(60 \text{ s/min})2\pi(6 \text{ in.})(1 \text{ in.}) \sin 30^\circ}$$
$$= 29.3 \text{ ft/s}$$

By using this value of W_2 in Eq. 5 we get

$$V_{\theta 2} = U_2 - W_2 \cos 30^\circ$$
$$= 90.3 \text{ ft/s} - (29.3 \text{ ft/s})(0.866) = 64.9 \text{ ft/s}$$

Equation 1 can now be used to obtain

$$\dot{W}_{\text{shaft}} = \dot{m} U_2 V_{\theta 2} = \frac{(0.00912 \text{ slug/s})(90.3 \text{ ft/s})(64.9 \text{ ft/s})}{[1 \text{ (slug} \cdot \text{ft/s}^2)/\text{lb}][550 \text{ (ft} \cdot \text{lb)/(hp} \cdot \text{s)}]}$$

with BG units.

With EE units

$$\dot{W}_{\text{shaft}} = \frac{(0.294 \text{ lbm/s})(90.3 \text{ ft/s})(64.9 \text{ ft/s})}{[32.174 \text{ (lbm} \cdot \text{ft)/(lb/s}^2)][550 \text{ (ft} \cdot \text{lb)/(hp} \cdot \text{s)}]}$$

In either case

$$\dot{W}_{\text{shaft}} = 0.097 \text{ hp} \qquad \text{(Ans)}$$

COMMENT Note that the "+" was used with the $U_2 V_{\theta 2}$ product because U_2 and $V_{\theta 2}$ are in the same direction. This result, 0.097 hp, is the power that needs to be delivered through the fan shaft for the given conditions. Ideally, all of this power would go into the flowing air. However, because of fluid friction, only some of this power will produce useful effects (e.g., movement and pressure rise) on the air. How much useful effect depends on the efficiency of the energy transfer between the fan blades and the fluid. Also, extra power would be needed from the motor to overcome friction in shaft bearings and other mechanical resistance.

5.3.1 Derivation of the Energy Equation

The ***first law of thermodynamics*** for a system is, in words,

Time rate of increase of the total stored energy of the system	=	net time rate of energy addition by heat transfer into the system	+	net time rate of energy addition by work transfer into the system

In symbolic form, this statement is

$$\frac{D}{Dt}\int_{\text{sys}} e\rho\, d\mathcal{V} = \left(\sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}}\right)_{\text{sys}} + \left(\sum \dot{W}_{\text{in}} - \sum \dot{W}_{\text{out}}\right)_{\text{sys}}$$

or

$$\frac{D}{Dt}\int_{\text{sys}} e\rho\, d\mathcal{V} = (\dot{Q}_{\underset{\text{in}}{\text{net}}} + \dot{W}_{\underset{\text{in}}{\text{net}}})_{\text{sys}} \tag{5.55}$$

Some of these variables deserve a brief explanation before proceeding further. The total stored energy per unit mass for each particle in the system, e, is related to the internal energy per unit mass, $\check{u}$, the kinetic energy per unit mass, $V^2/2$, and the potential energy per unit mass, gz, by the equation

$$e = \check{u} + \frac{V^2}{2} + gz \tag{5.56}$$

The net ***rate of heat transfer*** into the system is denoted with $\dot{Q}_{\text{net in}}$, and the net rate of work transfer into the system is labeled $\dot{W}_{\text{net in}}$. Heat transfer and work transfer are considered "+" going into the system and "−" coming out. [It should be noted that the opposite sign convention is often used for work (see Ref. 3 for example).]

Equation 5.55 is valid for inertial and noninertial reference systems. We proceed to develop the control volume statement of the first law of thermodynamics. For the control volume that is coincident with the system at an instant of time

$$(\dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{net}\\ \text{in}}})_{\text{sys}} = (\dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{net}\\ \text{in}}})_{\substack{\text{coincident}\\ \text{control volume}}} \tag{5.57}$$

Furthermore, for the system and the contents of the coincident control volume that is fixed and nondeforming, the Reynolds transport theorem (Eq. 4.19 with the parameter b set equal to e) allows us to conclude that

$$\frac{D}{Dt}\int_{\text{sys}} e\rho\, d\mathcal{V} = \frac{\partial}{\partial t}\int_{\text{cv}} e\rho\, d\mathcal{V} + \int_{\text{cs}} e\rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA \tag{5.58}$$

or in words,

Time rate of increase of the total stored energy of the system	=	time rate of increase of the total stored energy of the contents of the control volume	+	net rate of flow of the total stored energy out of the control volume through the control surface

Combining Eqs. 5.55, 5.57, and 5.58, we get the control volume formula for the first law of thermodynamics:

$$\boxed{\frac{\partial}{\partial t}\int_{\text{cv}} e\rho \, d\mathcal{V} + \int_{\text{cs}} e\rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = (\dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{net}\\ \text{in}}})_{\text{cv}}} \tag{5.59}$$

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega$$

$$\dot{W}_{\substack{\text{shaft}\\ \text{net in}}} = \sum \dot{W}_{\substack{\text{shaft}\\ \text{in}}} - \sum \dot{W}_{\substack{\text{shaft}\\ \text{out}}} \tag{5.60}$$

$$\sigma = -p \tag{5.61}$$

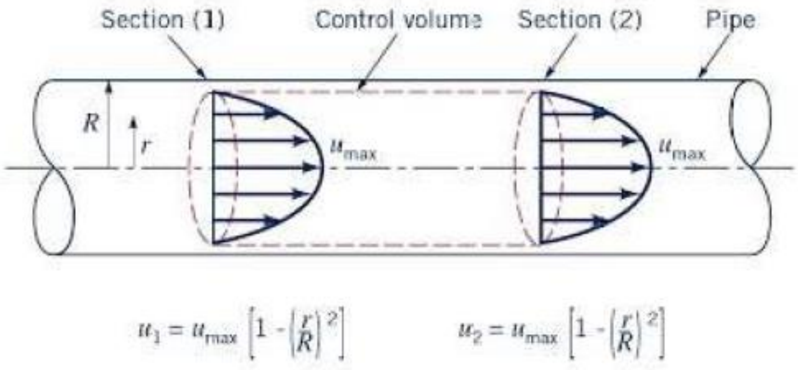
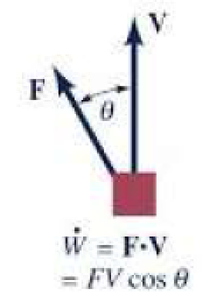


Figure 5.6 **Simple, fully developed pipe flow.**



The power transfer, $\dot{W}$, associated with a force $\mathbf{F}$ acting on an object moving with velocity $\mathbf{V}$ is given by the dot product $\mathbf{F} \cdot \mathbf{V}$. This is illustrated by the figure in the margin. Hence, the power transfer associated with normal stresses acting on a single fluid particle, $\delta \dot{W}_{\text{normal stress}}$, can be evaluated as the dot product of the normal stress force, $\delta \mathbf{F}_{\text{normal stress}}$, and the fluid particle velocity, $\mathbf{V}$, as

$$\delta \dot{W}_{\text{normal stress}} = \delta \mathbf{F}_{\text{normal stress}} \cdot \mathbf{V}$$

If the normal stress force is expressed as the product of local normal stress, $\sigma = -p$, and fluid particle surface area, $\hat{\mathbf{n}}\, \delta A$, the result is

$$\delta \dot{W}_{\text{normal stress}} = \sigma \hat{\mathbf{n}}\, \delta A \cdot \mathbf{V} = -p \hat{\mathbf{n}}\, \delta A \cdot \mathbf{V} = -p \mathbf{V} \cdot \hat{\mathbf{n}}\, \delta A$$

For all fluid particles on the control surface of Fig. 5.6 at the instant considered, power transfer due to fluid normal stress, $\dot{W}_{\text{normal stress}}$, is

$$\dot{W}_{\substack{\text{normal}\\ \text{stress}}} = \int_{\text{cs}} \sigma \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \int_{\text{cs}} -p \mathbf{V} \cdot \hat{\mathbf{n}}\, dA \qquad \textbf{(5.62)}$$

Work transfer can also occur at the control surface because of tangential stress forces. Rotating shaft work is transferred by tangential stresses in the shaft material. For a fluid particle, shear stress force power, $\delta \dot{W}_{\text{tangential stress}}$, can be evaluated as the dot product of tangential stress force, $\delta \mathbf{F}_{\text{tangential stress}}$, and the fluid particle velocity, $\mathbf{V}$. That is,

$$\delta \dot{W}_{\text{tangential stress}} = \delta \mathbf{F}_{\text{tangential stress}} \cdot \mathbf{V}$$

Using the information we have developed about power, we can express the first law of thermodynamics for the contents of a control volume by combining Eqs. 5.59, 5.60, and 5.62 to obtain

$$\frac{\partial}{\partial t}\int_{\mathrm{cv}} e\rho \, d\forall + \int_{\mathrm{cs}} e\rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \dot{Q}_{\substack{\mathrm{net}\\ \mathrm{in}}} + \dot{W}_{\substack{\mathrm{shaft}\\ \mathrm{net\ in}}} - \int_{\mathrm{cs}} p\mathbf{V}\cdot\hat{\mathbf{n}}\, dA \qquad \mathbf{(5.63)}$$

Finally, we multiply and divide by ρ inside the pressure integral, substitute the stored energy from Eq. 5.56, and group terms to obtain the ***energy equation***:

$$\boxed{\frac{\partial}{\partial t}\int_{\mathrm{cv}} e\rho \, d\forall + \int_{\mathrm{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \dot{Q}_{\substack{\mathrm{net}\\ \mathrm{in}}} + \dot{W}_{\substack{\mathrm{shaft}\\ \mathrm{net\ in}}}} \qquad \mathbf{(5.64)}$$

5.3.2 Application of the Energy Equation

In Eq. 5.64, the integrand of

$$\int_{\mathrm{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

can be nonzero only where fluid crosses the control surface ($\mathbf{V} \cdot \hat{\mathbf{n}} \neq 0$). Otherwise, $\mathbf{V} \cdot \hat{\mathbf{n}}$ is zero and the integrand is zero for that portion of the control surface. If the properties within parentheses, $\check{u}$, p/ρ, $V^2/2$, and gz, are all assumed to be uniformly distributed over the flow cross-sectional areas involved, the integration becomes simple and gives

$$\int_{\mathrm{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \sum_{\substack{\text{flow}\\ \text{out}}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \dot{m} - \sum_{\substack{\text{flow}\\ \text{in}}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \dot{m} \qquad \textbf{(5.65)}$$

Furthermore, if there is only one stream entering and leaving the control volume, then

$$\int_{\mathrm{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{out}} \dot{m}_{\text{out}} - \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{in}} \dot{m}_{\text{in}} \qquad \textbf{(5.66)}$$

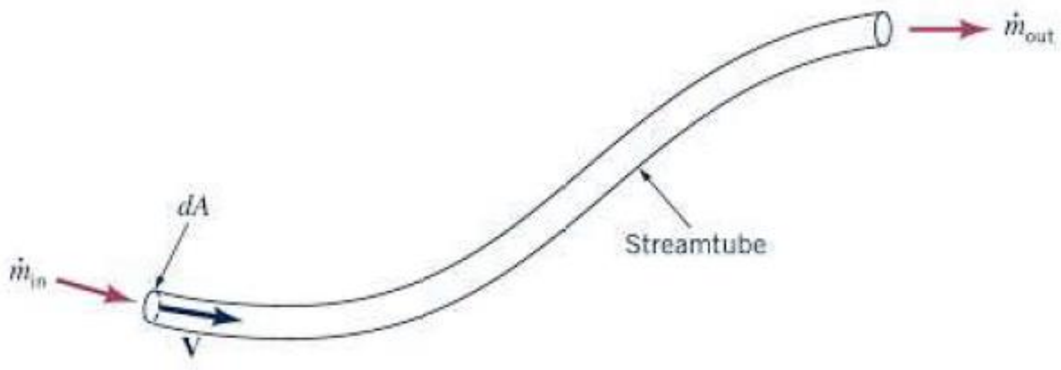
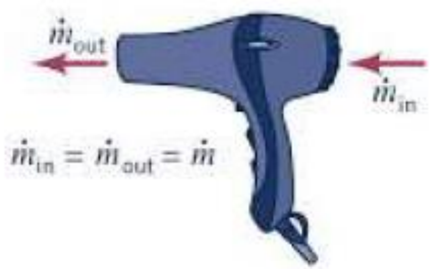


Figure 5.7 **Streamtube flow.**

$$\dot{m}\left[\check{u}_{out} - \check{u}_{in} + \left(\frac{p}{\rho}\right)_{out} - \left(\frac{p}{\rho}\right)_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{shaft\\net\ in}} \qquad \textbf{(5.67)}$$

We call Eq. 5.67 the *one-dimensional energy equation for steady-in-the-mean flow.*



$$\check{h} = \check{u} + \frac{p}{\rho} \qquad \textbf{(5.68)}$$

$$\dot{m}\left[\check{h}_{out} - \check{h}_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{shaft\\net\ in}} \qquad (5.69)$$

Equation 5.69 is often used for solving compressible flow problems.

EXAMPLE 5.20 Energy—Pump Power

GIVEN A pump delivers water at a steady rate of 300 gal/min as shown in Fig. E5.20. Just upstream of the pump [section (1)] where the pipe diameter is 3.5 in., the pressure is 18 psi. Just downstream of the pump [section (2)] where the pipe diameter is 1 in., the pressure is 60 psi. The change in water elevation across the pump is zero. The rise in internal energy of water, $\check{u}_2 - \check{u}_1$, associated with a temperature rise across the pump is 93 ft · lbf/lbm. The pumping process is considered to be adiabatic.

FIND Determine the power (hp) required by the pump.

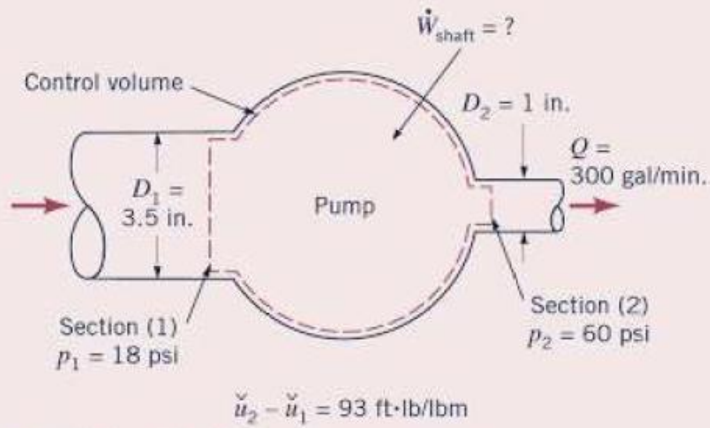


Figure E5.20

SOLUTION

We include in our control volume the water contained in the pump between its entrance and exit sections. Application of Eq. 5.67 to the contents of this control volume on a time-average basis yields

$$\dot{m}\left[\check{u}_2 - \check{u}_1 + \left(\frac{p}{\rho}\right)_2 - \left(\frac{p}{\rho}\right)_1 + \frac{V_2^2 - V_1^2}{2} + \overbrace{g(z_2 - z_1)}^{0 \text{ (no elevation change)}}\right]$$

$$= \underbrace{\dot{Q}_{\substack{\text{net}\\ \text{in}}}}_{0 \text{ (adiabatic flow)}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}} \qquad (1)$$

We can solve directly for the power required by the pump, $\dot{W}_{\text{shaft net in}}$, from Eq. 1, after we first determine the mass flowrate, $\dot{m}$, the speed of flow into the pump, V_1, and the speed of the flow out of the pump, V_2. All other quantities in Eq. 1 are given in the problem statement. From Eq. 5.6, we get

$$\dot{m} = \rho Q = \frac{(1.94 \text{ slugs/ft}^3)(300 \text{ gal/min})(32.174 \text{ lbm/slug})}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})}$$

$$= 41.8 \text{ lbm/s} \qquad (2)$$

Also from Eq. 5.6,

$$V = \frac{Q}{A} = \frac{Q}{\pi D^2/4}$$

so

$$V_1 = \frac{Q}{A_1} = \frac{(300 \text{ gal/min})4\ (12 \text{ in./ft})^2}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})\pi(3.5 \text{ in.})^2}$$

$$= 10.0 \text{ ft/s} \qquad (3)$$

and

$$V_2 = \frac{Q}{A_2} = \frac{(300 \text{ gal/min})4\,(12 \text{ in./ft})^2}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})\pi\,(1 \text{ in.})^2}$$

$$= 123 \text{ ft/s} \tag{4}$$

Substituting the values of Eqs. 2, 3, and 4 and values from the problem statement into Eq. 1 we obtain

$$\dot{W}_{\substack{\text{shaft}\\ \text{net in}}} = (41.8 \text{ lbm/s})\bigg[(93 \text{ ft} \cdot \text{lb/lbm})$$
$$+ \frac{(60 \text{ psi})(144 \text{ in.}^2/\text{ft}^2)}{(1.94 \text{ slugs/ft}^3)(32.174 \text{ lbm/slug})}$$
$$- \frac{(18 \text{ psi})(144 \text{ in.}^2/\text{ft}^2)}{(1.94 \text{ slugs/ft}^3)(32.174 \text{ lbm/slug})}$$
$$+ \frac{(123 \text{ ft/s})^2 - (10.0 \text{ ft/s})^2}{2[32.174\,(\text{lbm}\cdot\text{ft})/(\text{lb}\cdot\text{s}^2)]}\bigg]$$
$$\times \frac{1}{[550(\text{ft}\cdot\text{lb/s})/\text{hp}]} = 32.2 \text{ hp} \qquad \textbf{(Ans)}$$

COMMENT Of the total 32.2 hp, internal energy change accounts for 7.09 hp, the pressure rise accounts for 7.37 hp, and the kinetic energy increase accounts for 17.8 hp.

An actual pump would require slightly more than 32.2 hp owing to mechanical friction loss in bearings and shaft seals. The sum of the pressure rise, the kinetic energy increase, and any increase in elevation is sometimes called *water horsepower.*

If the flow is truly steady throughout, so that no work is done, one-dimensional, and only one fluid stream is involved, then setting the shaft work to zero, the energy equation becomes

$$\dot{m}\left[\breve{u}_{\text{out}} - \breve{u}_{\text{in}} + \left(\frac{p}{\rho}\right)_{\text{out}} - \left(\frac{p}{\rho}\right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} \quad \textbf{(5.70)}$$

We call Eq. 5.70 the *one-dimensional, steady-flow energy equation.*

$$\dot{m}\left[\breve{h}_{\text{out}} - \breve{h}_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} \quad \textbf{(5.71)}$$

EXAMPLE 5.22 Energy—Temperature Change

GIVEN The 420-ft waterfall shown in Fig. E5.22*a* involves steady flow from one large body of water to another.

FIND Determine the temperature change associated with this flow.

SOLUTION

To solve this problem, we consider a control volume consisting of a small cross-sectional streamtube from the nearly motionless surface of the upper body of water to the nearly motionless surface of the lower body of water as is sketched in Fig. E5.22*b*. We need to determine $T_2 - T_1$. This temperature change is related to the change of internal energy of the water, $\check{u}_2 - \check{u}_1$, by the relationship

$$T_2 - T_1 = \frac{\check{u}_2 - \check{u}_1}{\check{c}} \tag{1}$$

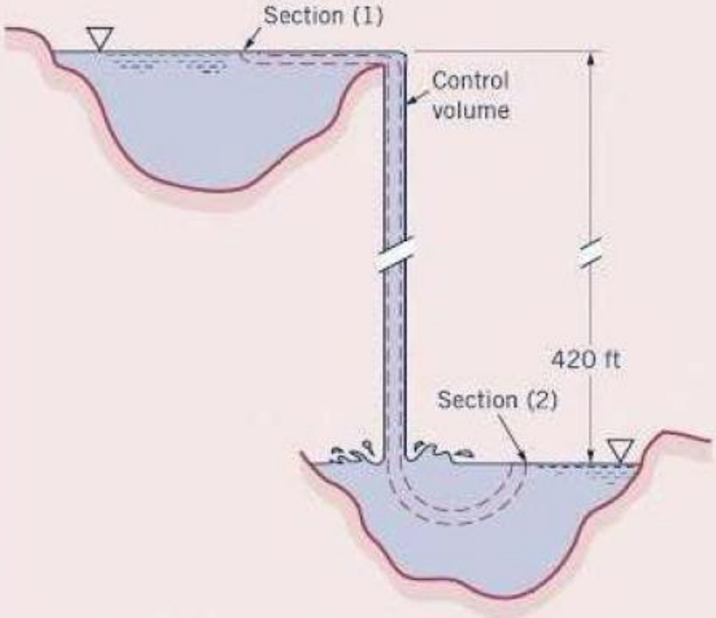


Figure E5.22*b*

where $\check{c} = 1$ Btu/(lbm · °R) is the specific heat of water. The application of Eq. 5.70 to the contents of this control volume leads to

$$\dot{m}\left[\check{u}_2 - \check{u}_1 + \left(\frac{p}{\rho}\right)_2 - \left(\frac{p}{\rho}\right)_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} \qquad (2)$$

We assume that the flow is adiabatic. Thus $\dot{Q}_{\text{net in}} = 0$. Also, because the flow is incompressible ($\rho_1 = \rho_2$) and atmospheric pressure prevails at sections (1) and (2) ($p_1 = p_2$).

$$\left(\frac{p}{\rho}\right)_1 = \left(\frac{p}{\rho}\right)_2 \qquad (3)$$

Furthermore,

$$V_1 = V_2 = 0 \qquad \textbf{(4)}$$

because the surface of each large body of water is considered motionless. Thus, Eqs. 1 through 4 combine to yield

$$T_2 - T_1 = \frac{g(z_1 - z_2)}{\check{c}}$$

so that with

$$\check{c} = [1 \text{ Btu/(lbm} \cdot {}^\circ\text{R)}]\ (778 \text{ ft} \cdot \text{lb/Btu})$$
$$= [778 \text{ ft} \cdot \text{lb/(lbm} \cdot {}^\circ\text{R)}]$$

$$T_2 - T_1 = \frac{(32.2 \text{ ft/s}^2)(420 \text{ ft})}{[778 \text{ ft} \cdot \text{lb/(lbm} \cdot {}^\circ\text{R)}][32.2 \text{ (lbm} \cdot \text{ft)/(lb} \cdot \text{s}^2)]}$$

$$= 0.540\ {}^\circ\text{R} \qquad \textbf{(Ans)}$$

COMMENT Note that it takes a considerable change of potential energy to produce even a small increase in temperature.

5.3.3 The Mechanical Energy Equation and the Bernoulli Equation

$$\dot{m}\left[\check{u}_{\text{out}} - \check{u}_{\text{in}} + \frac{p_{\text{out}}}{\rho} - \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\substack{\text{net}\\\text{in}}} \tag{5.72}$$

Dividing Eq. 5.72 by the mass flowrate, $\dot{m}$, and rearranging terms we obtain

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} - (\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\substack{\text{net}\\\text{in}}}) \tag{5.73}$$

where

$$q_{\substack{\text{net}\\\text{in}}} = \frac{\dot{Q}_{\text{net in}}}{\dot{m}}$$

$$p_{\text{out}} + \frac{\rho V_{\text{out}}^2}{2} + \gamma z_{\text{out}} = p_{\text{in}} + \frac{\rho V_{\text{in}}^2}{2} + \gamma z_{\text{in}} \tag{5.74}$$

Now we divide Eq. 5.74 by density, ρ, and obtain

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} \tag{5.75}$$

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\substack{\text{net}\\ \text{in}}} = 0 \tag{5.76}$$

when the steady incompressible flow is frictionless. For steady incompressible flow with friction, we learn from experience (the second law of thermodynamics; see Sec. 5.4 for details) that

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\substack{\text{net}\\ \text{in}}} > 0 \tag{5.77}$$

In Eqs. 5.73 and 5.75, we can consider the combination of variables

$$\frac{p}{\rho} + \frac{V^2}{2} + gz$$

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\substack{\text{net}\\ \text{in}}} = \text{loss} \tag{5.78}$$

For a frictionless flow, Eqs. 5.73 and 5.75 tell us that loss equals zero.

It is often convenient to express Eq. 5.73 in terms of loss as

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} - \text{loss} \tag{5.79}$$

EXAMPLE 5.23 Energy—Effect of Loss of Available Energy

GIVEN As shown in Fig. E5.23*a*, air flows from a room through two different vent configurations: a cylindrical hole in the wall having a diameter of 120 mm and the same diameter cylindrical hole in the wall but with a well-rounded entrance. The room pressure is held constant at 1.0 kPa above atmospheric pressure. Both vents exhaust into the atmosphere. As discussed in Section 8.4.2, the loss in available energy associated with flow through the cylindrical vent from the room to the vent exit is $0.5V_2^2/2$ where V_2 is the uniformly distributed exit velocity of air. The loss in available energy associated with flow through the rounded entrance vent from the room to the vent exit is $0.05V_2^2/2$, where V_2 is the uniformly distributed exit velocity of air.

FIND Compare the volume flowrates associated with the two different vent configurations.

SOLUTION

We use the control volume for each vent sketched in Fig. E5.23*a*. What is sought is the flowrate, $Q = A_2V_2$, where A_2 is the vent exit cross-sectional area, and V_2 is the uniformly distributed exit velocity. For both vents, application of Eq. 5.79 leads to

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + \overbrace{\cancel{gz_2} = \frac{p_1}{\rho} + \underbrace{\cancel{\frac{V_1^2}{2}}}_{0\,(V_1 \approx 0)} + \cancel{gz_1}}^{0\text{ (no elevation change)}} - {}_1\text{loss}_2 \qquad (1)$$

where ${}_1\text{loss}_2$ is the loss between sections (1) and (2). Solving Eq. 1 for V_2 we get

$$V_2 = \sqrt{2\left[\left(\frac{p_1 - p_2}{\rho}\right) - {}_1\text{loss}_2\right]} \qquad (2)$$

Since

$${}_1\text{loss}_2 = K_L\frac{V_2^2}{2} \qquad (3)$$

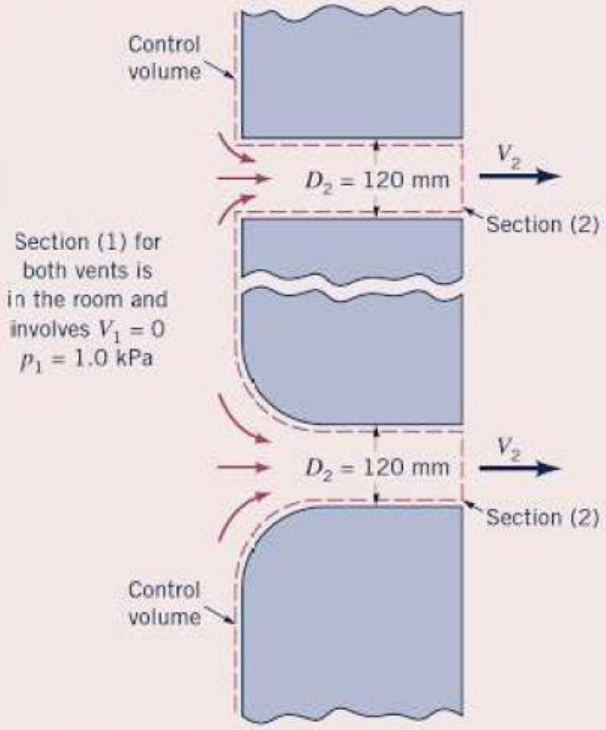


Figure E5.23*a*

where K_L is the loss coefficient ($K_L = 0.5$ and 0.05 for the two vent configurations involved), we can combine Eqs. 2 and 3 to get

$$V_2 = \sqrt{2\left[\left(\frac{p_1 - p_2}{\rho}\right) - K_L \frac{V_2^2}{2}\right]} \tag{4}$$

Solving Eq. 4 for V_2 we obtain

$$V_2 = \sqrt{\frac{p_1 - p_2}{\rho[(1 + K_L)/2]}} \tag{5}$$

Therefore, for flowrate, Q, we obtain

$$Q = A_2 V_2 = \frac{\pi D_2^2}{4}\sqrt{\frac{p_1 - p_2}{\rho[(1 + K_L)/2]}} \tag{6}$$

For the rounded entrance cylindrical vent, Eq. 6 gives

$$Q = \frac{\pi(120 \text{ mm})^2}{4(1000 \text{ mm/m})^2} \times \sqrt{\frac{(1.0 \text{ kPa})(1000 \text{ Pa/kPa})[1(\text{N/m}^2)/(\text{Pa})]}{(1.23 \text{ kg/m}^3)[(1 + 0.05)/2][1(\text{N}\cdot\text{s}^2)/(\text{kg}\cdot\text{m})]}}$$

or

$$Q = 0.445 \text{ m}^3/\text{s} \qquad \textbf{(Ans)}$$

For the cylindrical vent, Eq. 6 gives us

$$Q = \frac{\pi(120 \text{ mm})^2}{4(1000 \text{ mm/m})^2} \times \sqrt{\frac{(1.0 \text{ kPa})(1000 \text{ Pa/kPa})[1(\text{N/m}^2)/(\text{Pa})]}{(1.23 \text{ kg/m}^3)[(1 + 0.5)/2][1(\text{N}\cdot\text{s}^2)/(\text{kg}\cdot\text{m})]}}$$

or

$$Q = 0.372 \text{ m}^3/\text{s} \qquad \textbf{(Ans)}$$

COMMENT By repeating the calculations for various values of the loss coefficient, K_L, the results shown in Fig. E5.23*b* are obtained. Note that the rounded entrance vent allows the passage of more air than does the cylindrical vent because the loss associated with the rounded entrance vent is less than that for the cylindrical one. For this flow the pressure drop, $p_1 - p_2$, has two purposes: (1) overcome the loss associated with the flow, and (2) produce the kinetic energy at the exit. Even if there were no loss (i.e., $K_L = 0$), a pressure drop would be needed to accelerate the fluid through the vent.

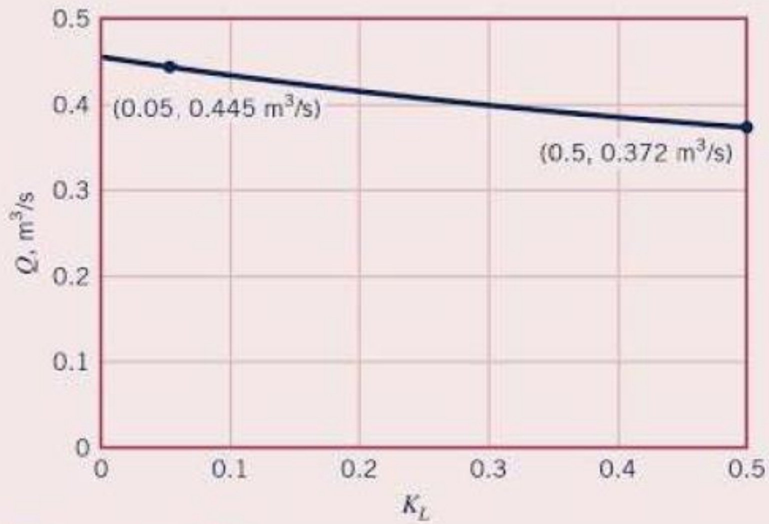


Figure E5.23*b*

An important group of fluid mechanics problems involves one-dimensional, incompressible, steady-in-the-mean flow with friction and shaft work. Included in this category are flows through pumps, blowers, fans, and turbines. For this kind of flow, Eq. 5.67 becomes

$$\dot{m}\left[\check{u}_{\text{out}} - \check{u}_{\text{in}} + \frac{p_{\text{out}}}{\rho} - \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}} \tag{5.80}$$

Dividing Eq. 5.80 by mass flowrate and using the work per unit mass, $w_{\substack{\text{shaft}\\ \text{net in}}} = \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}/\dot{m}$, we obtain

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\substack{\text{shaft}\\ \text{net in}}} - (\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\substack{\text{net}\\ \text{in}}}) \tag{5.81}$$

Since the flow is incompressible, Eq. 5.78 shows that $\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}}$ equals the loss of useful energy and Eq. 5.81 can be expressed as

$$\boxed{\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss}} \tag{5.82}$$

It is sometimes called the ***mechanical energy equation***

Minimizing loss is a central goal of fluid mechanical design.

EXAMPLE 5.24 Energy—Fan Work and Efficiency

GIVEN An axial-flow ventilating fan driven by a motor that delivers 0.4 kW of power to the fan blades produces a 0.6-m-diameter axial stream of air having a speed of 12 m/s. The flow far upstream of the fan has negligible speed.

FIND Determine how much of the work to the air actually produces useful effects, that is, fluid motion and a rise in available energy. Estimate the aerodynamic efficiency of this fan.

SOLUTION

We select a fixed and nondeforming control volume as is illustrated in Fig. E5.24. The application of Eq. 5.82 to the contents of this control volume leads to

0 (atmospheric pressures cancel) 0 ($V_1 \approx 0$)

$$w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss} = \left(\frac{\cancel{p_2}}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}\right) - \left(\frac{\cancel{p_1}}{\rho} + \frac{\cancel{V_1^2}}{2} + \cancel{gz_1}\right) \quad \textbf{(1)}$$

0 (no elevation change)

where $w_{\text{shaft net in}}$ − loss is the amount of work added to the air that produces a useful effect. Equation 1 leads to

$$w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss} = \frac{V_2^2}{2} = \frac{(12\text{ m/s})^2}{2[1(\text{kg}\cdot\text{m})/(\text{N}\cdot\text{s}^2)]}$$

$$= 72.0\ \text{N}\cdot\text{m/kg} \qquad \textbf{(2)} \quad \text{(Ans)}$$

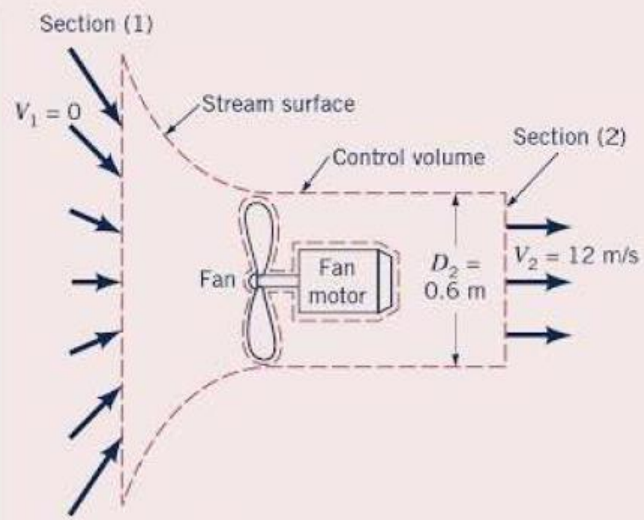


Figure E5.24

A reasonable estimate of *efficiency*, η, would be the ratio of amount of work that produces a useful effect, Eq. 2, to the amount of work delivered to the fan blades. That is,

$$\eta = \frac{w_{\substack{\text{shaft}\\\text{net in}}} - \text{loss}}{w_{\substack{\text{shaft}\\\text{net in}}}} \quad \textbf{(3)}$$

To calculate the efficiency, we need a value of $w_{\text{shaft net in}}$, which is related to the power delivered to the blades, $\dot{W}_{\text{shaft net in}}$. We note that

$$w_{\substack{\text{shaft}\\\text{net in}}} = \frac{\dot{W}_{\substack{\text{shaft}\\\text{net in}}}}{\dot{m}} \quad \textbf{(4)}$$

where the mass flowrate, $\dot{m}$, is (from Eq. 5.6)

$$\dot{m} = \rho A V = \rho \frac{\pi D_2^2}{4} V_2 \quad \textbf{(5)}$$

For fluid density, ρ, we use 1.23 kg/m^3 (standard air) and, thus, from Eqs. 4 and 5 we obtain

$$w_{\substack{\text{shaft}\\\text{net in}}} = \frac{\dot{W}_{\substack{\text{shaft}\\\text{net in}}}}{(\rho \pi D_2^2/4)V_2}$$

$$= \frac{(0.4 \text{ kW})[1000 \text{ (Nm)}/(\text{skW})]}{(1.23 \text{ kg/m}^3)[(\pi)(0.6 \text{ m})^2/4](12 \text{ m/s})}$$

or

$$w_{\substack{\text{shaft}\\\text{net in}}} = 95.8 \text{ N}\cdot\text{m/kg} \quad \textbf{(6)}$$

From Eqs. 2, 3, and 6 we obtain

$$\eta = \frac{72.0 \text{ N}\cdot\text{m/kg}}{95.8 \text{ N}\cdot\text{m/kg}} = 0.752 \quad \textbf{(Ans)}$$

COMMENT Note that only 75% of the power that was delivered to the air resulted in useful effects and, thus, 25% of the shaft power is lost to air friction. The power input to the motor would be more than 0.4 kW because of electrical losses in the motor and mechanical friction in the bearings.

If Eq. 5.82, which involves energy per unit mass, is multiplied by fluid density, ρ, we obtain

$$p_{\text{out}} + \frac{\rho V_{\text{out}}^2}{2} + \gamma z_{\text{out}} = p_{\text{in}} + \frac{\rho V_{\text{in}}^2}{2} + \gamma z_{\text{in}} + \rho w_{\substack{\text{shaft}\\ \text{net in}}} - \rho(\text{loss}) \tag{5.83}$$

where $\gamma = \rho g$ is the specific weight of the fluid. Equation 5.83 involves *energy per unit volume* and the units involved are identical with those used for pressure ($\text{ft} \cdot \text{lb/ft}^3 = \text{lb/ft}^2$ or $\text{N} \cdot \text{m/m}^3 = \text{N/m}^2$).

If Eq. 5.82 is divided by the acceleration of gravity, g, we get

$$\boxed{\frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_s - h_L} \tag{5.84}$$

where

$$h_s = w_{\text{shaft net in}}/g = \frac{\dot{W}_{\substack{\text{shaft}\\ \text{net in}}}}{\dot{m}g} = \frac{\dot{W}_{\substack{\text{shaft}\\ \text{net in}}}}{\gamma Q} \tag{5.85}$$

We can define a total head, H, as follows:

$$H = \frac{p}{\gamma} + \frac{V^2}{2g} + z$$

$$H_{\text{out}} = H_{\text{in}} + h_s - h_L$$

EXAMPLE 5.25 Energy—Head Loss and Power Loss

GIVEN The pump shown in Fig. E5.25*a* adds 10 horsepower to the water as it pumps water from the lower lake to the upper lake. The elevation difference between the lake surfaces is 30 ft and the head loss is 15 ft.

FIND Determine

(a) the flowrate and

(b) the power loss associated with this flow.

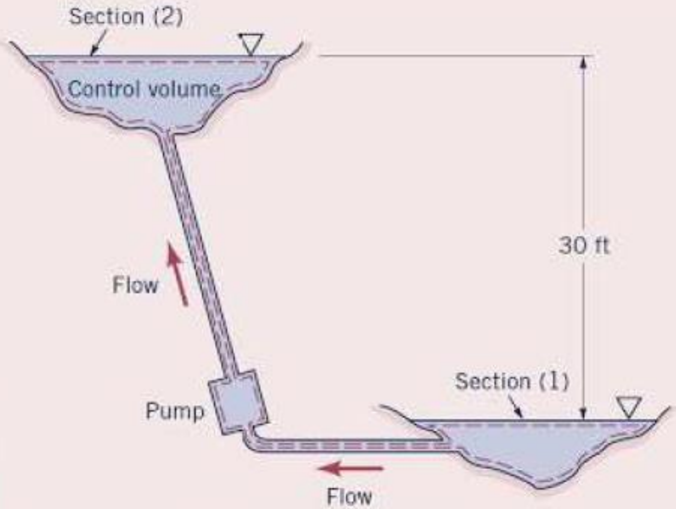


Figure E5.25*a*

SOLUTION

(a) The energy equation (Eq. 5.84) for this flow is

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_s - h_L \qquad (1)$$

where points 2 and 1 (corresponding to "out" and "in" in Eq. 5.84) are located on the lake surfaces. Thus, $p_2 = p_1 = 0$ and $V_2 = V_1 = 0$ so that Eq. 1 becomes

$$h_s = h_L + z_2 - z_1 \tag{2}$$

where $z_2 = 30$ ft, $z_1 = 0$, and $h_L = 15$ ft. The pump head is obtained from Eq. 5.85 as

$$\begin{aligned} h_s &= \dot{W}_{\text{shaft net in}}/\gamma Q \\ &= (10\text{ hp})(550\text{ ft}\cdot\text{lb/s/hp})/(62.4\text{ lb/ft}^3)\,Q \\ &= 88.1/Q \end{aligned}$$

where h_s is in ft when Q is in ft^3/s.

Hence, from Eq. 2,

$$88.1/Q = 15\text{ ft} + 30\text{ ft}$$

or

$$Q = 1.96\text{ ft}^3\text{/s} \qquad \textbf{(Ans)}$$

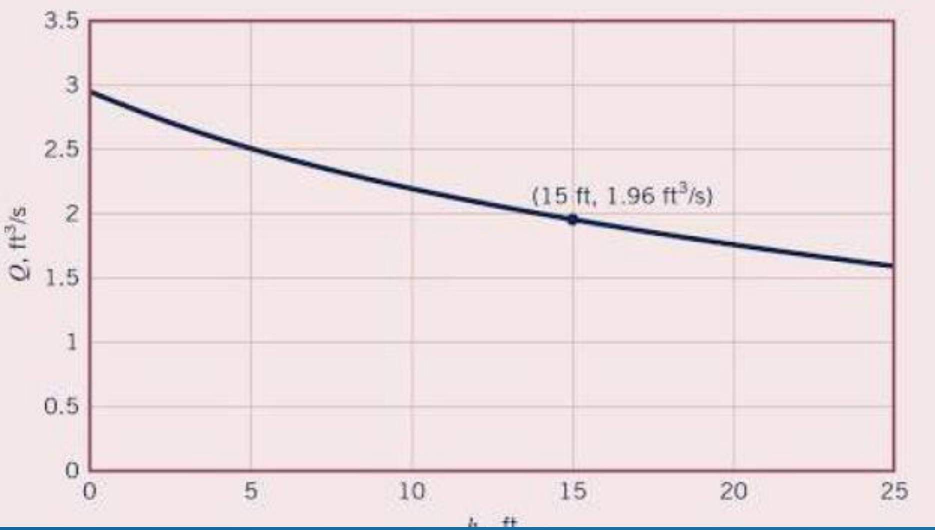
(b) The power lost due to friction can be obtained from Eq. 5.85 as

$$\begin{aligned} \dot{W}_{\text{loss}} &= \gamma Q h_L = (62.4\text{ lb/ft}^3)(1.96\text{ ft}^3\text{/s})(15\text{ ft}) \\ &= 1830\text{ ft}\cdot\text{lb/s}\ (1\text{ hp}/550\text{ ft}\cdot\text{lb/s}) \\ &= 3.33\text{ hp} \qquad \textbf{(Ans)} \end{aligned}$$

COMMENTS The remaining 10 hp − 3.33 hp = 6.67 hp that the pump adds to the water is used to lift the water from the lower to the upper lake. This energy is not "lost," but it is stored as potential energy.

By repeating the calculations for various head losses, h_L, the results shown in Fig. E5.25*b* are obtained. Note that as the head loss increases, the flowrate decreases because an increasing portion of the 10 hp supplied by the pump is lost and, therefore, not available to lift the fluid to the higher elevation.

Note that in this example the purpose of the pump is to lift the water (a 30-ft head) and overcome the head loss (a 15-ft head); it does not, overall, alter the water's pressure or velocity.



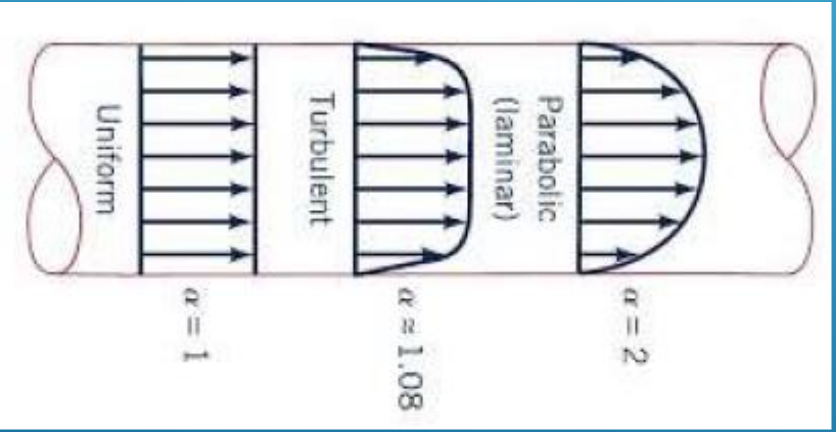
5.3.4 Application of the Energy Equation to Nonuniform Flows

The forms of the energy equation discussed in Sections 5.3.2 and 5.3.3 are applicable to one-dimensional flows, flows that are approximated with uniform velocity distributions where fluid crosses the control surface.

If the velocity profile at any section where flow crosses the control surface is not uniform, inspection of the energy equation for a control volume, Eq. 5.64, suggests that the integral

$$\int_{cs} \frac{V^2}{2} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$

will require special attention.



For one stream of fluid entering and leaving the control volume, we can define the relationship

$$\int_{cs} \frac{V^2}{2}\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \dot{m}\left(\frac{\alpha_{out}\overline{V}_{out}^2}{2} - \frac{\alpha_{in}\overline{V}_{in}^2}{2}\right)$$

where α is the ***kinetic energy coefficient*** and $\overline{V}$ is the average velocity defined earlier in Eq. 5.7. From the above we can conclude that

$$\frac{\dot{m}\alpha\overline{V}^2}{2} = \int_A \frac{V^2}{2}\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

for flow through surface area A of the control surface. Thus,

$$\alpha = \frac{\displaystyle\int_A (V^2/2)\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA}{\dot{m}\overline{V}^2/2} \tag{5.86}$$

Therefore, for nonuniform velocity profiles, the energy equation on an energy per unit mass basis for the incompressible flow of one stream of fluid through a control volume that is steady in the mean is

$$\boxed{\frac{p_{out}}{\rho} + \frac{\alpha_{out}\overline{V}_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{\alpha_{in}\overline{V}_{in}^2}{2} + gz_{in} + w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss}} \tag{5.87}$$

On an energy per unit volume basis we have

$$p_{\text{out}} + \frac{\rho\alpha_{\text{out}}\overline{V}_{\text{out}}^2}{2} + \gamma z_{\text{out}} = p_{\text{in}} + \frac{\rho\alpha_{\text{in}}\overline{V}_{\text{in}}^2}{2} + \gamma z_{\text{in}} + \rho w_{\substack{\text{shaft}\\ \text{net in}}} - \rho(\text{loss}) \qquad \textbf{(5.88)}$$

and on an energy per unit weight or head basis we have

$$\boxed{\frac{p_{\text{out}}}{\gamma} + \frac{\alpha_{\text{out}}\overline{V}_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{\alpha_{\text{in}}\overline{V}_{\text{in}}^2}{2g} + z_{\text{in}} + \frac{w_{\substack{\text{shaft}\\ \text{net in}}}}{g} - h_L} \qquad \textbf{(5.89)}$$

EXAMPLE 5.26 Energy—Effect of Nonuniform Velocity Profile

GIVEN The small fan shown in Fig. E5.26 moves air at a mass flowrate of 0.1 kg/min. Upstream of the fan, the pipe diameter is 60 mm, the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient, α_1, is equal to 2.0. Downstream of the fan, the pipe diameter is 30 mm, the flow is turbulent, the velocity profile is quite uniform, and the kinetic energy coefficient, α_2, is equal to 1.08. The rise in static pressure across the fan is 0.1 kPa, and the fan motor draws 0.14 W.

FIND Compare the value of loss calculated: **(a)** assuming uniform velocity distributions, **(b)** considering actual velocity distributions.

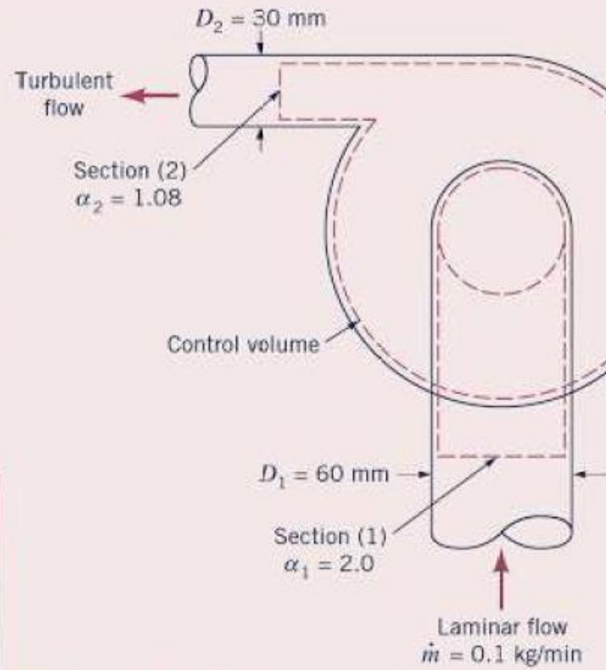


Figure E5.26

SOLUTION

Application of Eq. 5.87 to the contents of the control volume shown in Fig. E5.26 leads to

$$\frac{p_2}{\rho}+\frac{\alpha_2\bar{V}_2^2}{2}+\overbrace{g\cancel{z_2}=\frac{p_1}{\rho}+\frac{\alpha_1\bar{V}_1^2}{2}+g\cancel{z_1}}^{0\ \text{(change in } gz \text{ is negligible)}}$$

$$-\ \text{loss}+w_{\substack{\text{shaft}\\ \text{net in}}} \qquad (1)$$

or solving Eq. 1 for loss we get

$$\text{loss} = w_{\substack{\text{shaft}\\ \text{net in}}} - \left(\frac{p_2 - p_1}{\rho}\right) + \frac{\alpha_1 \overline{V}_1^2}{2} - \frac{\alpha_2 \overline{V}_2^2}{2} \qquad \textbf{(2)}$$

To proceed further, we need values of $w_{\text{shaft net in}}$, $\overline{V}_1$, and $\overline{V}_2$. These quantities can be obtained as follows. For shaft work

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{\text{power to fan motor}}{\dot{m}}$$

or

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{(0.14\ \text{W})[(1\ \text{N}\cdot\text{m/s})/\text{W}]}{0.1\ \text{kg/min}}(60\ \text{s/min})$$

$$= 84.0\ \text{N}\cdot\text{m/kg} \qquad \textbf{(3)}$$

For the average velocity at section (1), $\overline{V}_1$, from Eq. 5.11 we obtain

$$\overline{V}_1 = \frac{\dot{m}}{\rho A_1}$$

$$= \frac{\dot{m}}{\rho(\pi D_1^2/4)} \qquad \textbf{(4)}$$

$$= \frac{(0.1\ \text{kg/min})\ (1\ \text{min/60 s})\ (1000\ \text{mm/m})^2}{(1.23\ \text{kg/m}^3)[\pi(60\ \text{mm})^2/4]}$$

$$= 0.479\ \text{m/s}$$

For the average velocity at section (2), $\overline{V}_2$,

$$\overline{V}_2 = \frac{(0.1\ \text{kg/min})\ (1\ \text{min/60 s})\ (1000\ \text{mm/m})^2}{(1.23\ \text{kg/m}^3)[\pi(30\ \text{mm})^2/4]}$$

$$= 1.92\ \text{m/s} \qquad \textbf{(5)}$$

(a) For the assumed uniform velocity profiles ($\alpha_1 = \alpha_2 = 1.0$), Eq. 2 yields

$$\text{loss} = w_{\substack{\text{shaft}\\ \text{net in}}} - \left(\frac{p_2 - p_1}{\rho}\right) + \frac{\overline{V}_1^2}{2} - \frac{\overline{V}_2^2}{2} \qquad \textbf{(6)}$$

Using Eqs. 3, 4, and 5 and the pressure rise given in the problem statement, Eq. 6 gives

$$\text{loss} = 84.0\ \frac{\text{N}\cdot\text{m}}{\text{kg}} - \frac{(0.1\ \text{kPa})(1000\ \text{Pa/kPa})(1\ \text{N/m}^2/\text{Pa})}{1.23\ \text{kg/m}^3}$$

$$+\frac{(0.479\text{ m/s})^2}{2[1\ (\text{kg}\cdot\text{m})/(\text{N}\cdot\text{s}^2)]}-\frac{(1.92\text{ m/s})^2}{2[1\ (\text{kg}\cdot\text{m})/(\text{N}\cdot\text{s}^2)]}$$

or

$$\begin{aligned}\text{loss} &= 84.0\ \text{N}\cdot\text{m/kg} - 81.3\ \text{N}\cdot\text{m/kg}\\ &\quad + 0.115\ \text{N}\cdot\text{m/kg} - 1.84\ \text{N}\cdot\text{m/kg}\\ &= 0.975\ \text{N}\cdot\text{m/kg} \qquad \textbf{(Ans)}\end{aligned}$$

(b) For the actual velocity profiles ($\alpha_1 = 2$, $\alpha_2 = 1.08$), Eq. 1 gives

$$\text{loss} = w_{\substack{\text{shaft}\\ \text{net in}}} - \left(\frac{p_2 - p_1}{\rho}\right) + \alpha_1\frac{\overline{V}_1^2}{2} - \alpha_2\frac{\overline{V}_2^2}{2} \qquad \textbf{(7)}$$

If we use Eqs. 3, 4, and 5 and the given pressure rise, Eq. 7 yields

$$\text{loss} = 84\ \text{N}\cdot\text{m/kg} - \frac{(0.1\text{ kPa})(1000\text{ Pa/kPa})(1\text{ N/m}^2\text{/Pa})}{1.23\text{ kg/m}^3}$$

$$+\frac{2(0.479\text{ m/s})^2}{2[1\ (\text{kg}\cdot\text{m})/(\text{N}\cdot\text{s}^2)]}-\frac{1.08(1.92\text{ m/s})^2}{2[1\ (\text{kg}\cdot\text{m})/(\text{N}\cdot\text{s}^2)]}$$

or

$$\begin{aligned}\text{loss} &= 84.0\ \text{N}\cdot\text{m/kg} - 81.3\ \text{N}\cdot\text{m/kg}\\ &\quad + 0.230\ \text{N}\cdot\text{m/kg} - 1.99\ \text{N}\cdot\text{m/kg}\\ &= 0.940\ \text{N}\cdot\text{m/kg} \qquad \textbf{(Ans)}\end{aligned}$$

COMMENT The difference in loss calculated assuming uniform velocity profiles and actual velocity profiles is not large compared to $w_{\text{shaft net in}}$ for this fluid flow situation.

5.3.5 Combination of the Energy Equation and the Moment-of-Momentum Equation[3]

If Eq. 5.82 is used for one-dimensional incompressible flow through a turbomachine, we can use Eq. 5.54, developed in Section 5.2.4 from the moment-of-momentum equation (Eq. 5.42), to evaluate shaft work. This application of both Eqs. 5.54 and 5.82 allows us to ascertain the amount of loss that occurs in incompressible turbomachine flows as is demonstrated in Example 5.28.

5.4 Second Law of Thermodynamics—Irreversible Flow[4]

The second law of thermodynamics affords us with a means to verify the inequality

$$\check{u}_2 - \check{u}_1 - q_{\underset{\text{in}}{\text{net}}} \geq 0 \qquad \textbf{(5.90)}$$

for steady, incompressible, one-dimensional flow with friction (see Eqs. 5.73 and 5.77). In this section we continue to develop the notion of loss of useful energy for flow with friction.

5.4.1 Semi-infinitesimal Control Volume Statement of the Energy Equation

If we apply the one-dimensional, steady flow energy equation, Eq. 5.70, to the contents of a control volume that is infinitesimally thin as illustrated in Fig. 5.8, the result is

$$\dot{m}\left[d\check{u} + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g\,(dz)\right] = \delta\dot{Q}_{\substack{\text{net}\\ \text{in}}} \tag{5.91}$$

For all pure substances including common engineering fluids, such as air, water, and oil, the following relationship among fluid properties, called the *first Tds equation,* is valid (see, for example, Ref. 3).

$$T\,ds = d\check{u} + pd\left(\frac{1}{\rho}\right) \tag{5.92}$$

where T is the absolute temperature and s is the *entropy* per unit mass. (See Chapter 11 and Ref. 3 for more information about entropy.)

Combining Eqs. 5.91 and 5.92 we get

$$\dot{m}\left[T\,ds - pd\left(\frac{1}{\rho}\right) + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g\,dz\right] = \delta\dot{Q}_{\substack{\text{net}\\ \text{in}}}$$

or, dividing through by $\dot{m}$ and letting $\delta q_{\substack{\text{net}\\ \text{in}}} = \delta\dot{Q}_{\substack{\text{net}\\ \text{in}}}/\dot{m}$, we obtain

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz = -\left(T\,ds - \delta q_{\substack{\text{net}\\ \text{in}}}\right) \tag{5.93}$$

5.4.2 Semi-infinitesimal Control Volume Statement of the Second Law of Thermodynamics

A general statement of the second law of thermodynamics is (see Ref. 3)

$$\frac{D}{Dt}\int_{\text{sys}} s\rho \, d\mathcal{V} \geq \sum \left(\frac{\delta \dot{Q}_{\text{in}}^{\text{net}}}{T}\right)_{\text{sys}} \tag{5.94}$$

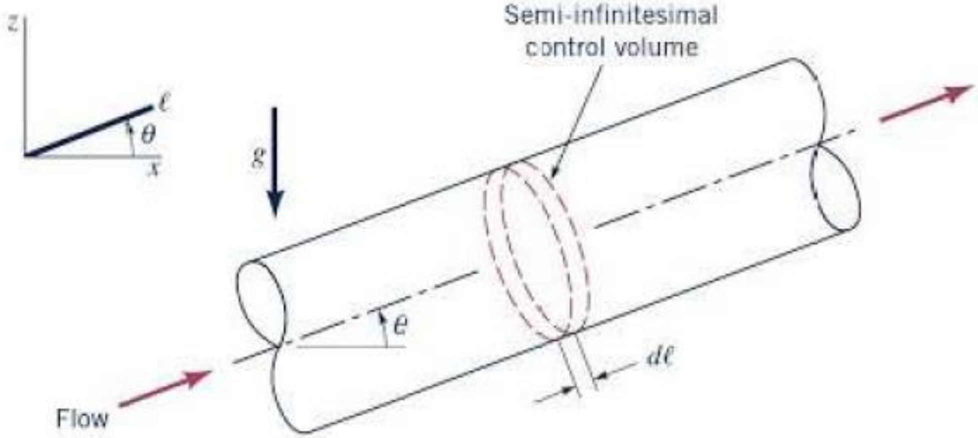


Figure 5.9 Semi-infinitesimal control volume.

$$\sum\left(\frac{\delta \dot{Q}_{\substack{\text{net}\\\text{in}}}}{T}\right)_{\text{sys}} = \sum\left(\frac{\delta \dot{Q}_{\substack{\text{net}\\\text{in}}}}{T}\right)_{\text{cv}} \quad \textbf{(5.95)}$$

With the help of the Reynolds transport theorem (Eq. 4.19) the system time derivative can be expressed for the contents of the coincident fixed and nondeforming control volume. Using Eq. 4.19, we obtain

$$\frac{D}{Dt}\int_{\text{sys}} s\rho \, d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} s\rho \, d\forall + \int_{\text{cs}} s\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \quad \textbf{(5.96)}$$

Eqs. 5.94, 5.95, and 5.96 combine to give

$$\frac{\partial}{\partial t}\int_{\text{cv}} s\rho \, d\forall + \int_{\text{cs}} s\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \geq \sum\left(\frac{\delta \dot{Q}_{\substack{\text{net}\\\text{in}}}}{T}\right)_{\text{cv}} \quad \textbf{(5.97)}$$

At any instant for steady flow

$$\frac{\partial}{\partial t}\int_{\text{cv}} s\rho \, d\forall = 0 \quad \textbf{(5.98)}$$

If the flow consists of only one stream through the control volume and if the properties are uniformly distributed (one-dimensional flow), Eqs. 5.97 and 5.98 lead to

$$\dot{m}(s_{\text{out}} - s_{\text{in}}) \geq \sum \frac{\delta \dot{Q}_{\substack{\text{net}\\\text{in}}}}{T} \quad \textbf{(5.99)}$$

For the infinitesimally thin control volume of Fig. 5.8, Eq. 5.99 yields

$$\dot{m} \, ds \geq \sum \frac{\delta \dot{Q}_{\substack{\text{net}\\\text{in}}}}{T} \quad \textbf{(5.100)}$$

If all of the fluid in the infinitesimally thin control volume is considered as being at a uniform temperature, T, then from Eq. 5.100 we get

$$T\,ds \geq \delta q_{\substack{\text{net}\\ \text{in}}}$$

or

$$T\,ds - \delta q_{\substack{\text{net}\\ \text{in}}} \geq 0 \qquad \textbf{(5.101)}$$

The equality is for any reversible (frictionless) process; the inequality is for all irreversible (friction) processes.

The relationship between entropy and heat transfer is different for reversible and irreversible processes.

5.4.3 Combination of the Equations of the First and Second Laws of Thermodynamics

Combining Eqs. 5.93 and 5.101, we conclude that

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz\right] \geq 0 \tag{5.102}$$

$$\boxed{-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz\right] = \delta(\text{loss}) = (T\,ds - \delta q_{\substack{\text{net}\\\text{in}}})} \tag{5.103}$$

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz = 0 \tag{5.104}$$

$$\boxed{-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz\right] = \delta(\text{loss}) - \delta w_{\substack{\text{shaft}\\\text{net in}}}} \tag{5.105}$$

Equations 5.103 and 5.105 are valid for incompressible and compressible flows. If we combine Eqs. 5.92 and 5.103, we obtain

$$d\breve{u} + pd\left(\frac{1}{\rho}\right) - \delta q_{\substack{\text{net}\\ \text{in}}} = \delta(\text{loss}) \qquad \textbf{(5.106)}$$

For incompressible flow, $d(1/\rho) = 0$ and, thus, from Eq. 5.106,

$$d\breve{u} - \delta q_{\substack{\text{net}\\ \text{in}}} = \delta(\text{loss}) \qquad \textbf{(5.107)}$$

Integrating for a finite control volume, we obtain

$$\breve{u}_{\text{out}} - \breve{u}_{\text{in}} - q_{\substack{\text{net}\\ \text{in}}} = \text{loss}$$

which is the same conclusion we reached earlier (see Eq. 5.78) for incompressible flows.

For compressible flow, $d(1/\rho) \neq 0$, and thus when we apply Eq. 5.106 to a finite control volume we obtain

$$\breve{u}_{\text{out}} - \breve{u}_{\text{in}} + \int_{\text{in}}^{\text{out}} pd\left(\frac{1}{\rho}\right) - q_{\substack{\text{net}\\ \text{in}}} = \text{loss} \qquad \textbf{(5.108)}$$

indicating that $\breve{u}_{\text{out}} - \breve{u}_{\text{in}} - q_{\text{net in}}$ is not equal to loss.