IENG112

Transportation Problem

Evaluation of potential facilities with Transportation Problem (TP)

The Transportation Problem

One typical application of the transportation problem is transporting stone from mines to road constructions. Even asphalt roads in general and particularly the asphalt itself are produced mainly from stone. In the original form of the transportation problem it is assumed that the total quantity offered by the mines equals the total quantity required by the road constructions. It is also assumed that there is no difference between the qualities of the different mines. In that situation the most economic way to organize the transportation is to minimize the total transportation cost. The variables (unknowns) of the model are the quantities transported from a particular mine to a particular road construction. The total transportation cost is the objective function of the optimization problem. It is supposed to be linear, i.e. the transportation cost from any mine to any road construction is proportional to the transported quantity. There are two sets of linear constraints. For all mines it must be true that the total quantity transported from the mine must be equal to the quantity offered by the mine. Similarly for all road construction it must be true that the total quantity which is transported to it must be equal to its demand. Finally all variables must be nonnegative. A negative value means that the stone is transported from the road construction to the mine what has no sense.

The key properties of stone are the followings. It is a single product produced and consumed on several places. Stone is produced and used in quantities which are significantly larger than the capacity of any kind of transportation device. It can be divided arbitrarily, i.e. it is bulk. If a product has these properties then its transportation can be optimized by the transportation problem.

Example. *Two stone mines supply three road constructions with stone. The production of the mine in million tons is 7 and 12. The quantities required by the road constructions are 5, 6 and 8. The unit costs of the transportation are the followings:*

3	*4*	*5*
7	*6*	*8*

Give the linear programming model of the problem.

Denote by x_{ij} the quantity which is transported from mine i to road construction j ($i=1,2$, $j=1,2,3$). As the total offered quantity (7+12=19) equals the total demand (5+6+8=19), everything must be transported from the mines:

$$x_{11}+x_{12}+x_{13} = 7$$

$$x_{21}+x_{22}+x_{23} = 12.$$

Similarly, the demands of the road constructions must be satisfied, i.e.

$$x_{11} + x_{21} = 5$$

$$x_{12} + x_{22} = 6$$

$$x_{13} + x_{23} = 8.$$

The transported quantities are nonnegative:

$$x_{ij} \geq 0 \quad i=1,2,\ j=1,2,3.$$

The transportation cost is proportional to the transported quantity in each relation. Hence the total transportation cost which is the objective function of the problem is

$$\min(3x_{11} + 4x_{12} + 5x_{13} + 7x_{21} + 6x_{22} + 8x_{23}).$$

Example of the textbook. Production facilities: Tulsa, Tempe. Distribution centers: San Francisco, Lincoln, Cleveland, Phoenix, and Washington DC. Potential new facilities: Youngstown, and Sparta. Transportation and production costs:

	Washington DC	Cleveland	Lincoln	San Francisco	Phoenix	Weekly capacity	Pr. cost
Tulsa	5.00	3.00	2.00	3.00	2.00	7,000	$75
Tempe	6.50	5.00	3.50	1.50	0.20	5,500	$70
Youngstown	1.50	0.50	1.80	6.50	5.00	12,500	$70
Sparta	3.80	5.00	8.00	7.50	8.00	12,500	$67
Weekly demand	5,000	6,000	4,000	7,000	3,000		

Evaluation of Youngstown with the "Least cost" heuristics of TP.

	Washington DC	Cleveland	Lincoln	San Francisco	Phoenix	Weekly capacity
Tulsa	80.00	78.00	77.00 *6* 2,500	78.00 *7* 4,500	77.00	7,000
Tempe	76.50	75.00	73.50	71.50 *3* 2,500	70.20 *1* 3,000	5,500
Youngstown	71.50 *4* 5,000	70.50 *2* 6,000	71.80 *5* 1,500	76.50	75.00	12,500
Weekly demand	5,000	6,000	4,000	7,000	3,000	