**EASTERN MEDITERRANEAN UNIVERSITY**



**Department of Industrial Engineering**

**IENG314/MANE314 Operations Research II**

**HOMEWORK 1 Spring 2021-22**

1. Consider a continuous random variable **X** with the density function (called the *exponential density*)



1. Find *c*.
2. Find and sketch the c.d.f for **X**.
3. Find the mean and variance of **X**. (*Hint:* Use integration by parts.)
4. Find *P*(0.4**X**  1.3).
5. Compute expected value of *h(X)=***X***2-5***X***.*
6. Assume that **X** is a normal random variable with mean 5 and variance 2 what is the mean and variance of **Y**=3**X**+5, respectively.
7. Suppose that **X** is a random variable with the following density function

*f* (*x*)

1. Find c. **b.** Find *P{X>1.7}.*
2. Let **X** be the following discrete random variable: *P*(**X =-**1)= *P*(**X=**0)= *P*(**X =** 1) . Let **Y=** **X**2. Show that *cov*(**X**, **Y**)= 0, but **X** and **Y** are not independent random variables.
3. Of all 60-year-old women, 0.5% have breast cancer. If a woman has breast cancer, a mammogram will give a positive indication for cancer 98% of the time. If a woman does not have breast cancer, a mammogram will give a positive indication for cancer 1% of the time. If a 60-year-old woman’s mammogram gives a positive indication for cancer, what is the probability that she has cancer?
4. In a multiple choice test a student may know the correct answer or guess it. Assume that he knows the correct answer with probability *p* then *1-p* is the probability that he guess the answer. If the number of choice be *m,* clearly the probability of guess correct answer is . What is the probability that the student answer a question given that he know the correct answer? Compute the mentioned probability if *m=6* and *p=0.55.*
5. Consider two urns. The first contains three white and seven black balls, and the second contains five white and five black balls. We flip a fair coin and then draw a ball from the first urn or the second urn depending on whether the outcome was heads or tails. What is the conditional probability that the outcome of the toss was heads given that a white ball was selected?
6. An urn contains 10 red balls and 30 blue balls.
7. Suppose you draw 4 balls from the urn. Let **X***i* be the number of red balls drawn on the *i*th ball (**X***i* = 0 or 1). After each ball is drawn, it is put back into the urn. Are the random variables **X**1, **X**2, **X**3, and **X**4 independent random variables?

**b .** Repeat part (a) for the case in which the balls are not put back in the urn after being drawn.

1. I have 100 units of a product in stock. The demand **X** for the item is a continuous random variable with the following density function

*f* (*x*)

1. Find the probability that supply is insufficient to meet demand.
2. What is the expected number of items sold? What is the variance of the number of items sold?
3. In the Toy manufacturing company, suppose the product acceptance probabilities are not known. But the following data is known.

*Anticipated First Year Profit ($)*

*Product Line*

|  |  |  |  |
| --- | --- | --- | --- |
| Product  Acceptance | Full | Partial | Minimal |
| *Good* | *10* | *70* | *50* |
| *Fair* | *50* | *45* | *40* |
| *Poor* | *-25* | *-15* | *0* |

Determine the optimal decision under each of following decision criteria.

|  |  |
| --- | --- |
| 1. Maximin criterion | 1. Maximax criterion |
| 1. Minimax regret criterion | 1. Expected value criterion |

1. Daily demand for chocolate bars at the Gillis Grocery has a mean of 100 and a variance of 3,000 (chocolate bars)2. At present, the store has 3,500 chocolate bars in stock. What is the probability that the store will run out of chocolate bars during the next 30 days? Also, how many should Gillis have on hand at the beginning of a 30-day period if the store wants to have only a 1% chance of running out during the 30-day period? Assume that the demands on different days are independent random variables.
2. Sodaco is considering producing a new product: Chocovan soda. Sodaco estimates that the annual demand for Chocovan, **D** (in thousands of cases), has the following mass function: *P*(**D** = 30) = .20, *P*(**D** = 50) = .40, *P*(**D** = 80) = .40. Each case of Chocovan sells for $5 and incurs a variable cost of $3. It costs $800,000 to build a plant to produce Chocovan. Assume that if $1 is received every year (forever), this is equivalent to receiving $10 at the present time. Considering the reward for each action and state of the world to be in terms of net present value. Show how this problem fits into the state of world model.
3. We are going to invest $1,000 for a period of 6 months. Two potential investments are available: T-bills and gold. If the $1,000 is invested in T-bills, we are certain to end the 6-month period with $1,296. If we invest in gold, there is a 0.75 chance that we will end the 6-month period with $400 and a 0.25 chance that we will end the 6-month period with $10,000. If we end up with *x* dollars, our utility function is given by . Should we invest in gold or T-bills?
4. We now have $97,000 in assets and are given a choice between investment 1 and investment 2. With investment 1, 75% of the time we increase our asset position by $297,000, and 25% of the time we decrease our asset position by$90,000. With investment 2, 35% of the time we increase our asset position by $595,000, 50% of the time we decrease our asset position by $5,000, and15% of the time we increase our asset position by $715,000. Our utility function for final asset position *x* is *u*(*x*). We are given the following values for *u*(*x*): *u*(5000) = 0, *u*(690,000) = 0.83, *u*(810,000) = 0.95, *u*(90,000) = 0.38, *u*(1,000,000) = 1, *u*(490,000) =0.76.

**a)** Are we risk-averse, risk-seeking, or risk-neutral? Explain.

**b)** Will we prefer investment 1 or investment 2?