MENG541 Advanced Thermodynamics

CHAPTER 4 – EXERGY AND EXERGY ANALYSIS

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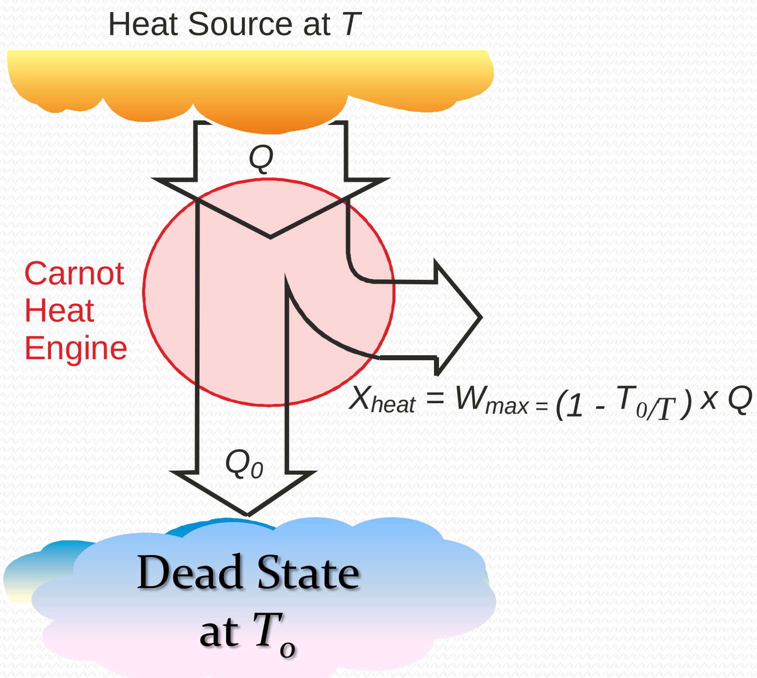
Chapter 4
Exergy and Exergy Analysis

Outline

- Fundamentals on Exergy
- Exergy Associated with KE and PE
- Irreversibility (Exergy Destruction)
- Second Law Efficiency
- Nonflow Exergy
- Exergy of a Flow stream
- Exergy by Heat, Work and Mass
- Exergy Balance

Exergy: Work Potential of Energy

- The exergy of a system is defined as the maximum shaft work that can be achieved by both the system and a specified reference environment
- Therefore exergy is a property of both the system and the environment



Exergy transfer by heat

$$X_{heat} \text{ or } X_Q = \left(1 - \frac{T_0}{T}\right) Q$$

Revision of Fundamentals

- Work = f (initial state, process path, final state)
- The specified *initial state* is constant
- Maximum work is obtained from *reversible* process
- To maximize the work output, final state = *dead state*
- Dead state means thermodynamic equilibrium of the system with the environment
- Exergy is destroyed whenever an *irreversible* process occurs
- Exergy transfer associated with *shaft work* is equal to the shaft work
- Exergy transfer associated with *heat transfer* is dependent on the temperature of process in relation to the temperature of the environment

Exergy Associated with KE and PE

- Kinetic and potential energies are forms of *mechanical energy*
- Hence they can be converted to work entirely, i.e. The work potential or exergy are themselves:

$$\text{exergy of kinetic energy:} \quad x_{ke} = \mathrm{ke} = \frac{\gamma^2}{2}$$

$$\text{exergy of potential energy:} \quad x_{pe} = \mathrm{pe} = g\,z$$

Exergy Associated with Electricity

- Just like shaft work, exergy associated with *electricity* is equal to electric energy itself.
- Hence, electric energy W_{el} and power $\dot{W}_{el}$ can be converted directly to X_{el} and $\dot{X}_{el}$ respectively:

$$\text{exergy of electric energy}: \quad x_{el} = w_{el}$$
$$\text{exergy power}: \quad \dot{x}_{el} = \dot{w}_{el}$$

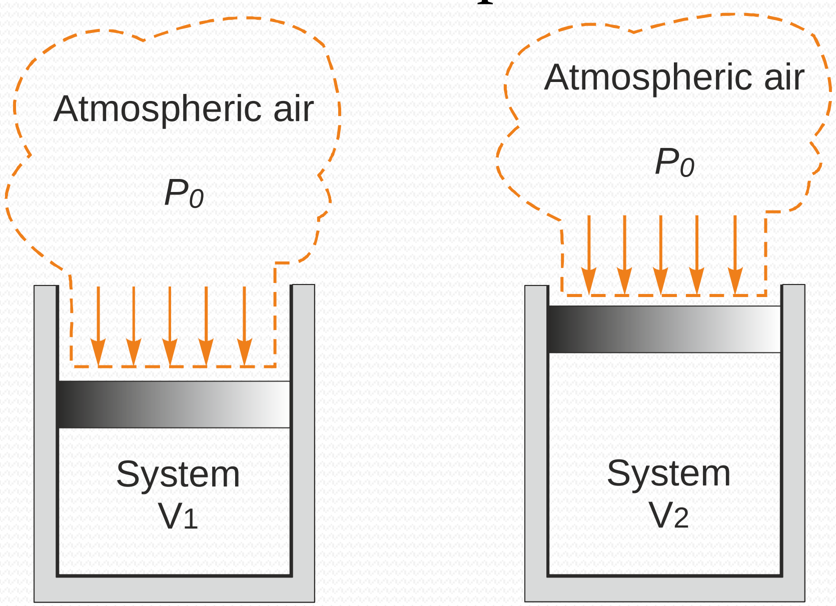
Surroundings Work

- Work produced by a work producing device (that involve moving boundary) is not always completely usable
- Work done by or against the surroundings is known as *surroundings work*, W_{surr}
- In a piston-cylinder device some work is used to push the atmospheric air out of the way
- In this example:

$$W_{surr} = P_0 (V_2 - V_1)$$

- Useful work:

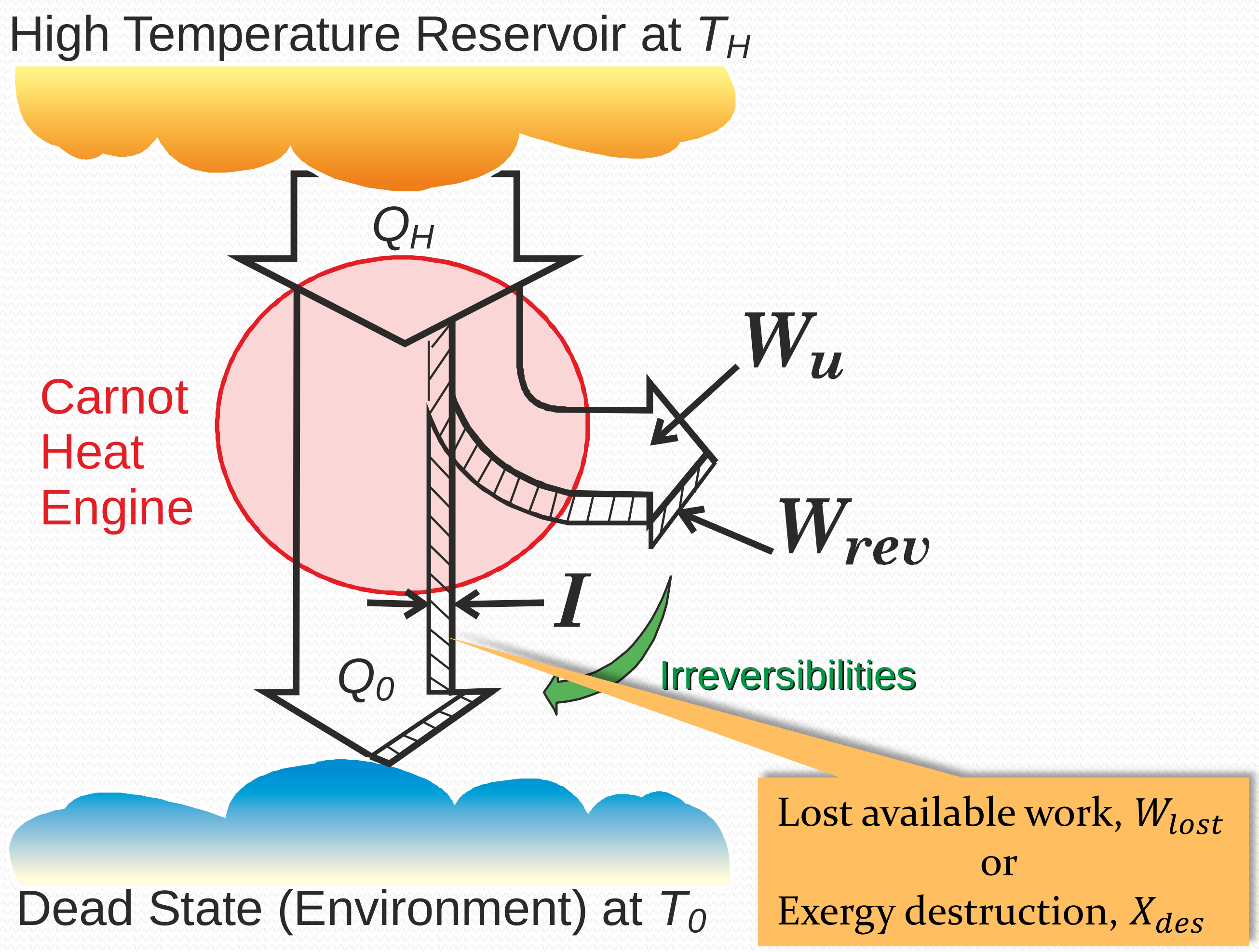
$$W_u = W - W_{surr}$$
$$= W - P_0 (V_2 - V_1)$$



Irreversibility (exergy destruction)

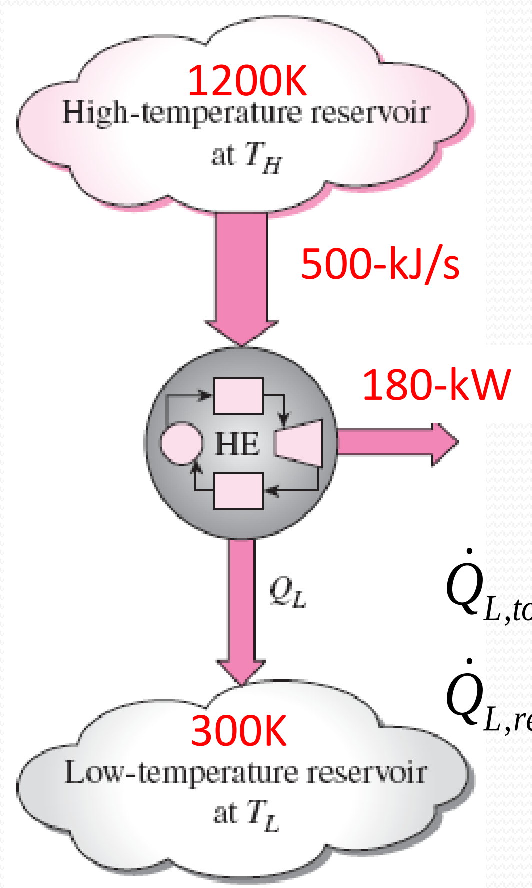
- **Reversible work** (W_{rev}) is defined as the *maximum useful work* that can be generated (or the *minimum work* that needs to be supplied) during a process
- When the final state of the process is the dead state then $W_{rev} = Exergy = X$
- The useful work (W_u) obtained in work producing devices is less than W_{rev} due to the **irreversibilities**
- **Irreversibility** is viewed as the lost opportunity to do work
- **Irreversibilities** (I) cause **exergy destruction**
- $I = X_{des} = W_{rev,out} - W_{u,out}$ **or** $W_{u,in} - W_{rev,in}$

I or X_{des} from a Heat Source



Example:

I or X_{des} of a Heat Engine



$$\dot{W}_{rev} = \eta_{th,rev}\dot{Q}_{in} = \left(1 - \frac{T_L}{T_H}\right)\dot{Q}_{in}$$

$$\dot{W}_{rev} = \left(1 - \frac{300\text{K}}{1200\text{K}}\right)(500\text{kW}) = \quad \mathbf{375\ kW}$$

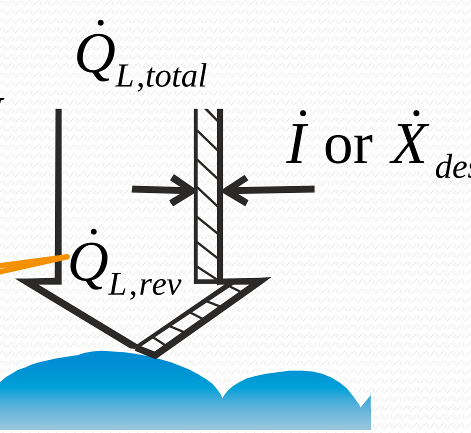
The rate of irreversibility or exergy destruction:

$$\dot{X}_{des} = \dot{I} = \dot{W}_{rev} - \dot{W}_u = 375 - 180 = \mathbf{195\ kW}$$

$$\dot{Q}_{L,total} = \dot{Q}_H - \dot{W}_u = 500 - 180 = 320\ \text{kW}$$

$$\dot{Q}_{L,rev} = \dot{Q}_H - \dot{W}_{rev} = 500 - 375 = 125\ \text{kW}$$

$$\dot{I} = 320 - 125 = 195\ \text{kW}$$



This is not available for converting to work

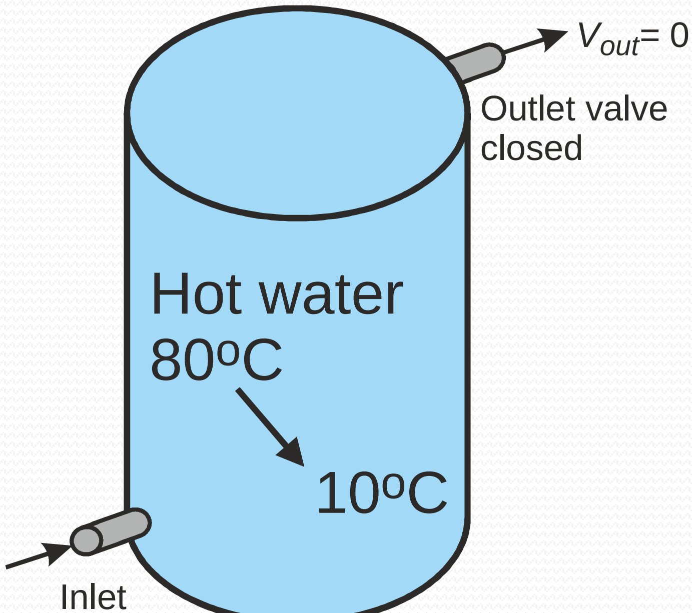
Example taken from Çengel 7th Ed. p.425

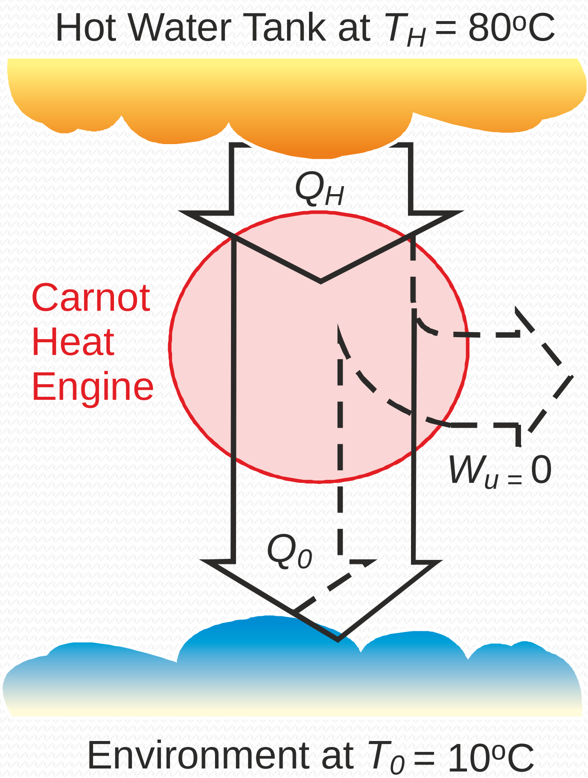
Example:

X_{des} from a Hot Water Tank

- When the water is not used the work potential is completely wasted

Exergy stored in the tank is completely destroyed, $I = X_{des} = W_{rev} - \cancelto{0}{W_u} = W_{rev}$



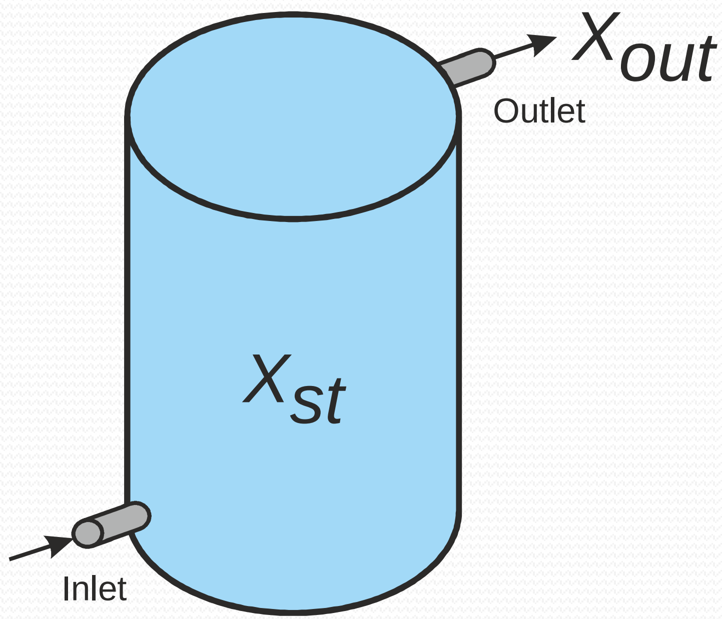


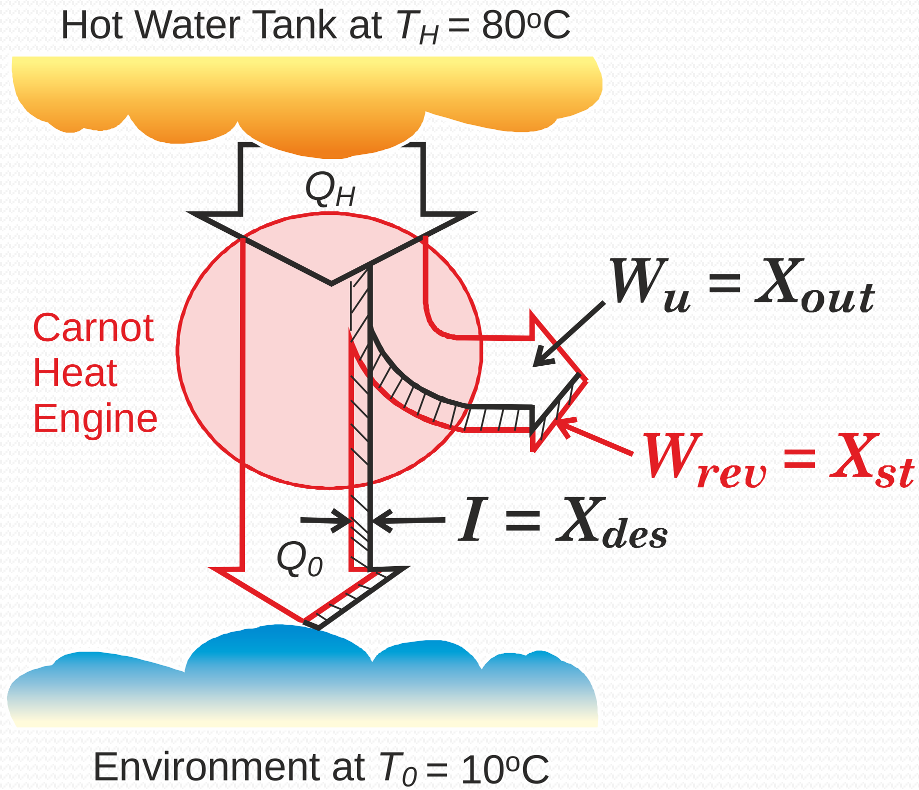
Example:

X_{des} from a Hot Water Tank

- When the water is used the $\boldsymbol{X_{des}}$ can be expressed as:

Exergy destroyed, $I = X_{des} = W_{rev} - W_u = X_{st} - X_{out}$

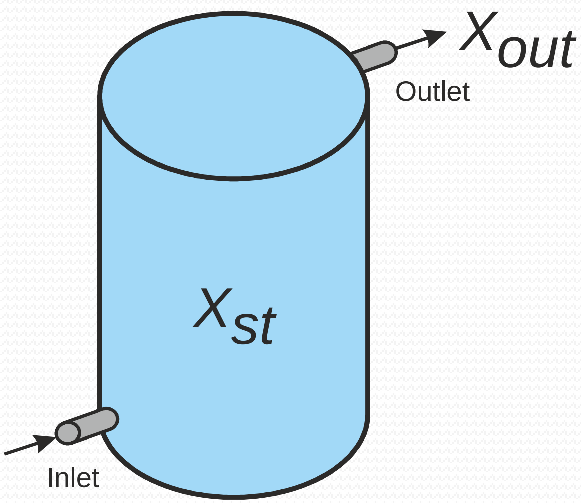




Second-Law Efficiency, η_{II}

- Second-law efficiency is defined as the ratio of the actual thermal efficiency to the maximum possible (reversible) thermal efficiency under the same conditions:
- For heat engines: $\eta_{II} = \dfrac{\eta_{th}}{\eta_{th,rev}}$
- For work producing devices: $\eta_{II} = \dfrac{W_u}{W_{rev}}$
- For work consuming devices: $\eta_{II} = \dfrac{W_{rev}}{W_u}$
- For refrigerators and heat pumps: $\eta_{II} = \dfrac{COP}{COP_{rev}}$

Example:
Hot Water Usage from a Tank



$$\eta_{II} = \frac{X_{out}}{X_{st}}$$

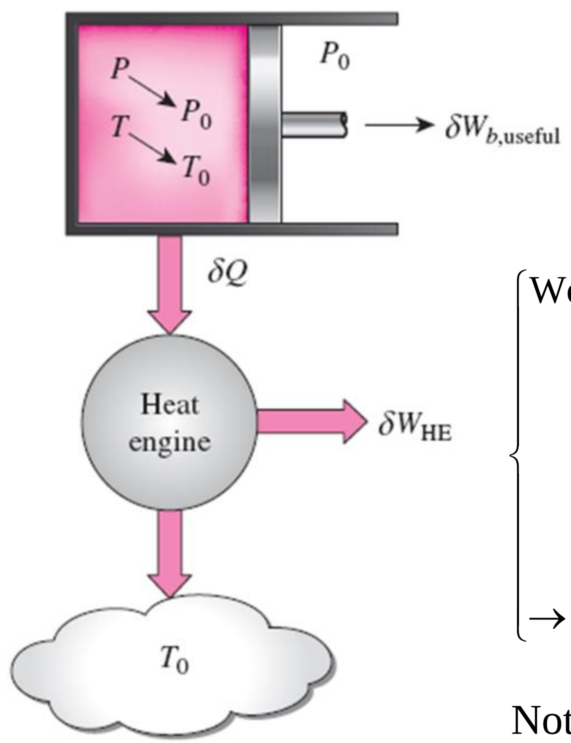
where X_{out} is the useful exergy extracted from the tank and X_{st} is the exergy stored in the tank

Also note that:

$$\eta_{II} = 1 - \frac{X_{des}}{X_{st}}$$

If all the stored exergy is destroyed, then $\eta_{II} = 0$
If no exergy destruction takes place (reversible case) then $\eta_{II} = 1$ (maximum). This means that $W_u = W_{rev}$

Nonflow Exergy: *Exergy of a fixed mass*



Any useful work is due to pressure above atmospheric pressure:

$$\rightarrow \delta W = P\,dV$$

$$= \underbrace{(P - P_0)\,dV}_{\delta W_{b,useful}} + P_0\,dV$$

Work potential due to heat transfer:

$$\rightarrow \delta W_{HE} = \left(1 - \frac{T_0}{T}\right)\delta Q$$

$$= \delta Q - \underbrace{\frac{T_0}{T}\delta Q}_{-T_0 dS}$$

$$\rightarrow \delta Q = \delta W_{HE} - T_0\,dS$$

Note that: $\delta W_{\text{total useful}} = \delta W_{HE} + \delta W_{b,useful}$

Substitute δQ and δW in the energy equation:

$$-\delta Q - \delta W = dU$$

$$\rightarrow -(\delta W_{HE} - T_0\,dS) - (\delta W_{b,useful} + P_0 dV) = dU$$

$$\rightarrow -\delta W_{\text{total useful}} + T_0\,dS - P_0 dV = dU$$

$$\rightarrow \delta W_{\text{total useful}} = -dU - P_0 dV + T_0\,dS$$

Nonflow Exergy: *Exergy of a fixed mass*

Equation obtained :

$$\rightarrow \delta W_{\text{total useful}} = -dU - P_0 dV + T_0 dS$$

Integrating from given state to dead state (0 subscript) :

$$\rightarrow \underbrace{W_{\text{total useful}}}_{\text{Availability or Exergy}} = (U - U_0) + P_0(V - V_0) - T_0(S - S_0)$$

On a unit mass basis the nonflow exergy can be expressed as :

$$\rightarrow \phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$$

Including the kinetic energy and potential energy terms :

$$\rightarrow \phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{1}{2}\gamma^2 + gz$$

where g is gravitational acceleration, γ is velocity and z is elevation.

The exergy change of a nonflow system (from state 1 to 2) :

$$\rightarrow \Delta\phi = \phi_2 - \phi_1 = (u_2 - u_1) + P_0(v_2 - v_1) - T_0(s_2 - s_1) + \frac{1}{2}(\gamma_2^2 - \gamma_1^2) + g(z_2 - z_1)$$

$$= (e_2 - e_1) + P_0(v_2 - v_1) - T_0(s_2 - s_1)$$

where e is $(u + \gamma^2/2 + gz)$

Nonflow Exergy: *Exergy of a fixed mass*

For incompressible substances it is recalled that :

$$du = c\,dT, \quad dv = 0 \quad \text{and} \quad ds = \frac{c}{T}dT$$

For example, the nonflow exergy of a full tank of hot water can be evaluated from :

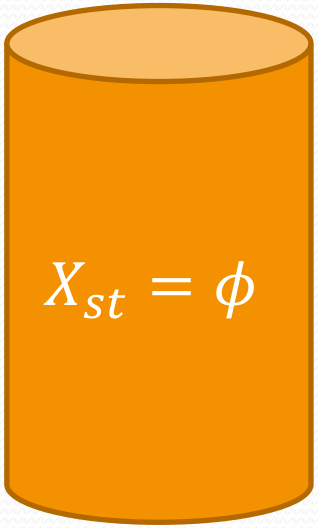
$$\rightarrow \phi = (u - u_0) + \cancelto{0}{P_0(v - v_0)} - T_0(s - s_0)$$

$$= (u - u_0) - T_0(s - s_0)$$

where u is the total specific internal energy and s is the total specific entropy in the tank.

Note 1 : Suffix "0" denotes the dead state.

Note 2 : Nonflow exergy is the exergy stored in the tank, therefore $X_{st} = \phi$

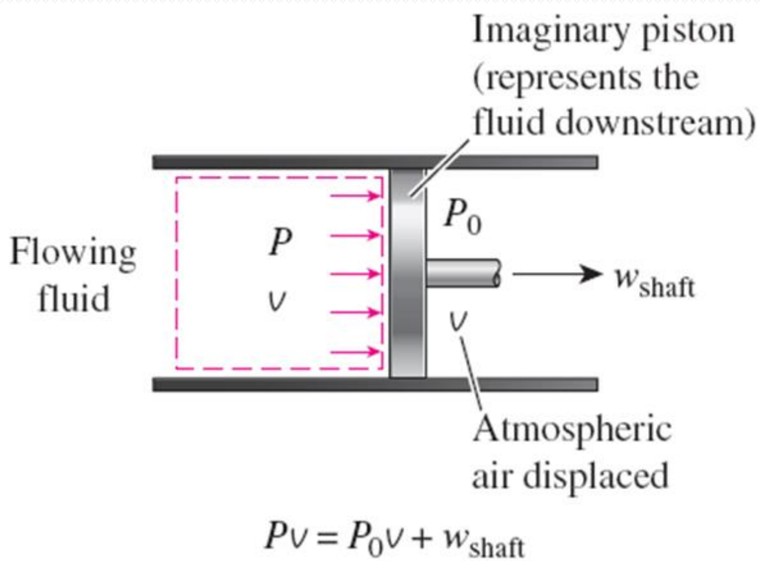


Flow Exergy: *Exergy of a flow stream*

For flowing fluids **flow energy** or **flow work** was defined before. This is the energy needed to maintain flow in a control volume, such that $w_{flow} = Pv$.

The flow work is done against the fluid upstream in excess of the boundary work against the atmosphere such that exergy associated with this flow work:

$$x_{flow} = Pv - P_o v = (P - P_o)v$$



The *exergy* associated with *flow energy* is the useful work that would be delivered by an imaginary piston in the flow section.

Flow Exergy: *Exergy of a flow stream*

Exergy of a flow stream:

$$x_{\text{flowing fluid}} = x_{\text{nonflowing fluid}} + x_{\text{flow}}$$

$$= (u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{1}{2}\gamma^2 + gz + (P - P_0)v$$

$$= (u + Pv) - (u_0 + Pv_0) - T_0(s - s_0) + \frac{1}{2}\gamma^2 + gz$$

$$= (h - h_0) - T_0(s - s_0) + \frac{1}{2}\gamma^2 + gz$$

Therefore exergy for a flow stream:

$$\rightarrow \psi = (h - h_0) - T_0(s - s_0) + \frac{1}{2}\gamma^2 + gz$$

The exergy change of a fluid stream (from state 1 to 2):

$$\rightarrow \Delta\psi = \psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) + \frac{1}{2}(\gamma_2^2 - \gamma_1^2) + g(z_2 - z_1)$$

Example: Exergy change during a compression process

Refrigerant 134a is to be compressed from 0.14 MPa and -10°C to 0.8 MPa and 50°C.

Environment conditions are 20°C and 95 kPa.

Inlet state :

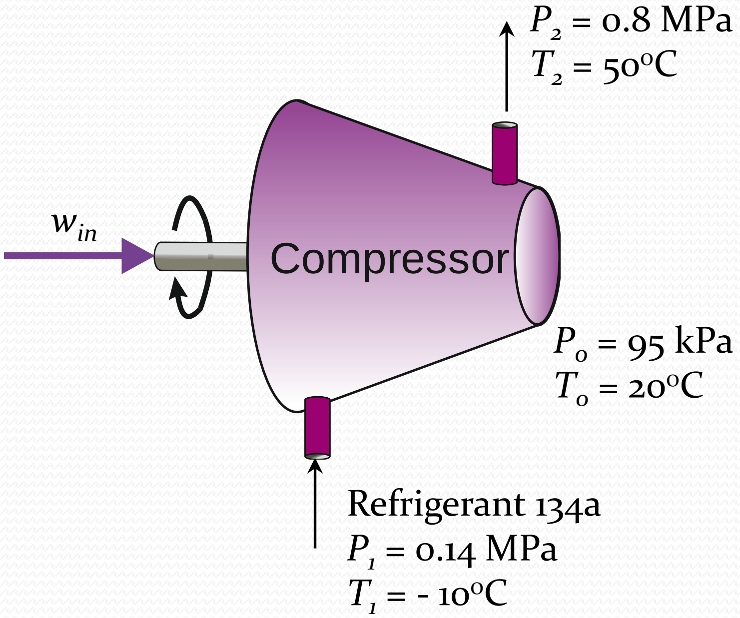
$$\left.\begin{aligned} P_1 &= 0.14\ \text{MPA} \\ T_1 &= -10^\circ\text{C} \end{aligned}\right\} h_1 = 246.36\ \text{kJ/kg} \ \text{ and } \ s_1 = 0.9724\ \text{kJ/kg}\cdot\text{K}$$

Exit state :

$$\left.\begin{aligned} P_1 &= 0.8\ \text{MPA} \\ T_1 &= 50^\circ\text{C} \end{aligned}\right\} h_1 = 286.69\ \text{kJ/kg} \ \text{ and } \ s_1 = 0.9802\ \text{kJ/kg}\cdot\text{K}$$

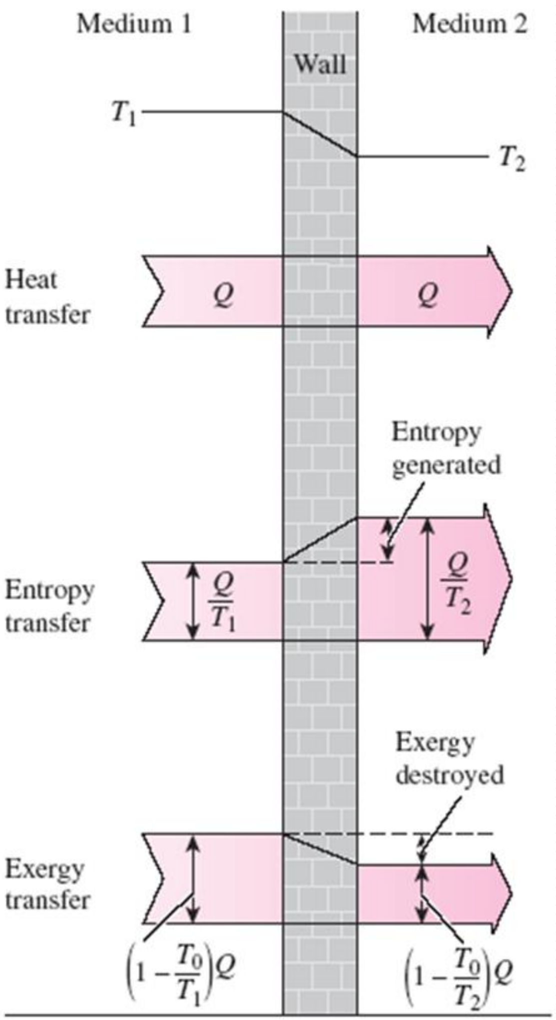
The exergy change of the refrigerant is determined from :

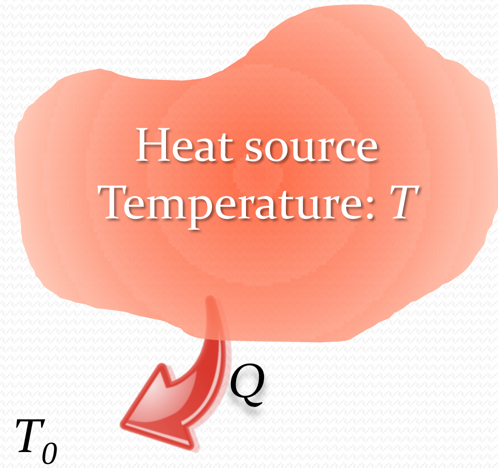
$$\begin{aligned} \Delta\psi = \psi_2 - \psi_1 &= (h_2 - h_1) - T_0(s_2 - s_1) + \frac{1}{2}(\gamma_2^2 - \gamma_1^2) + g(z_2 - z_1) \\ &= (h_2 - h_1) - T_0(s_2 - s_1) \\ &= (286.69 - 246.36)\ \text{kJ/kg} - (293\ \text{K})\{(0.9802 - 0.9724)\ \text{kJ/kg}\cdot\text{K}\} \\ &= 38.0\ \text{kJ/kg} \end{aligned}$$



This represents the minimum work input ($w_{in,min}$) required to compress the refrigerant to the specified state.

Exergy transfer by heat, X_Q





The maximum work can be obtained by a carnot engine :

$$X_Q = \underbrace{\left(1-\frac{T_0}{T}\right)}_{\text{Carnot efficiency}} Q \quad \left\{\begin{array}{l}\text{Exergy transfer}\\ \text{by heat}\end{array}\right.$$

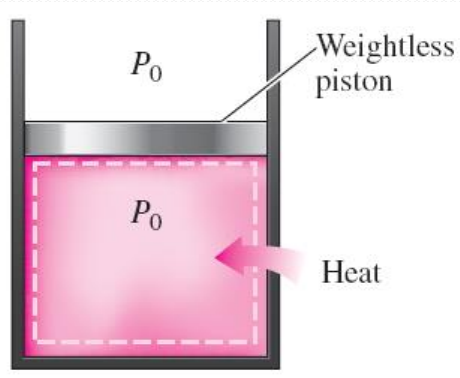
When temperature is not constant :

$$X_Q = \int \left(1-\frac{T_0}{T}\right) \delta Q$$

Exergy transfer by work, X_W

$$X_W \begin{cases} W - W_{surr} & \text{(for boundary work)} \\ W & \text{(for other forms of work)} \end{cases}$$

Note that $W_{surr} = P_0(V_2 - V_1)$



There is no useful work transfer associated with boundary work when the pressure of the system is maintained constant at atmospheric pressure.

Exergy transfer by mass, X_{mass}

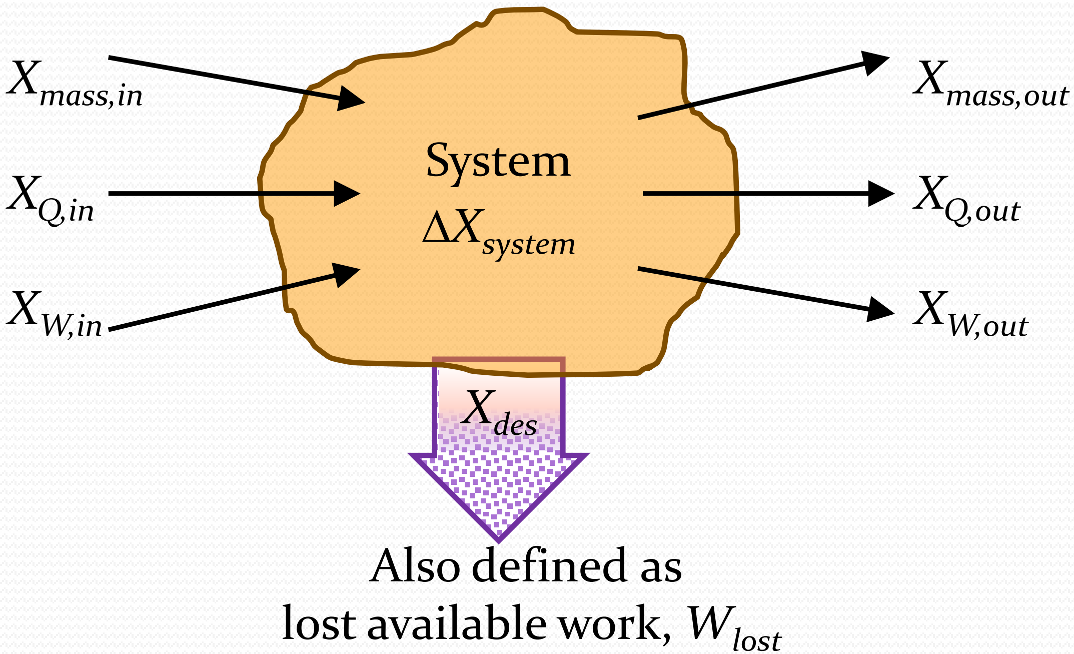
When mass, *m*, enters or leaves a system the amount of exergy that accompanies it:

$$X_{mass} = m\psi$$

Mechanisms of Exergy Balance

$$\begin{pmatrix}\text{Total exergy}\\ \text{entering the}\\ \text{system}\end{pmatrix} - \begin{pmatrix}\text{Total exergy}\\ \text{leaving the}\\ \text{system}\end{pmatrix} - \begin{pmatrix}\text{Total}\\ \text{exergy}\\ \text{destroyed}\end{pmatrix} = \begin{pmatrix}\text{Change in the}\\ \text{total exergy of}\\ \text{the system}\end{pmatrix}$$

$$\underbrace{X_{in}}_{\substack{\text{Exergy entering}\\ \text{the system by mass flow,}\\ \text{heat and work transfers}}} - \underbrace{X_{out}}_{\substack{\text{Exergy exiting}\\ \text{the system by mass flow,}\\ \text{heat and work transfers}}} - \underbrace{X_{des}}_{\substack{\text{Exergy destroyed}\\ \text{during the process}}} = \Delta X_{system}$$

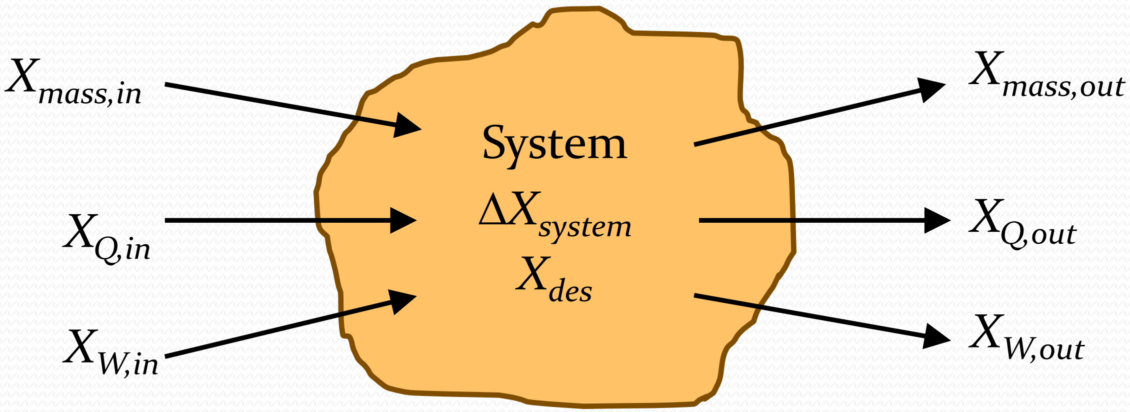


Exergy Balance: Closed Systems

$$\begin{pmatrix}\text{Total exergy}\\ \text{entering the}\\ \text{system}\end{pmatrix} - \begin{pmatrix}\text{Total exergy}\\ \text{leaving the}\\ \text{system}\end{pmatrix} - \begin{pmatrix}\text{Total}\\ \text{exergy}\\ \text{destroyed}\end{pmatrix} = \begin{pmatrix}\text{Change in the}\\ \text{total exergy of}\\ \text{the system}\end{pmatrix}$$

$$\underbrace{X_{in}}_{\substack{\text{Exergy entering}\\ \text{the system by mass flow,}\\ \text{heat and work transfers}}} - \underbrace{X_{out}}_{\substack{\text{Exergy exiting}\\ \text{the system by mass flow,}\\ \text{heat and work transfers}}} - \underbrace{X_{des}}_{\substack{\text{Exergy destroyed}\\ \text{during the process}}} = \Delta X_{system}$$

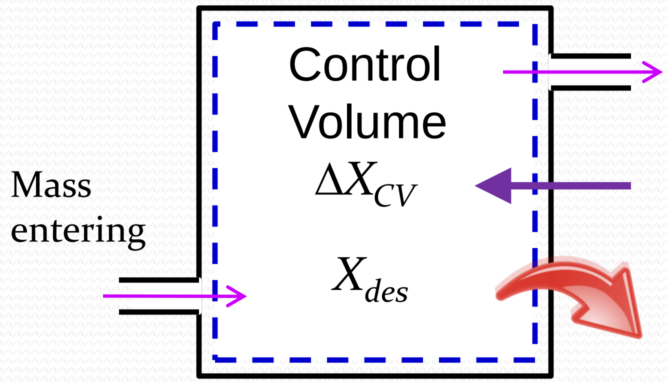
A closed system does not involve any mass flow



Exergy Balance: Control Volumes

$$\begin{pmatrix}\text{Total exergy}\\ \text{entering the}\\ \text{system}\end{pmatrix} - \begin{pmatrix}\text{Total exergy}\\ \text{leaving the}\\ \text{system}\end{pmatrix} - \begin{pmatrix}\text{Total}\\ \text{exergy}\\ \text{destroyed}\end{pmatrix} = \begin{pmatrix}\text{Change in the}\\ \text{total exergy of}\\ \text{the system}\end{pmatrix}$$

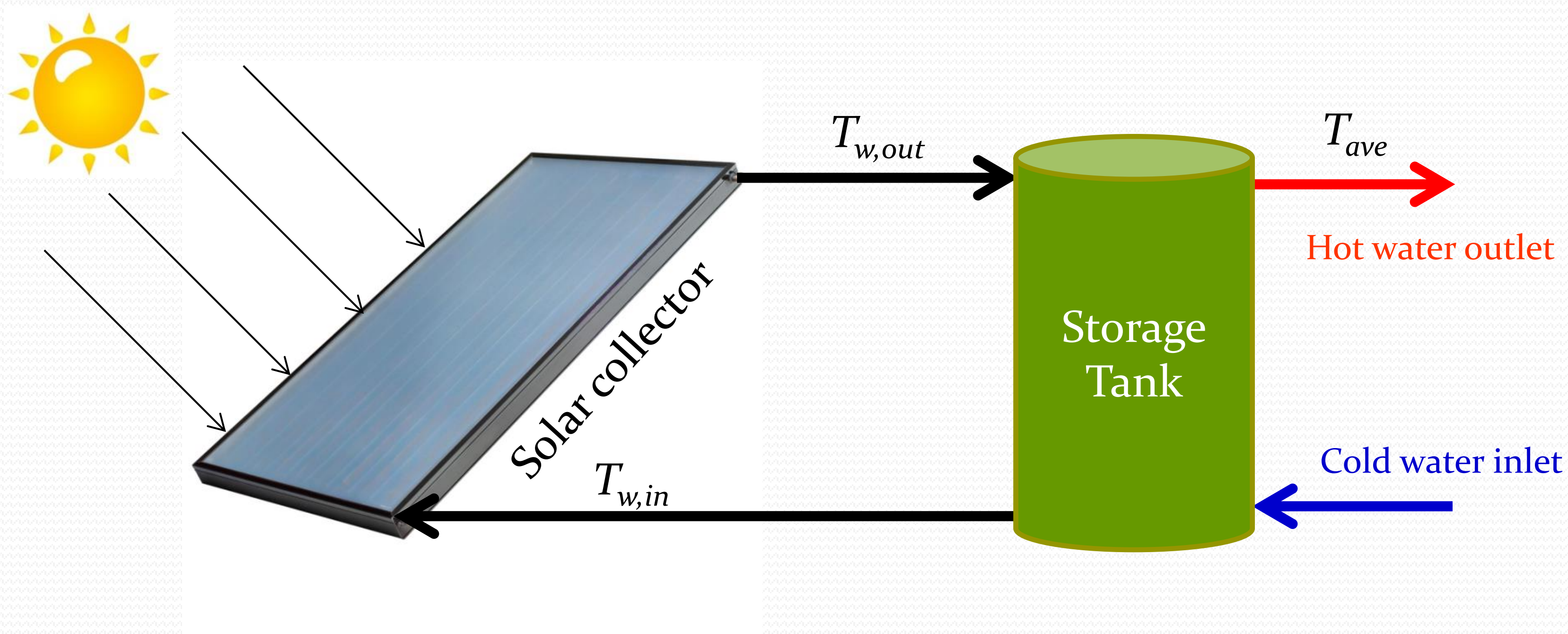
$$\underbrace{X_{in}}_{\substack{\text{Exergy entering}\\ \text{the system by mass flow,}\\ \text{heat and work transfers}}} - \underbrace{X_{out}}_{\substack{\text{Exergy exiting}\\ \text{the system by mass flow,}\\ \text{heat and work transfers}}} - \underbrace{X_{des}}_{\substack{\text{Exergy destroyed}\\ \text{during the process}}} = \Delta X_{system}$$



Procedure for Exergy Analysis

- Subdivide the process under consideration into sections as desired
- Conduct conventional energy analysis
- Select a reference environment
- Evaluate energy and exergy values relative to the environment
- Set up the exergy balance and determine exergy destruction
- Define first and second law efficiencies of the system
- Interpretation of results and conclusions

Example: Solar Water Heating System from Hepbasli*



$$\eta_{II} = \frac{\dot{X}_{output}}{\dot{X}_{input}} = 1 - \frac{\dot{X}_{des}}{\dot{X}_{input}} = \frac{\dot{X}_{output}}{\dot{X}_{sun}}$$

*Hepbasli A. Renewable and Sustainable Energy Reviews 2008;12

Solar Collector

The instantaneous exergy efficiency of solar collector :

$$\eta_{\mathrm{II,col}} = \frac{\text{Increased exergy of water}}{\text{Exergy of the solar radiation}} = \frac{\dot{X}_u}{\dot{X}_{col}}$$

where

$$\dot{X}_u = \dot{m}_w[(h_{w,out} - h_{w,in}) - T_0(s_{w,out} - s_{w,in})]$$

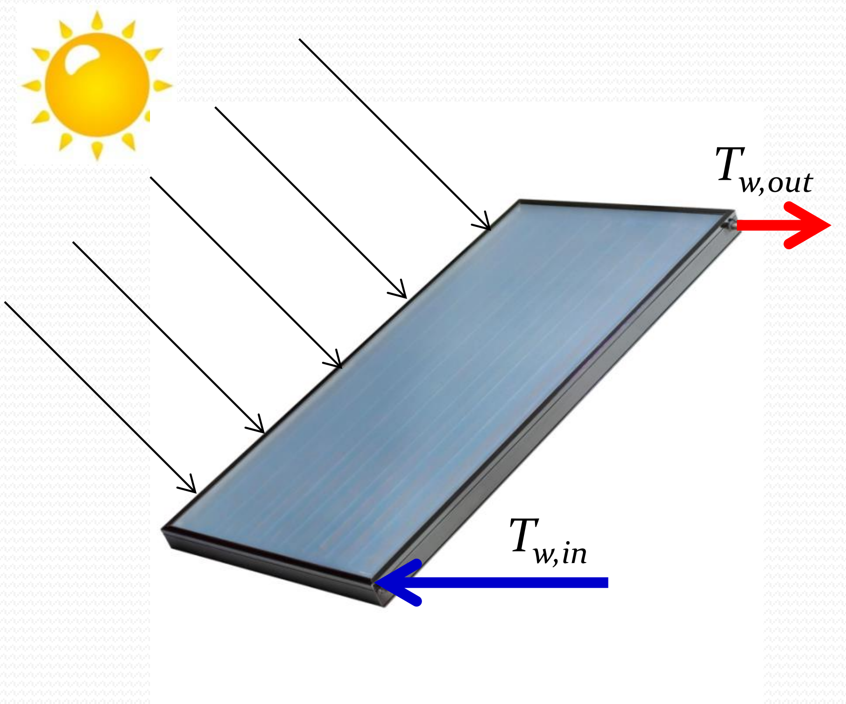
(Note that $s = dq/T = CdT/T = C\ln T$)

$$\rightarrow = \dot{m}_w C_w \left\{ (T_{w,out} - T_{w,in}) - T_0\left(\ln\frac{T_{w,out}}{T_{w,in}} \right) \right\}$$

$$\rightarrow = \dot{Q}_u \left\{ 1 - \frac{T_0}{T_{w,out} - T_{w,in}} \left(\ln\frac{T_{w,out}}{T_{w,in}} \right) \right\}$$

and

$$\dot{X}_{col} = \underbrace{A}_{\text{Area}} \underbrace{I_T}_{\substack{\text{Total}\\ \text{global}\\ \text{irradiance}}} \underbrace{r_{srad,\max}}_{\substack{\text{The maximum}\\ \text{exergy-to-}\\ \text{energy ratio}\\ \text{for radiation}}}$$

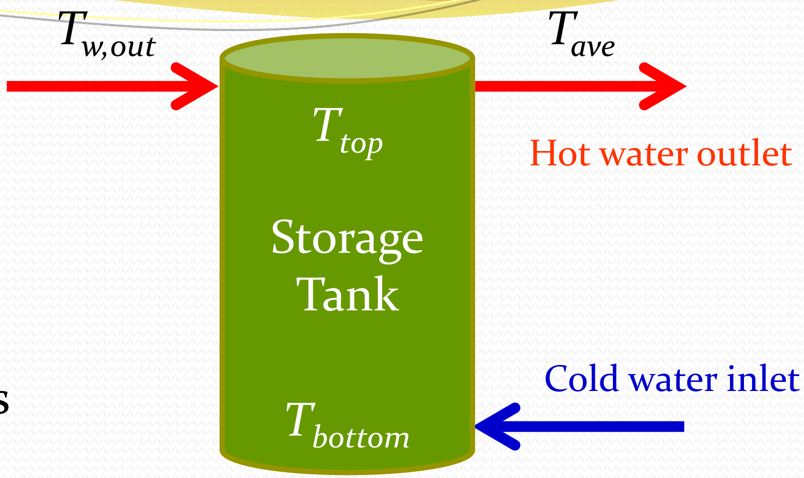


According to Petela*

$$r_{srad,\max} = 1 + \frac{1}{3}\left(\frac{T_0}{T}\right)^4 - \frac{4}{3}\frac{T_0}{T}$$

*Petela R. Exergy of undiluted thermal radiation. Solar Energy 2003;74

Storage Tank



Exergy from the storage tank to the end-user as presented by Xiaowu et al*:

$$\dot{X}_{output} = \dot{m}_w C_w (T_{ave} - T_0) - \dot{m}_w C_w T_0 \left(\ln \frac{T_{top}}{T_0} - 1 \right) - \frac{T_{bottom} T_0 \dot{m}_w C_w}{T_{top} - T_{bottom}} \ln\left(\frac{T_{top}}{T_{bottom}} \right)$$

Exergy from the collector to the storage tank as presented by Xiaowu et al*:

$$\dot{X}_{col\rightarrow \mathrm{tank}} = \dot{m}_w C_w \left[(T_{w,out} - T_0) \right] - T_0 \left(\ln \frac{T_{w,out}}{T_0} \right)$$

*Xiaowu et al. Exergy analysis of domestic-scale solar water heatersRenewable and Sustainable Energy Reviews 2005;9