



## Linear ADT-1: Restricted Lists Stacks, Queues

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## Linear ADTs

### Restricted Lists

- Stack
- Queue
  - Circular queue
  - Priority queue

### General Lists

- Arrays
- Linked list
- Circular list
- Doubly linked list



### Using Stacks: Checking Expression Validity

- Consider the following expression

$$[a + b * \{c/d(1-n) + e/f (1+n)\} / (g - \text{sqrt}[(b^{**2}) - 4*a*c]/2)]$$

The slide contains a logo in the top left, a title "Using Stacks: Checking Expression Validity", a bullet point "Consider the following expression", and a mathematical expression in square brackets.



## Using Stacks: Checking Expression Validity

```

bool ckeck_validity (char [ ] exp) {
    char next_char, popped_char;
    stack S;
    bool valid = true;
    while (not_empty(exp)) {
        next_char = get_next(exp);
        if ((next_char == '(' or (next_char == '{' or (next_char == '[')) then
            push (S, next_char);
        if ((next_char == ')' or (next_char == '}' or (next_char == ']')) then {
            popped_char = pop (S);
            if (next_char <> popped_char)
                return (valid = false);
        }
    }
    If (not_empty(S)) then
        valid = false;
    return valid;
}

```



## Stack

- Linear LIFO organization
- An attribute **top** always pointing to the most recently inserted data item
- Basic operations
  - void **push(x, S)** : Insert element x into S
  - item **pop(S)** : Return the last element inserted into S
  - boolean **isStackEmpty(S)**: Return yes if S is empty



## Applications of Stacks

- Direct applications
  - Delimiter matching
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine
  - Expression evaluations
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

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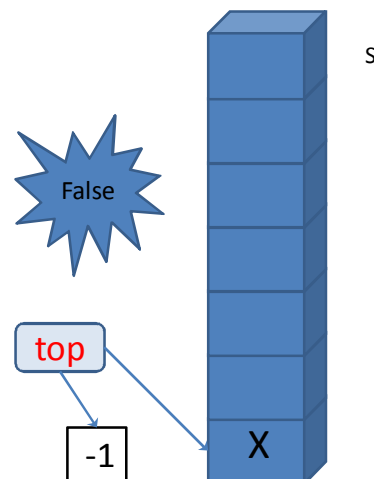
## Stack: An Array Implementation


```
createStack(S): Define an array S
                 for some fixed size N,
                 top ← -1
```

```
push(x,S): if top = N-1 then error
           else top ← top + 1
           S[top] ← x
```

```
isEmpty(S): return (top < 0)
```

```
pop(S): if isEmpty() then error
        else item ← S[top]
        top ← top - 1
        return (item)
```

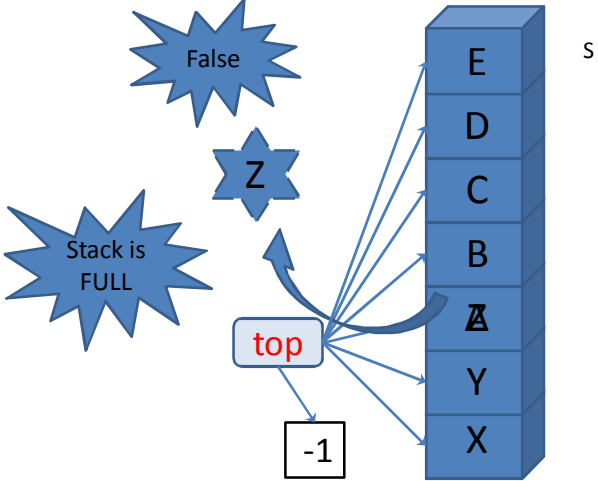





### Stack Animation: Array Implementation

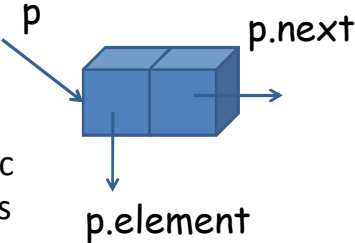
```

createStack()
push (X, S)
push (Y, S)
push (Z, S)
pop (S)
isEmpty(S)
push (A, S)
push (B, S)
push (C, S)
push (D, S)
push (E, S)
push (F, S)
...
    
```

### Stack: Pointer Implementation

- Pointer facilitate dynamic implementation of a data structure
- Data is organized in a dynamic structure comprising of nodes where each node
  - is identified by a reference (pointer)
  - contains a data element
  - may points to next node/null

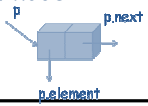




## Stack: Pointer Implementation

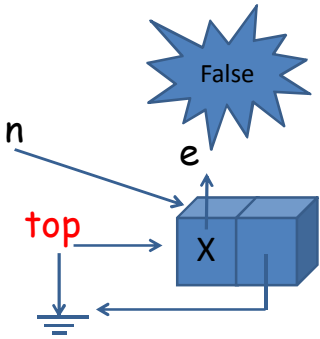
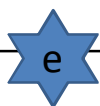
createStack(S): Define a pointer **top**,  
 $top \leftarrow null$

push(x,S):  $n = \text{create new node}$   
 $n.\text{element} \leftarrow x$   
 $n.\text{next} \leftarrow top$   
 $top \leftarrow n$



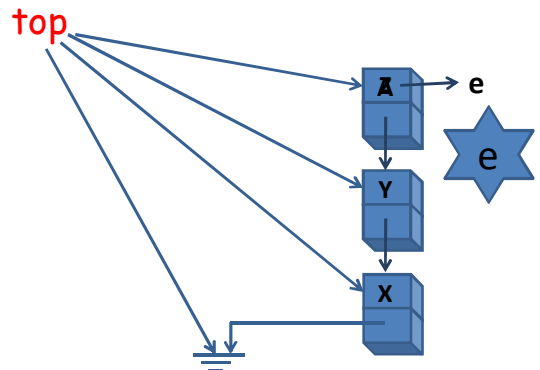
isEmpty(S): return ( $top == null$ )

pop(S): if isEmpty() then error  
 else  $e \leftarrow top.\text{element}$   
 $top \leftarrow top.\text{next}$   
 return (e)



## Stack Animation: Pointer Implementation

createStack(S)  
 push (X, S)  
 push (Y, S)  
 push (Z, S)  
 pop (S)  
 isEmpty(S)  
 push (A, S)  
 push (B, S)  
 push (C, S)  
 push (D, S)  
 push (E, S)  
 push (F, S)  
 ...





## Stack: The running time

```
createStack(S): Define a pointer top,
                top ← null
```

```
push(x,S): n = create new node
           n.element ← x
           n.next ← top
           top ← n
```

```
isStackEmpty (S): return (top == null)
```

```
pop(S): if isStackEmpty() then error
        else e ← top.element
          top ← top.next
          return (e)
```

- Each operation takes  $O(1)$  time



## Stack: C implementation using Array

```
#define MAX 10
struct stack {
    int arr[MAX];
    int top;
} STACK;
```

```
void createStack (STACK *s);
int stackEmpty (STACK *)
int stackFull (STACK *)
void push (STACK *, int item);
int pop (STACK *);
```

```
int stackFull (STACK *s) {
    if (s-> top == MAX - 1)
        return (1);
    else return (0);
}
```

```
int stackEmpty (STACK *s) {
    if (s-> top == -1)
        return (1);
    else return (0);
}
```

```
void createStack (STACK *s) {
    s-> top = -1;
}
```

```
void push (STACK *s, int item) {
    if (stackFull(s)) {
        printf ("\nStack is full.");
        return;
    }
    s-> top++;
    s-> arr[s-> top] = item;
}
```

```
int pop (struct stack *s) {
    int item;
    if (stackEmpty(s)) {
        printf ("\nStack is empty.");
        return NULL;
    }
    item = s-> arr[s-> top];
    s-> top--;
    return item;
}
```



## Stack: C implementation using Pointer

```

struct node {
    int data;
    struct node *link;
};
void createStack(struct node **)
void push ( struct node **, int );
int pop ( struct node **);
void delStack ( struct node **);
int stackEmpty ( struct node **);

```

```

int stackEmpty ( struct node **tos) {
    return ( *tos == NULL);
}

```

```

void createStack (struct node ** top) {
    top = NULL;
}

```

```

void push ( struct node **top, int item ) {
    struct node *temp;
    temp = (struct node*) malloc(sizeof(struct node));
    if ( temp == NULL )
        printf( "\nStack is full. " );
    temp -> data = item;
    temp -> link = *top;
    *top = temp;
}

```

```

int pop ( struct node **top) {
    struct node *temp;
    int item;
    if (stackEmpty(top)) {
        printf( "\nStack is empty. " );
        return 0;
    }
    temp = *top;
    item = temp -> data;
    *top = ( *top ) -> link;
    free ( temp );
    return item;
}

```



## Queue

- Linear FIFO organization
- An attribute **rear** points to the place the next data item to be inserted
- An attribute **front** points to the next data item to be removed
- Basic operations
  - void enqueue(x,Q) : Insert element x into Q
  - item dequeue(Q) : Return the last element inserted into Q
  - boolean isEmpty(Q): Return yes if Q is empty



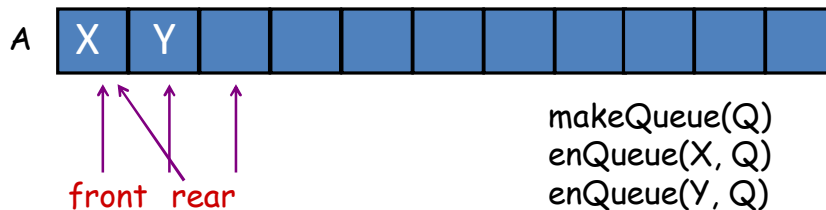


## Queue

- `void enqueue(x, Q)` : Insert **last** element x into Q
- `item dequeue(Q)` : Delete the **first** element in Q
- `boolean isEmpty(Q)`: Return yes if Q is empty
- `item front(Q)`: Return the first element in Q
- `int size(Q)`: Return the number of elements in the Q
- `Queue make-queue()`: Initialize a Q



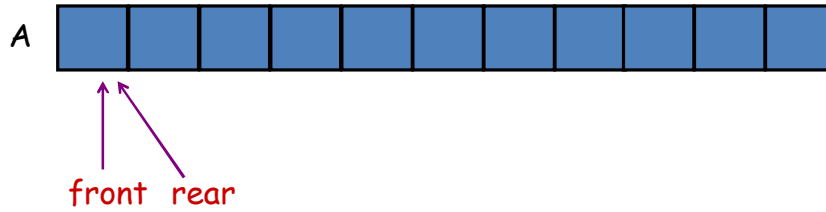
## Queue: Array Implementation



```
makeQueue(Q)
enqueue(X, Q)
enqueue(Y, Q)
enqueue(Z, Q)
dequeue(Q)
isEmpty(Q)
dequeue(Q)
enqueue(A, Q)
enqueue(B, Q)
...
```



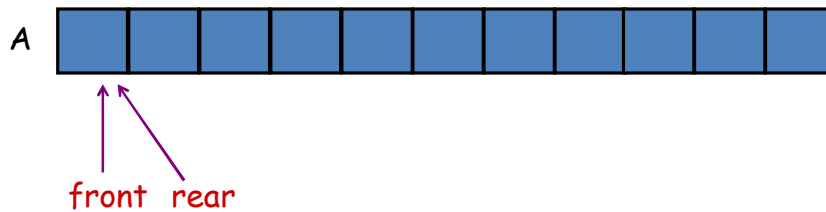
## Queue: Array Implementation



Empty Queue?  $front == rear$



## Circular Queue: Array Implementation



Empty Queue?  $front == rear$

Full Queue?  $front == (rear+1) \bmod N + 1$



## Circular Queues: Types of Array Implementations

- **front** is always at first position
- An array with two indices always increasing
- A circular array with **front** and **rear** and one position is left vacant
- A circular array with two indices and a boolean variable
- A circular array with two indices and a count (size) variable
- Special values for array indices



## Circular Array with **size()** function

```
isQueueEmpty(Q):  
return (front == rear)
```

```
front(Q):  
if (isQueueEmpty(Q)) then error  
else return A[front]
```

```
size(Q):  
if (rear >= front) then  
return (rear-front+1)  
else return (N-(front-rear))
```

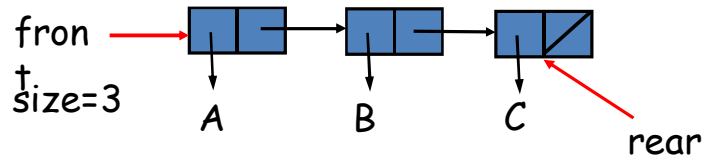
```
enQueue(x,Q): if size(Q) == N then error  
else A[rear] ← x  
rear ← (rear+1) mod N
```

```
pop(Q):  
if (isQueueempty(Q)) then error  
else e ← A[front]  
front ← (front + 1) mod N  
return (e)
```

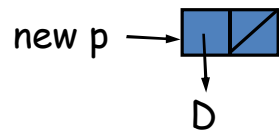
```
deQueue(Q):  
if (isQueueempty?(Q)) then error  
else e ← A[front]  
front ← (front + 1) mod N  
return (e)
```



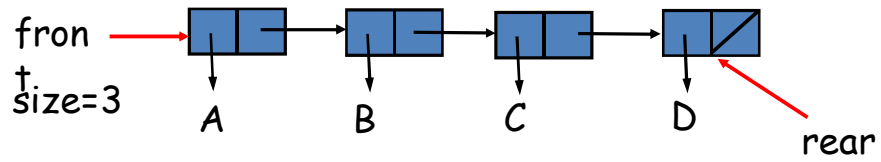
## Queue: Pointer Implementation



enqueue(D,Q)



## Implementation with lists



enqueue(D,Q)



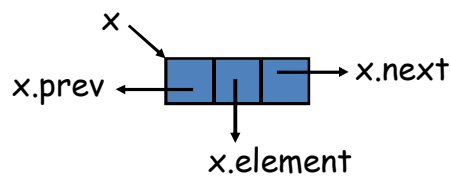
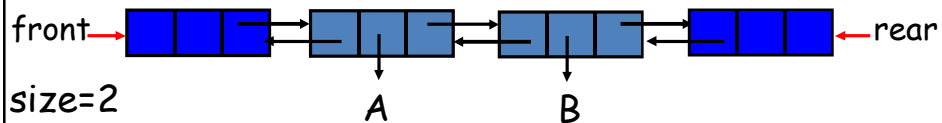
## Double Ended Queue (Deque)

`insertFront(x,D)` : Insert x as the first in D  
`deQueue(D)` : Delete the first element of D  
`enQueue(x,D)`: Insert x as the last in D  
`removeLast(D)`: Delete the last element of D  
`Size(D)`  
`isQueueEmpty?(D)`  
`make-deque()`



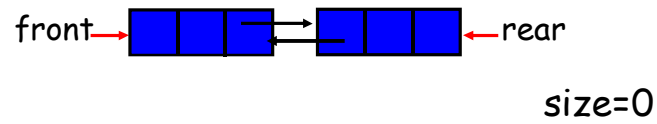
## Implementation of Deque with Doubly Linked Lists

We use two sentinels (dummy nodes) here to make the code simpler





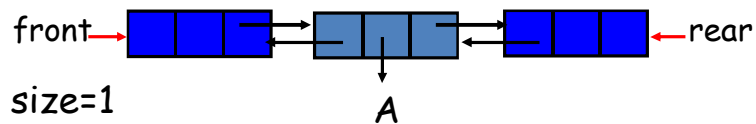
## Empty Deque?



`front.next == rear && rear.prev == front`



## Insert Element in the Deque



`insertFront(B,D):`

```
n = new node
n.element ← B
n.next ← front.next
(front.next).prev ← n
front.next ← n
n.prev ← front
size ← size + 1
```

Diagram illustrating the insertion of a new node  $n$  at the front of a doubly linked list. The list currently contains one node  $A$  (labeled  $size=1$ ). The  $front$  and  $rear$  pointers both point to node  $A$ . Node  $n$  has element  $B$ . The diagram shows the state before the insertion operation.

**insertFront(B,D):**

```

n = new node
n.element ← B
n.next ← front.next
(front.next).prev ← n
front.next ← n
n.prev ← front
size ← size + 1
    
```

Diagram illustrating the insertion of a new node  $n$  at the front of a doubly linked list. The list now contains two nodes:  $n$  (with element  $B$ ) and  $A$  (labeled  $size=1$ ). The  $front$  pointer now points to node  $n$ , and the  $rear$  pointer still points to node  $A$ . Node  $n$ 's next pointer points to node  $A$ , and node  $A$ 's previous pointer points to node  $n$ .

**insertFront(B,D):**

```

n = new node
n.element ← B
n.next ← front.next
(front.next).prev ← n
front.next ← n
n.prev ← front
size ← size + 1
    
```

$n \rightarrow$  [ ] [ ] [ ]  
 [ ] [ ] [ ]  $\xrightarrow{\text{front}}$  [ ] [ ] [ ]  $\xleftarrow{\text{rear}}$  [ ] [ ] [ ]  
 B A size=1

**insertFront(B,D):**  
*n = new node*  
*n.element ← B*  
*n.next ← front.next*  
*(front.next).prev ← n*  
*front.next ← n*  
*n.prev ← front*  
*size ← size + 1*

$n \rightarrow$  [ ] [ ] [ ]  
 [ ] [ ] [ ]  $\xrightarrow{\text{front}}$  [ ] [ ] [ ]  $\xleftarrow{\text{rear}}$  [ ] [ ] [ ]  
 B A size=2

**insertFront(B,D):**  
*n = new node*  
*n.element ← B*  
*n.next ← front.next*  
*(front.next).prev ← n*  
*front.next ← n*  
*n.prev ← front*  
*size ← size + 1*





## Restricted Linear ADT: Summary

- Two important forms of linear restricted ADT are Stacks and Queues
- Stack is LIFO and Queue is FIFO
- All operations are  $O(1)$
- Typically not used as search data structures
- Where to use them?