ANT colony Optimizations

Ex: TSP:

d:

0	10	12	11	14
10	0	13	15	8
12	13	0	9	14
11	15	9	0	16
14	8	14	16	0

5x5

5 cities.

step 1: Define the # of ants : $N = 3$

Define heuristic function h: $\frac{1}{d}$ → visibility of a city.

Evaporation coefficient $\rho = 0.5$

h:

0	0.1	0.833	0.909	0.0714
0.1	0	0.076	0.0667	0.1250
0.833	0.0769	0	0.1111	0.0714
0.0909	0.667	0.1111	0	0.0625
0.0714	0.125	0.0714	0.0625	0

5x5

Initial pheromone value τ_0:

τ_0:

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Possibility to visit other cities from <u>city 1</u>

$$P_k(r,s) = \begin{cases} [\tau(r,s)^{\alpha} \cdot \eta(r,s)^{\beta}] / \sum_{u \in M_k} \tau(r,u)^{\alpha} \cdot \eta(r,u)^{\beta}, & \text{if } s \in M_k \\ 0 & \text{otherwise} \end{cases}$$

$P_k(r,s)$ → Probability to move from city r, to city s.

η → visibility of cities r and s

$u \in M_k$ → unvisited cities.

Let $\alpha = 1$, $\beta = 2$.

start with <u>Ant 1</u>: starting city is <u>1</u>, column 1 of η becomes <u>0</u>.

$\tau(1,s)^1 \eta(1,s)^2$ from city 1 to all others.

$(1,2) \Rightarrow \tau(1,2)^1 \eta(1,2)^2 = 1 * (0.1)^2 = 0.01$

$(1,3) \Rightarrow \tau(1,3)^1 \eta(1,3)^2 = 1 * (0.0833)^2 = 0.0069$

$(1,4) \Rightarrow 0.0083$

$(1,5) \Rightarrow 0.051$

Total $\tau(1,s)^1 \eta(1,s)^2 = 0.0303$

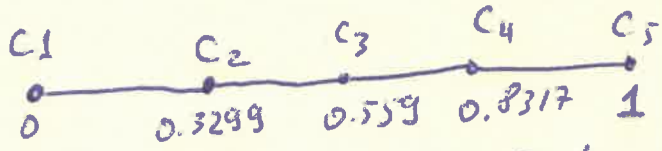
Ant 1: $P_1(1,s) = [\tau(1,s)^1 \eta(1,s)^2] / [\sum \tau(1,s)^1 \cdot \eta(1,s)^2]$

$p(1,2) = 0.3299$

$p(1,3) = 0.2291$

$p(1,4) = 0.2727$

$p(1,5) = 0.1683$



Generate a random number: $r = 0.6841$ → select <u>city 4</u>

column 4 of η will be zero.

Constructing a solution for ant 1:

1 4 ?

Now calculate τ from city 4 to all the others except city 1 & 4

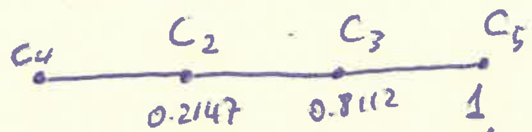
$\tau(4,s)^1 \cdot \eta(4,s)^2$

4,2 → 0.0044
4,3 → 0.0123
4,4 → 0
4,5 → 0.0039

Total = 0.0207

So $P_1(4,s) \Rightarrow$

$P_1(4,2) = 0.2147$
$P_1(4,3) = 0.5965$
$P_1(4,4) = 0$
$P_1(4,5) = 0.1887$



Let r = 0.4024 → city 3 <u>selected</u>.

So solution becomes ⇒ 1, 4, 3

repeat this porceses until a solution constructed.

let Ant1 : 1 4 3 5 2 1 → f : 52

Ant2 : 1 4 2 5 3 1 → f = 60

Ant3 : 1 4 5 2 3 1 → f = 60

** Now update pheromone level τ:

$$\tau_{r,s} = (1-\rho)\,\tau_{r,s} + \sum_{k=1}^{N} \Delta\tau^{k}_{r,s} \quad \rightsquigarrow \frac{1}{f^k}$$

start with <u>Ant 1:</u> $\Delta\tau^1_{15} = \frac{1}{f^1} = \frac{1}{52} = 0.0192$

τ : (1,4), (4,3), (3,5), (2,1) will change by $(1-0.5)*1 + 0.0192$

others by $(1-0.5) \cdot 1$.

so τ

0.5	0.5	0.5	0.5192	0.5
0.5192	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5192
0.5	0.5	0.5192	0.5	0.5
0.5	0.5192	0.5	0.5	0.5

For Ant2 1 4 2 5 3 1 $f = 60$ $\Delta\tau^2 = \frac{1}{60} = 0.0167$

Do not forget to use initial pheromone to update.

$\tau = (1-0.5) \cdot 1 + \left(\frac{1}{52} + \frac{1}{60}\right)$

τ

0.5	0.5	0.5	0.5359	0.5
0.5192	0.5	0.5	0.5	0.5167
0.5167	0.5	0.5	0.5	0.5192
0.5	0.5167	0.5192	0.5	0.5
0.5	0.5192	0.5167	0.5	0.5

$\left(\frac{1}{52} + \frac{1}{60} + \frac{1}{60}\right)$

for Ant3 1 4 5 2 3 1 $f = 60$ $\frac{1}{60} = 0.0167$

τ

0.5	0.5	0.5	0.5526	0.5
0.5192	0.5	0.5167	0.5	0.5167
0.5334	0.5	0.5	0.5	0.5192
0.5	0.5167	0.5192	0.5	0.5167
0.5	0.5359	0.5167	0.5	0.5

Best = 52 (