IENG112
Notes #11

Engineering Economy

Engineering Economy is applying “Scientific economics to the solution of practical problem”.

Time Value of Money is the most important concept of engineering economy. The change in the amount of money over a given time period is called the time value of money.

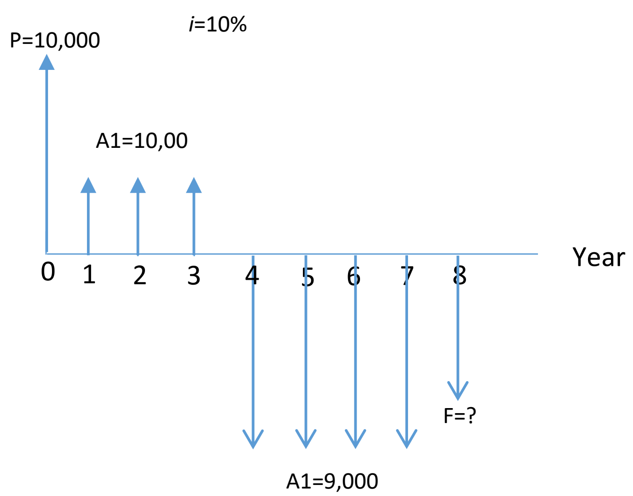
Terminology and Symbols
The equations and procedures of engineering economy utilize the following terms and symbols.

i = annual interest rate; % per year.
n = number of annual interest periods.
P = Amount of money at present or time 0.
A = series of consecutive, equal, end-of-period amounts of money.
F = Amount of money at some future time.

Time Diagram shows the flow of money for the various alternatives. All cash outflows (first cost of assets, operating costs, maintenance costs, income taxes, upgrade costs, etc.) are shown as arrows pointing upward, and all cash inflows (revenues, operating cost reductions, asset salvage value, etc.) are shown as arrows pointing downward.

Time diagram is a very useful tool in economic analysis, especially when the cash flow series is complex. It is a graphical representation of cash flows drawn on a time scale.

Example1. Construct a time diagram for the following cash flows: $10,000 outflow at time zero, $3,000 per year outflow in years 1 through 3 and $9,000 inflow in years 4 through 7 at an interest rate of 10% per year and an unknown future amount in year 8.



Interest Factors: The value of money is a function of time and interest rate.
If we wish to determine the future value (F) after n years from a single present amount of money (P) invested at t = 0, the amount F_1 accumulated 1 year hence at an interest rate of i percent per year will be
$F_1 = P + Pi = P(1 + i)$

At the end of the second year, the amount accumulated F_2 is the amount after 1 year plus the interest from the end of year 1 to the end of year 2 on the entire F_1.

$F_2 = F_1 + F_1 i = P(1+i) + P(1+i)i = P(1 + i + i + i^2) = P(1+2i+i^2) = P(1+i)^2$

Similarly, the amount of money accumulated at the end of year 3 will be;

$F_3 = F_2 + F_2 i$, substituting $P(1+i)^2$ for F_2 and simplifying, we get

$F_3 = P(1+i)^3$

It is evident by mathematical induction that the formula can be generalized for *n* years to

$F = P(1+i)^n$

The factor $(1+i)^n$ is called the single-payment compound amount factor but it is usually referred to as F/P factor. Reverse the situation to determine the **P** value for a stated amount of **F** that occurs **n** years in the future;

$$P = F\left[\frac{1}{(1+i)^n}\right]$$

Similar relationships can be derived for all combinations of P, F, and A.

$$P = A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] \quad \Longrightarrow \quad A = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

$$F = A\left[\frac{(1+i)^n-1}{i}\right] \quad \Longrightarrow \quad A = F\left[\frac{i}{(1+i)^n-1}\right]$$

Example2. How much will \$150 be worth 10 years from now at an annual interest rate of 12%?

$F = P(1+i)^n$ where n = 10, i = 0.12 and P = \$150

F = 150*(1+0.12) ^10 = \$456.90

Example3. How much would have to be set aside now to provide \$10,000 fifteen years from now at an annual interest rate of 12%?

$P = F\left[\frac{1}{(1+i)^n}\right]$= 10,000/1.12^15 = \$1,827

Example4. If \$100 is set aside at the end of each year for 8 years at an annual interest rate of 12%, what would it be worth at the end of the eight year?

A = \$100 (equal annual payments)

$F = A\left[\frac{(1+i)^n-1}{i}\right] = 100\left[\frac{(1+0.12)^8-1}{0.12}\right]$ = \$1,230

Example5. How much would be required at the end of each year for 10 years to repay a loan of \$1,500 now if the interest is 12% per year?

$A = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] = 1{,}500\left[\frac{0.12(1+0.12)^{10}}{(1+0.12)^{10}-1}\right]$ = \$265.50

Example6. How much would be required at the end of each year for 9 years to accumulate \$2,000 at the end of the ninth year if the interest is 12% per year?

$A = F\left[\frac{i}{(1+i)^n-1}\right] = 2{,}000\left[\frac{0.12}{(1+0.12)^9-1}\right] = \$135.40$

Example7. How much can be borrowed now, if it can be repaid by five equal end of year payments of $100 each? The annual interest rate is 12% per year.

$P = A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] = 100\left[\frac{(1+0.12)^5-1}{0.12(1+0.12)^5}\right] = \$360.48$

Comparison of Investment Options –Present Worth Analysis -

One of the methods used in evaluating the project alternatives is the Present Worth Analysis.

In this method the future amount of money for each alternative converted to its equivalent value at the end of year 0 considering the given interest rate. This equivalent amount at the end of year 0 is called the ***present worth amount.***

Example8. As president of Gadgets, Inc., one of your major decisions is to decide which one of two new automated machines should be purchased to replace a manual method now used on a product line. The product line will be discontinued entirely in 5 years. The manual method costs $10,000 per year. The new automated machine A costs $30,000 now (purchase price) and will cost $2,000 per year to operate over the next 5 years, at which time it will be worthless. Another new automated machine B is available that costs $40,000 now and will cost $1,000 per year to operate for the next 5 years, at which it will be worth $9,000 (Interest rate is 12% per year).

We should calculate the present worth of all three alternatives and select the best one;

Alternative 1- keep the current manual machine;

$P_1 = A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] = 10{,}000 * \frac{(1+0.12)^5-1}{0.12(1+0.12)^5} = \$36{,}048$

Alternative 2- buying machine A;

$P_2 = 30{,}000 + 2{,}000\left[\frac{(1+0.12)^5-1}{0.12(1+0.12)^5}\right] = \$37{,}209.60$

Alternative 3- buying machine B;

$P_3 = 40{,}000 + 1{,}000\left[\frac{(1+0.12)^4-1}{0.12(1+0.12)^4}\right]$ - 8,000 (1/(1+0.12)^5 = $38,498.2

It can be deduced that alternative 1 (stay with the manual method) is the best alternative.