CIVL471 DESIGN OF RC STRUCTURES

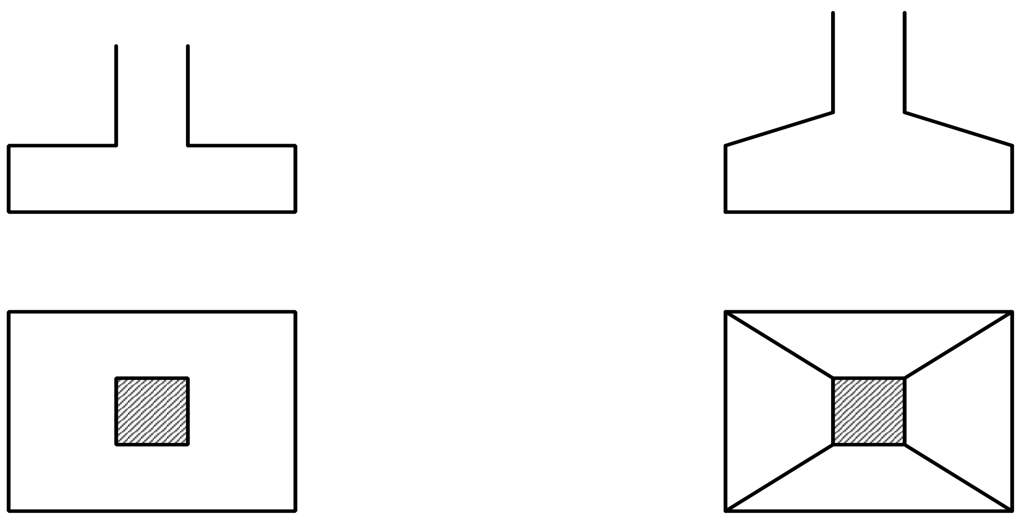
LECTURE NOTE #13

CHAPTER V

FOOTINGS and FOUNDATIONS (CONT.)

5.4 SINGLE COLUMN FOOTINGS

Bearing area of a single column footing is square, rectangular or occasionally circular. The shape of vertical cross-section is either rectangular or trapezoidal (See Fig. 511).



(a) (b)

Figure 5 11

It is obvious that they behave like slabs cantilevering in all directions. However bending moments and shear forces may be calculated in two principal directions and reinforcement can be provided in these two directions. Bending moments and shear forces used in design are calculated at the column faces as shown in Fig. 5 12.

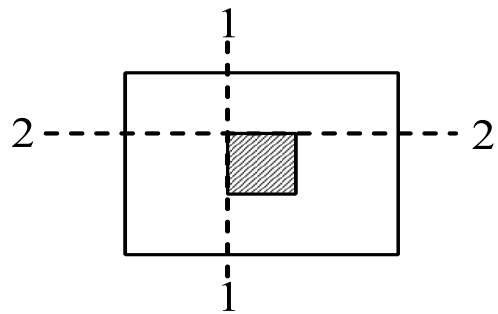


Figure 5 12

Single column footings should be designed for the combined action of axial force and bending moments but if moments are very small they may be neglected. Reinforcement is placed like slab reinforcement. Therefore effective depths in two directions are different but an average value may be used for both directions with no great mistake. Shear reinforcement is not practical and economical in footings. Therefore shear force should be resisted by concrete alone and the height of the footing should be selected accordingly.

In single column footings a second type of failure which is known as punching shear failure may be observed. Column load which is concentrated at the bottom of the column spreads out into the footing but generates also shear stresses around the column.

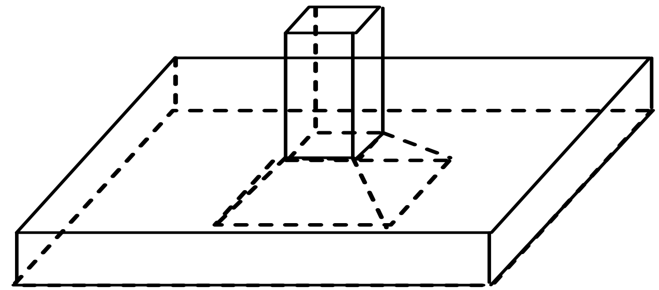


Figure 5.13

In Fig.5.13 such a failure shape is shown. It resembles a truncated pyramid, sloped sides making an angle approximately equal to 45°. In the case of circular columns this shape takes the form of a truncated cone.

In punching shear average shear stresses are calculated at the vertical planes d/2 away from the column faces. Horizontal projection of these planes is a rectangle and called "punching perimeter". Heights of the planes are equal to effective depth. In Fig. 5.14 punching perimeter is shown by dotted lines.

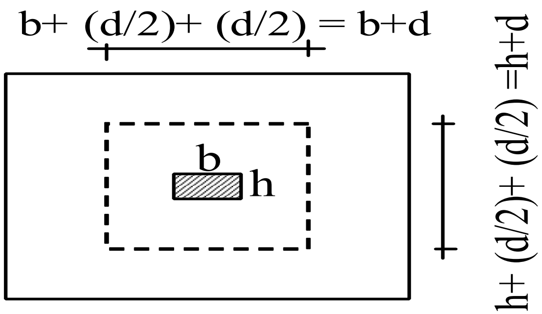


Figure 5.14

For rectangular columns with the dimensions a and b the total area of punching planes is $u_p d$ in which u_p (punching perimeter) = 2(h + b + 2d) and d is the effective length. Punching shear resistance is determined as,

$$V_{pr} = \gamma f_{ctd} u_p d \tag{5.1}$$

The value of the coefficient γ depends on the value of eccentricity. If the column load is concentric $\gamma = 1$. For eccentrically loaded column TS500 gives:

$$\gamma = \frac{1}{1 + \eta(e / W_m) u_p d} \tag{5.2}$$

where e is the eccentricity of the column load, W_m is the section modulus of the concrete area inside the punching perimeter.

$$\eta = 1/\sqrt{b_2 / b_1} \tag{5.3}$$

b_1 and b_2 are the side lengths of the perimeter. If the shape of the perimeter is irregular b_1 and b_2 are the side lengths of the smallest rectangle which takes the punching perimeter in. b_1 is the dimension in the direction of bending. Eqs.(5.2) and (5.3) are valid if $b_2 \geq 0.7 b_1$. For the internal columns γ can be computed approximately as follows:

$$\gamma = \frac{1}{1 + 1.5\dfrac{e_x + e_y}{\sqrt{b_1 b_2}}} \tag{5.4}$$

where e_x and e_y are the eccentricities in x and y directions. For circular columns:

$$\gamma = \frac{1}{1 + \dfrac{2e}{d + d_0}} \tag{5.5}$$

in which e is the eccentricity of the load and d_0 is the diameter of the column. Calculations of eccentricities should be based on 40% of the bending moment

Punching design force is the total upward soil pressure outside the punching perimeter and designated as V_{pd}. V_{pr} should be equal or greater than V_{pd}.

Punching perimeters for some columns which are not rectangular are illustrated in Fig. 5.15.

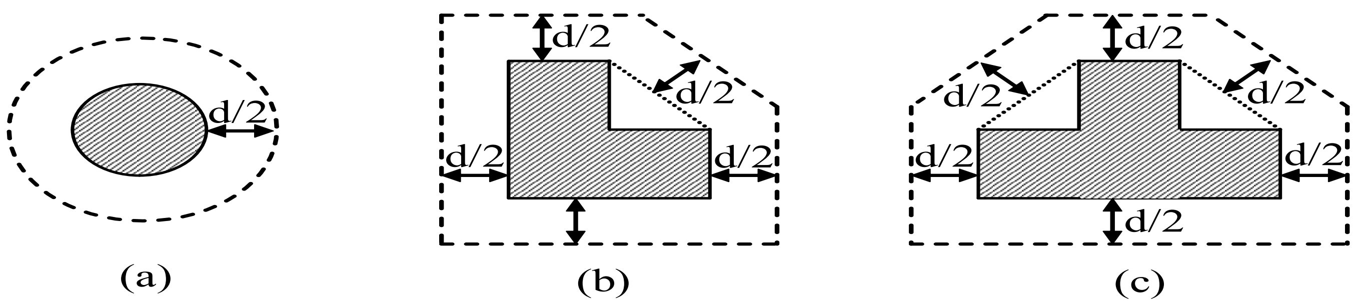


Figure 5.15

In eccentrically loaded footings soil pressure distribution is assumed linearly varying as shown in Fig.5.16. Stresses can be calculated by the elementary equations of strength of materials:

$$q_{max} = \frac{N}{AB} + \frac{M}{AB^2/6} \qquad q_{min} = \frac{N}{AB} - \frac{M}{AB^2/6} \tag{5.6}$$

q_{max} should be less than allowable soil stress and q_{min} should be positive. Tension stresses can not develop between the soil and the footing. Equation for q_{min} can be written as follows:

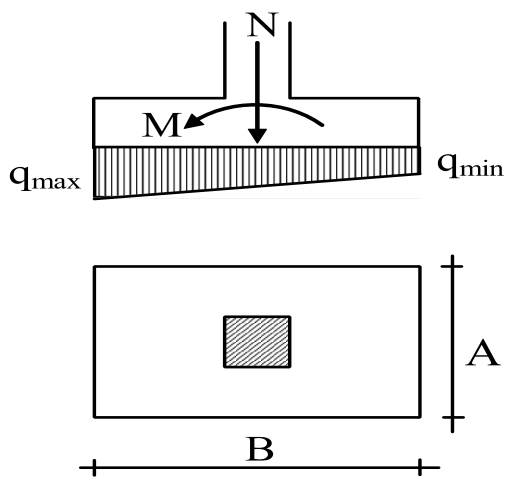


Figure 5.16

$$q_{min} = \frac{N}{AB}\left(1 - \frac{6e}{B}\right)$$

For positive values the term in parenthesis should be positive. If $B \geq 6e$, minimum soil pressure will not be negative. Alternatively, a triangular stress distribution may be assumed as shown in Fig. 5.17. Here it means that an opening between a small part of the footing and the soil is allowed.

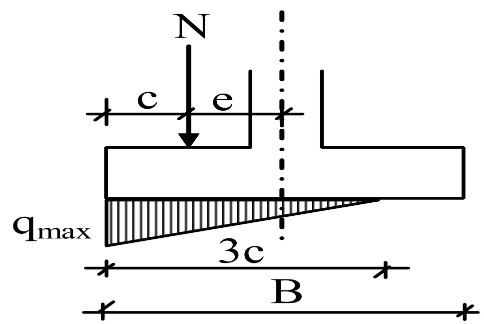


Figure 5.17

For the equilibrium conditions N should be acting on the line passing trough the centroid of the stress triangle. In other words the base of the triangle should be equal to 3c. The following equilibrium equation can be written if A is the width of the footing:

$$N = \frac{1}{2} q_{max} (3c) A \tag{5.7}$$

$$q_{max} = \frac{2N}{3cA} \tag{5.8}$$

where $c = B/2 - e = B/2 - M/N$
According to TS500 one side of the bearing area can not be less than 70 cm and minimum bearing area is 1 m^2. Minimum height of the footing is 25 cm. Clear concrete cover below the reinforcement should not be less than 5 cm. Minimum steel ratio in each direction is 0.002 and maximum spacing of bars is 25 cm.

Example 5.2

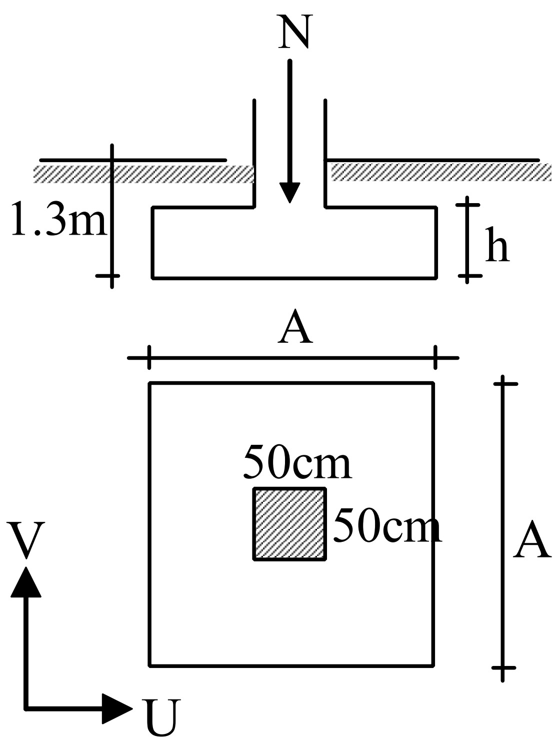


Figure 5.18

Dead Load: $N_d = 640$ kN
Live Load: $N_l = 450$ kN

$q_a = 200$ kN/m^2

Materials: C18 S220

Design the single column footing shown in Fig. 5.18

Solution:

Assuming an average unit weight for the material filled over the footing and the material of the footing (reinforced concrete), say 20 kN/m^3 the effective soil pressure can be calculated as,

$$q_{ef} = q_a - 1.3*20 = 200 - 26 = 174 \text{ kN/m}^2$$

Required minimum bearing area:

$$\min.A^2 = \frac{N_d + N_l}{q_{ef}} = \frac{640 + 450}{174} = 6.26 \text{ m}^2 \qquad \min.A = \sqrt{6.26} \approx 2.5 \text{ m}$$

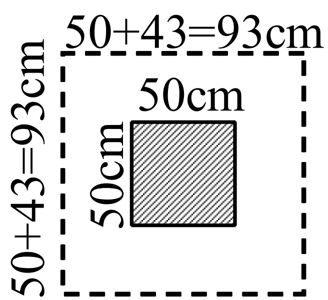
Selected A = 2.5 m

Design of the footing:
Design soil stress:

$$q_u = \frac{1.4*640 + 1.6*450}{2.5*2.5} = 258.6 \text{ kN/m}^2$$

A-Punching shear check:

Assume the height of the footing h = 50 cm. Assume an average d' = 7 cm $d_{av} = 50 - 7 = 43$ cm.



Punching perimeter: $u_p = 4*93 = 372$ cm

The footing area outside the punching perimeter:

$2.5*2.5 - 0.93*0.93 = 5.39\ m^2$

Punching design force: $V_{pd} = 5.39*258.6 = 1393.8$ kN

Punching resistance:

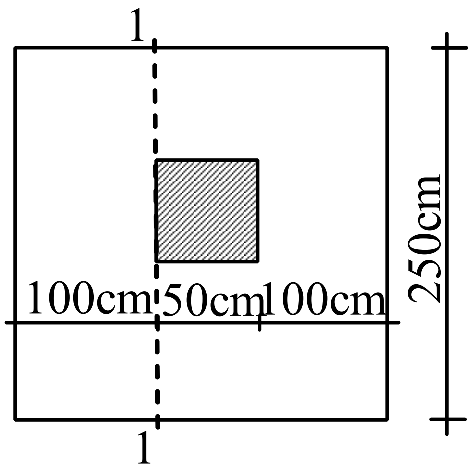
Since the load is concentric $\gamma = 1.00$:

$$V_{pr} = (1*3720*430) / 1000 = 1599.6 \text{ kN} > V_{pd}$$

It shows that selected height is sufficient for punching shear.

B-One-way shear check :

Since column load is concentric, column and the footing are square in plan and since average effective depth is used, one way shear check and bending moment calculations shall be done only in one direction.



$V_d = 1*2.5*258.6 = 646.6$ kN

$V_{cr} = (0.65*1*2500*430) / 1000 = 698.8$ kN
mm mm

$V_{cr} > V_d$ Dimensions are adequate for shear.

C-Bending resistance:

$M_d = 2.5*(258.6*1^2 / 2) = 323.25$ kN=m

$$R = \frac{M}{bd^2} = \frac{3232500}{250*43^2} = 6{,}99\,\text{kg/cm}^2 \qquad \rho = 0.0038 > \rho_{min} = 0.002$$

$A_s = 0.0038*250*43 = 40.85\ cm^2$ Selected: 13Ø20 ($40.83\ cm^2$)

Details are shown in Fig. 5.19.

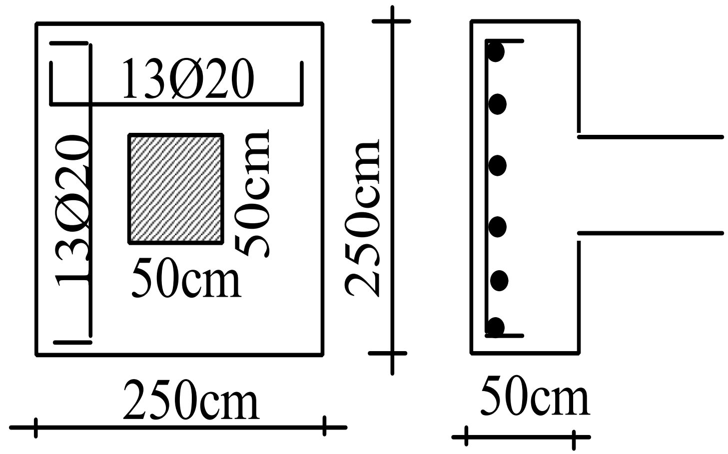


Figure 5.19

Check: Spacing of the bars assuming 5 cm concrete cover at each side:

$$s = (250 - 10) / 12 = 20 \text{ cm} < 25 \text{ cm} \qquad \text{OK.}$$

As it can be observed bars are bent upwards to increase the anchorage length.

Example 5.3

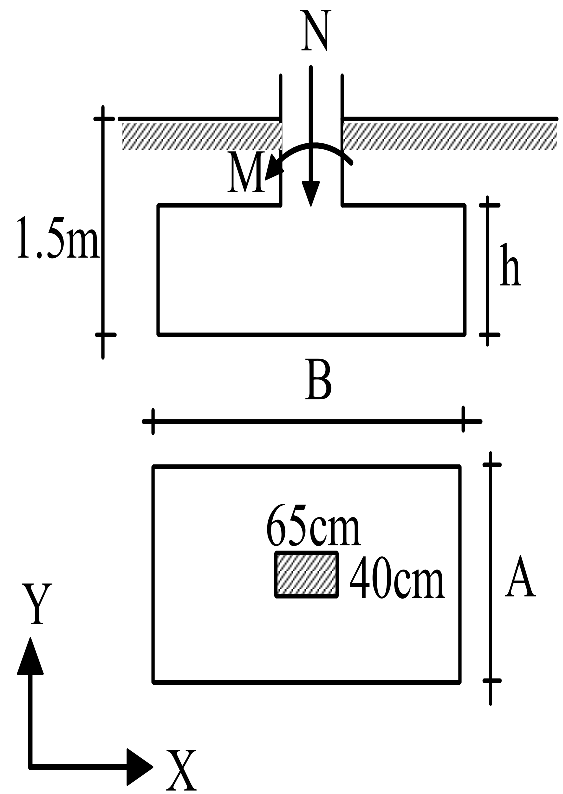


Figure 5.20

Dead load: $N_d = 245$ kN
$M_d = 128$ kN-m

Live load: $N_l = 150$ kN
$M_l = 68$ kN-m

$q_a = 210$ kN/m^2

Materials: C16 S220

Design this single column footing shown in Fig.5.20 (Unite weight of the earth fill above the footing is 18 kN/m^3)

Solution:

Service loads:

$$N = N_d + N_l = 245 + 150 = 395 \text{ kN} \quad , \quad M = M_d + M_l = 128 + 68 = 196 \text{kN-m}$$

$$e = M / N = 196 / 395 = 0.5 \text{ m}$$

There will not be tensile stresses if $B \geq 6e = 6*0.5 = 3.00$ m. Selected B = 3.20 m
Assume h = 50 cm

$$q_{ef} = 210 - 0.5*25 - (1.5 - 0.5)*18 = 179.5 \text{ kN/m}^2$$

min.A can be found by equating q_{max} (Eq.5.6) to q_{ef}:

$$\frac{N}{AB} + \frac{M}{AB^2 / 6} = q_{ef}$$

$$\frac{395}{3.2 * A} + \frac{6 * 196}{A * 3.2^2} = 179.5 \text{ kN/m}^2$$

$$A = \frac{395}{3.2 * 179.5} + \frac{6 * 196}{3.2^2 * 179.5} = 0.688 + 0.640 = 1.328 \text{ m}$$

Selected A = 1.35 m

Design:

Design soil pressures:

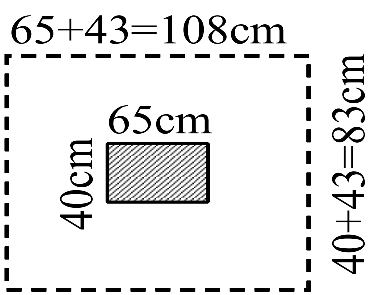
$$q_{u,max} = \frac{1.4*245+1.6*150}{1.35*3.2} + \frac{6*(1.4*128+1.6*68)}{1.35*3.2^2} = \frac{583}{4.32} + \frac{6*288}{13.824}$$

$$134.95 + 125 = 259.95 \text{ kN/m}^2 \approx 260 \text{ kN/m}^2$$

$$q_{u,min} = 134.95 - 125 = 9.95 \text{ kN/m}^2 \approx 10 \text{ kN/m}^2 > 0 \text{ OK}$$

A-Punching shear check:

Assuming $d'_{av} = 7$ cm $d = d_{av} = 50 - 7 = 43$ cm



Punching perimeter: $u_p = 2*(108 + 83) = 382$ cm

Footing area outside the punching perimeter:

$$1.35*3.2 - 1.08*0.83 = 3.42 \text{ m}^2$$

In punching force calculation average soil pressure shall be used since pressure distribution is not uniform.

$$q_{u,av} = (260 + 10) / 2 = 135 \text{ kN/m}^2$$

$$V_{pd} = 3.42*135 = 461.7 \text{ kN}$$

Resisting force:

$$\gamma = \frac{1}{1+1.5\frac{e_x + e_y}{\sqrt{b_x b_y}}} \qquad e_x = \frac{0.4M_y}{N} = \frac{0.4*288}{583} = 0.20\,\text{m} \qquad e_y = 0$$

$$\gamma = \frac{1}{1+1.5\frac{0.2}{\sqrt{1.08*0.83}}} = \frac{1}{1+1.5*0.21} = 0.76$$

V_{pr} = (0.76*0.9*3820*430) / 1000 = 1124 kN > 461.7 kN   OK.

B- One way shear check in X direction:

In this direction shear force should be calculated in section 1-1 as shown in Fig.5.21a and soil pressure at this section (σ_{u1}) should be calculated. Once σ_{u1} is calculated shear force is computed from the free body diagram drawn for the cantilever part at the left of the section 1-1 (Fig.5.21b).

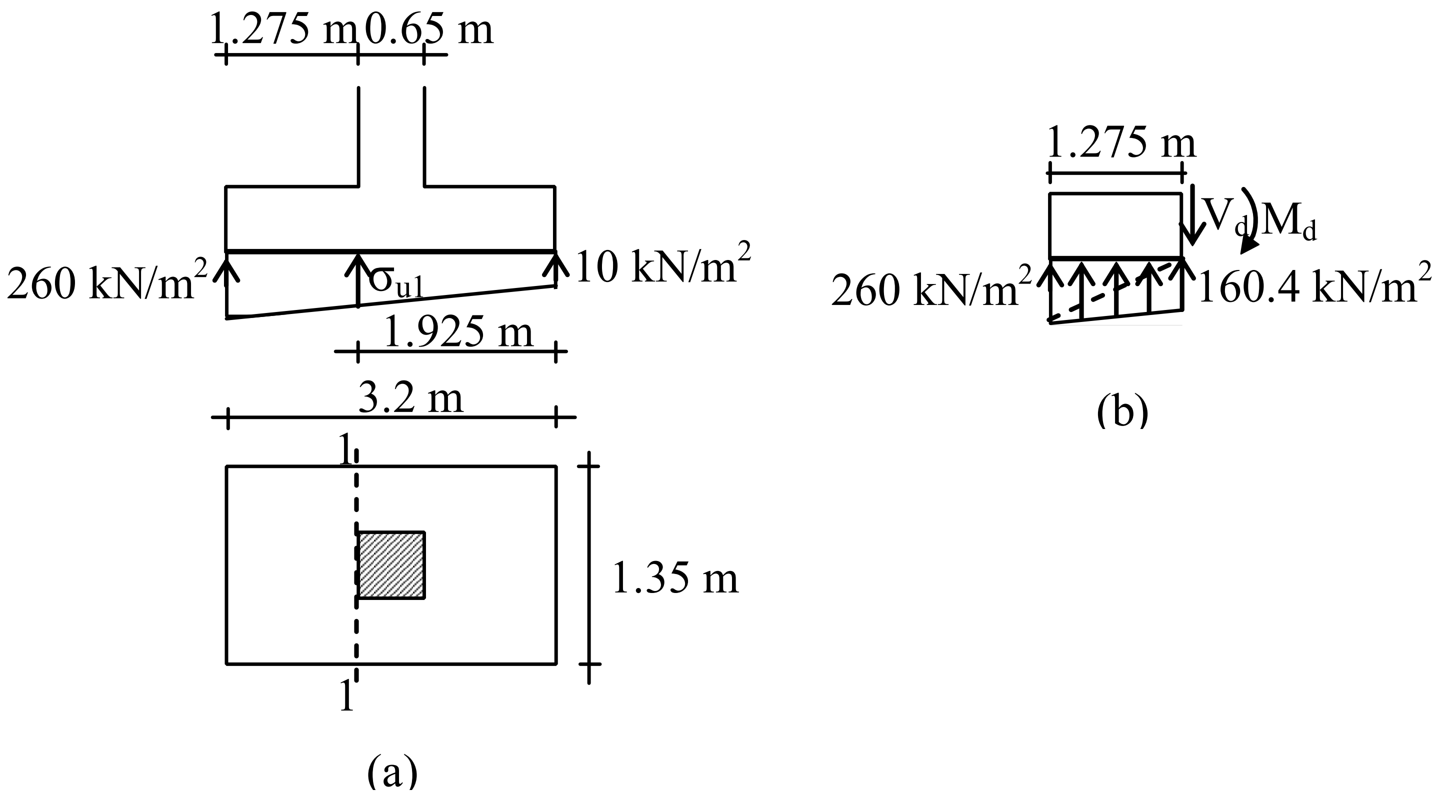


Figure 5.21

by using Similar triangles, the stress at section 1-1 can be calculated

$$\sigma_{u1} = 10 + \frac{260 - 10}{3.2} 1.925 = 160.4 \, \mathrm{kN/m^2}$$

$$V_d = 1.35 \frac{260 + 160.4}{2} 1.275 = 361.8 \text{ kN}$$

Using d_{av} :

$$V_{cr} = (0.65*0.9*1350*430) / 1000 = 339.6 \text{ kN} < 361.8 \text{ kN}$$

The thickness of the footing is not adequate. Let us increase h. Let h = 55 cm. Now $d_{av} = 55 - 7 = 48$ cm.

$$V_{cr} = 339.6*480/430 = 379.1 \text{ kN} > 361.8 \text{ kN} \quad \text{OK}$$

C- Bending in X direction:

$$M_d = 1.35\ (\frac{1}{2}*1{,}275*260*\frac{2}{3}*1{,}275+\frac{1}{2}*1{,}275*160{,}4*\frac{1}{3}*1{,}275\ )$$

$$= 1.35\ (\frac{260}{3}+\frac{160{,}4}{6})1{,}275^2 = 248{,}87 \text{ kN - m}$$

$$R = \frac{248{,}8*10^4}{135*48^2} = 8 \text{ Kg/cm}^2 \qquad \rho = 0.0044 > \rho_{min} = 0.002$$

$$A_s = 0.0044*135*48 = 28.51 \text{ cm}^2 \qquad \text{Selected: } 8Ø22\ (30.41 \text{ cm}^2)$$

Check: Spacing s = (135 − 10) / (8 − 1) = 18 cm < 25 cm OK.

D- One-way shear check in Y direction:

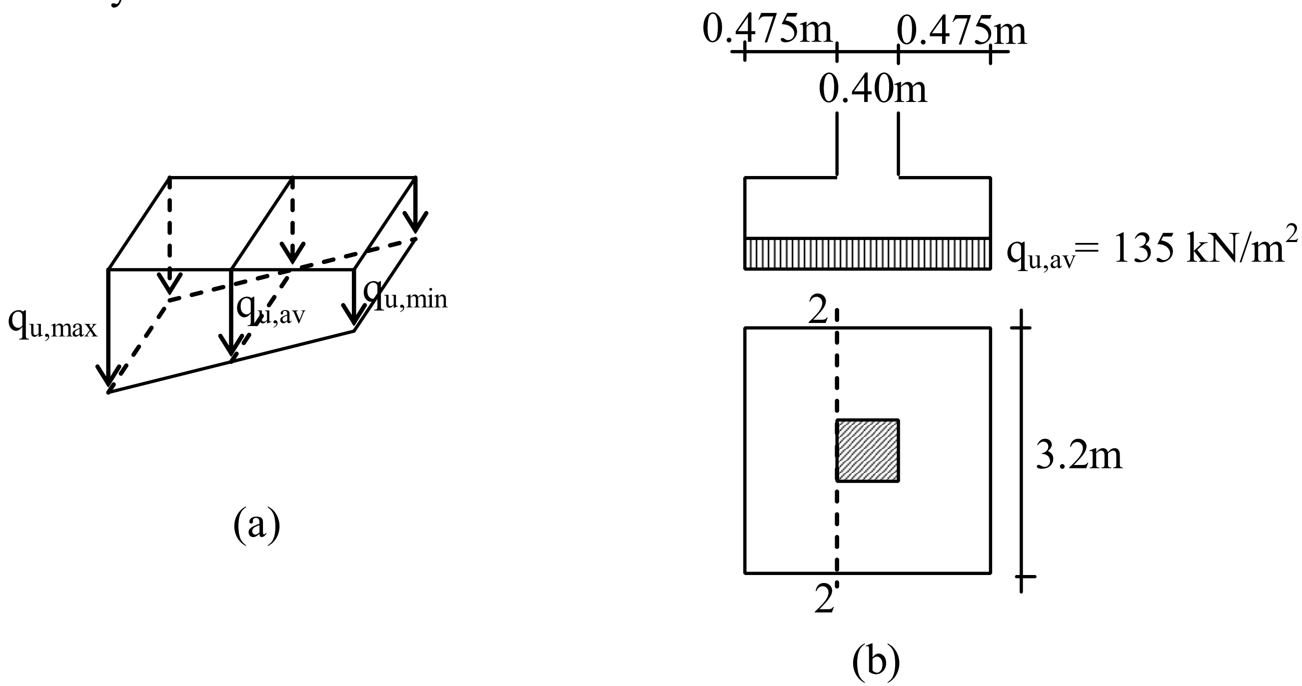


Figure 5.22

Figure 5.22a shows three dimensional distributions of soil pressures. It is easy to see that internal force in Y direction can be computed by using average soil pressure value. Such a calculation does not change the resultant force and the moment arm. Therefore internal forces in section 2-2 will be calculated by using average stresses as shown in Fig. 5.22b.

V_d = 3.2*0.475*135 = 202.5 kN

V_{cr} = (0.65*0.9*3200*480) / 1000 = 898.2 kN > V_d OK.

E-Bending in Y direction:

$$M_d = 3.2*135*0.475^2 / 2 = 48.74 \text{ kN-m}$$

$$R = \frac{487400}{320*48^2} = 0.66 \text{ Kg/cm}^2 \qquad \rho < \rho_{min} = 0.002$$

$$A_s = 0.002*320*48 = 30.72 \text{ cm}^2 \qquad \text{Selected } 16Ø16 \text{ (32.16 cm}^2\text{)}$$

Check: spacing s = (320 − 10) / (16 − 1) = 20.7 cm < 25 cm

Details are shown in Fig. 5.23.

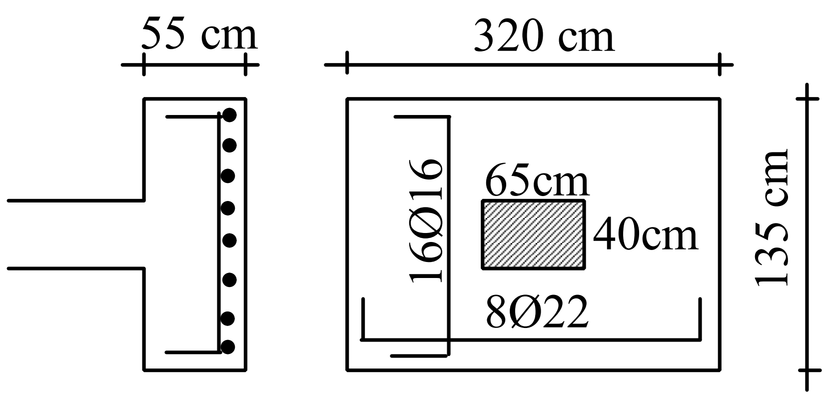


Figure 5.23

5.5 TIE BEAMS

In seismic zones footings should be tied in two directions by tie beams. They may be at level of footing or just above the footing as shown in Fig.5.24.

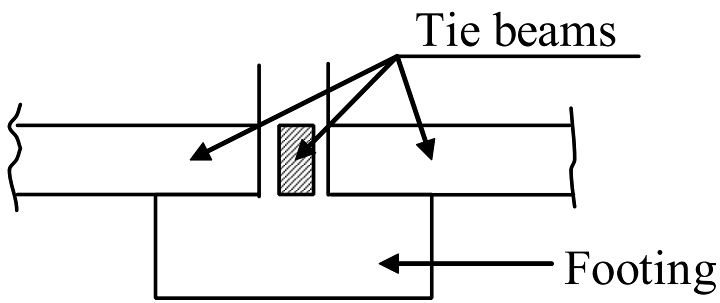


Figure 5.24

However if foundation is built on the rocks tie beams may not be needed. Tie beams improve the behavior of the structure when subjected to the horizontal loads. A tie beam should be capable of resisting a tensile force specified by earthquake regulations. This force which is a percentage of the column load depends on the soil type and the seismic zone. Besides, minimum area of the cross-section, dimension and reinforcement are also specified by the earthquake regulations. Thus enough rigidity is provided to the tie beams in order to reduce the differential settlement.