CIVL471 DESIGN OF RC STRUCTURES

LECTURE NOTE #4

CHAPTER IV

SELENDER COLUMNS

4.1 INTRODUCTION

A column is called slender if cross-sectional dimensions are relatively small compared to the height. In other words slenderness is not directly related to the length of the member. It is expressed in terms of L / r = length / radius of gyration. Radius of gyration is defined as $r = \sqrt{I / A}$ where I is the moment of inertia in bending direction and A is the area of the cross-section. In the previous chapter it was explained that short columns, that is, columns with small L / r values fail when materials reach to their strength values. On the other hand slender columns fail by buckling. Buckling is the sudden lateral deflection of an axially loaded member. Buckling loads are less than the crushing loads and much smaller if the member is eccentrically loaded or if there is an initial bending in the member for any other reason.

Initial bending increases the bending moments since it causes development of additional moments. These additional moments are called secondary (or second order) moments. Exact method for the calculation of second order moments is application of second order analysis to the frames. But some alternative methods which are approximate but very simple have been developed. One of them is "moment magnification factor" method which is proposed by TS500 as well. According to TS500 moment magnifier method can be used if slenderness ratio (kL / r) is less than 100. "kL" is known as the effective length of the column.

4.2 CONCENTRICALLY LOADED COLUMNS

Buckling (or critical) load at which a straigth member buckles is determined by the well-known Euler's formula:

$$N_{cr} = \frac{\pi^2 E_t I}{(kL)^2} \tag{4.1}$$

where E_t is the elasticity modulus of a completely elastic material (Young Modulus), I is the moment of inertia and kL is the effective length of the member. Corresponding strees is:

$$\sigma_{cr} = \frac{N_{cr}}{A} = \frac{\pi^2 E_t}{(kL/r)^2} = \frac{\pi^2 E_t}{\lambda^2} \tag{4.2}$$

λ is known as slenderness ratio. For a particular material, the relation between the critical stress and λ may be plotted as shown in Fig.4.1.

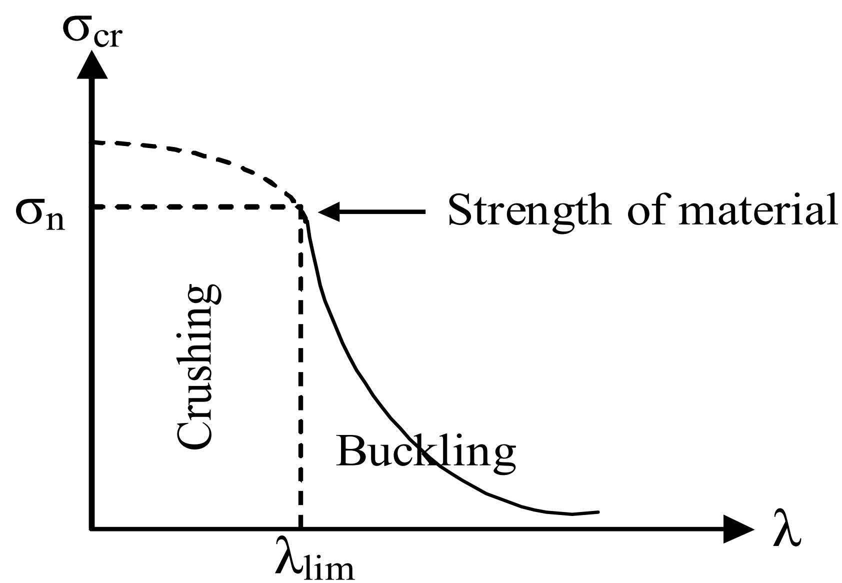


Figure 4.1

If λ_{lim} is the value of λ corresponding to the strength of the material, there can be two cases:

a) If $\lambda < \lambda_{lim}$ member fails by crushing, since stress reaches to strength value before reaching to critical stress.
b) If $\lambda > \lambda_{lim}$ member fails by buckling.

But in effect, this relation is not that simple in actual structures. First of all materials used in structures are neither completely elastic nor linear. Besides effective length of a column in a structure may only be determined approximately. It depends not only on the edge conditions but also on the relative lateral displacements of the frames. Lateral displacement of one end of a column relative to the other end is known as sidesway. Buckling shapes of the columns braced against the sidesway are illustrated in Fig.4.2.

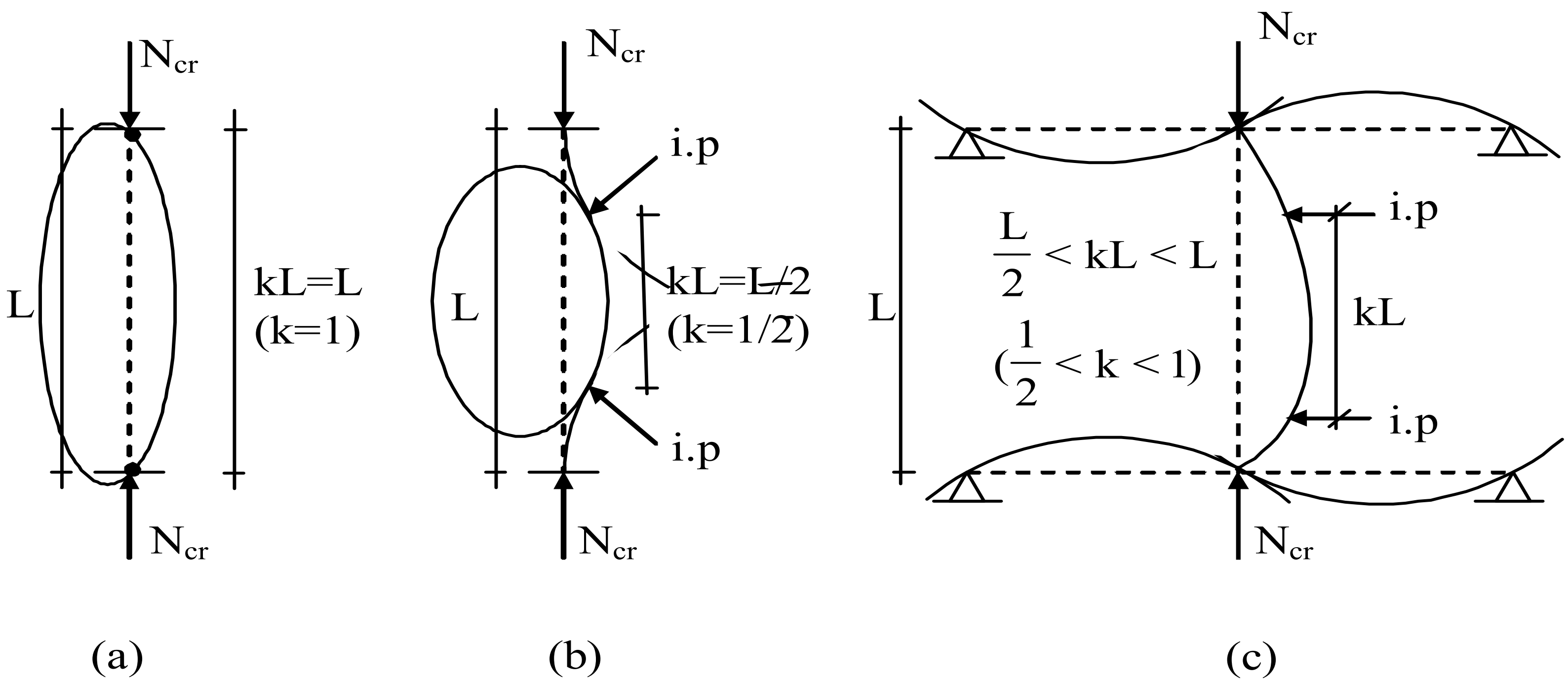


Figure 4.2

The column in Fig.4.2a is hinged at both ends. The effective length in this column is equal to the length of the column. That is, k =1. If both ends are fixed as shown in Fig.4.2b effective length is equal to the distance between the inflexion points (shown as i.p in the figure) which is L/2. That is k = 1/2. The column shown in Fig.4.2c represents a column in an actual structure. The ends of such a column is neither hinged nor fixed. The positions of inflexion points depend on the rigidities of the column relative to those of the beams. Effective buckling length varies between L and L/2. In other words k is always less than 1 or equal to 1.

Buckling shapes of the columns which are free to sidesway are shown in Fig.4.3.

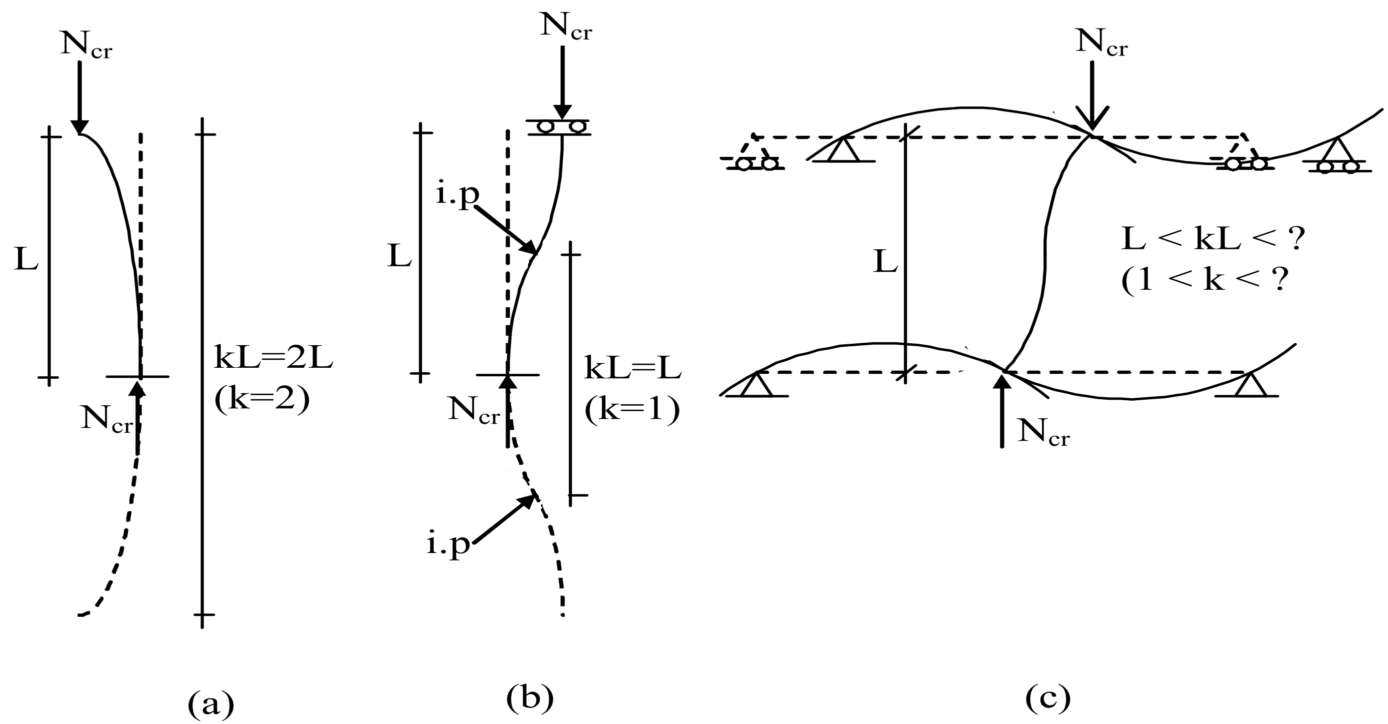


Figure 4.3

4.3 ECCENTRICALLY LOADED COLUMNS

It was already mentioned that columns are generally subjected to combined action of bending moment and axial load, and axial load increases the bending caused by bending moments. In Fig.4.4 two typical examples are shown. The column shown in Fig.4.4a is subjected to compression with equal eccentricities at the ends whereas the column shown in Fig.4.4c is subjected to

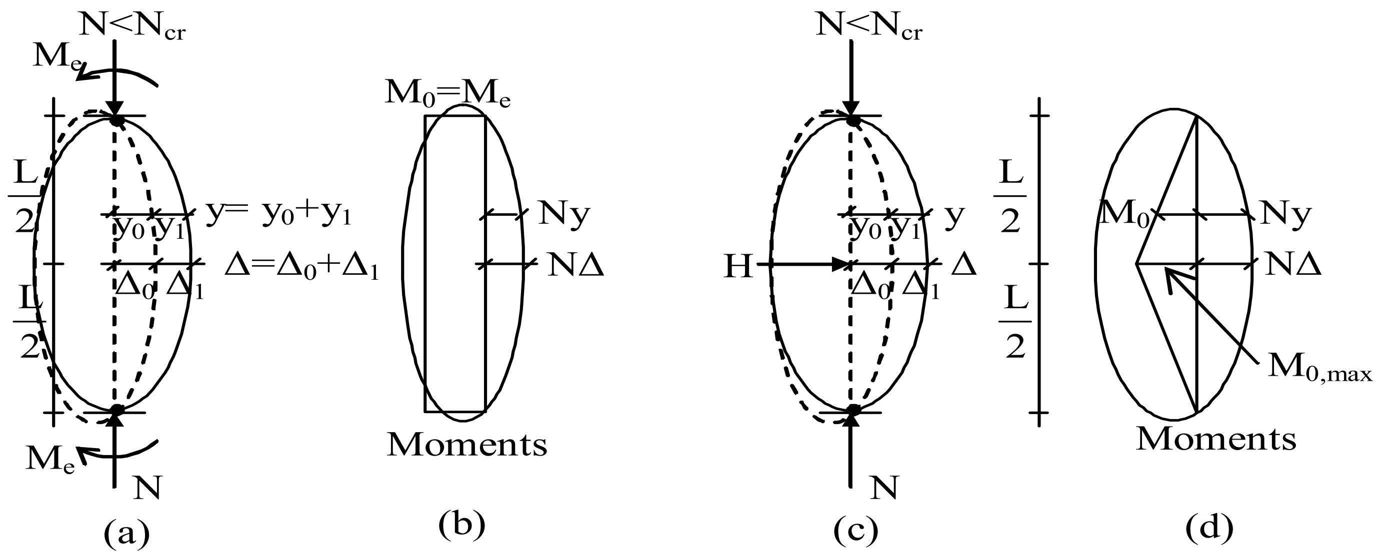


Figure 4.4

compression and a lateral load at the middle. If there were not axial loads members would bend as shown by dashed curves. These bending shapes are due to initial moments M_0. However, axial loads increase the moments as shown in Figs.4.4b and d and as a result the deflections. The solid curves are the bending shapes of the members under the simultaneous action of M_0 and Ny. Total moments in the columns are:

$$M = M_0 + Ny \tag{4.3}$$

This effect of the initial curvature on the moments is called NΔ effect. Timoshenko and Gere derived the following equation for deflection calculations of the members shown in Fig. 4.4.

$$y = y_0 \frac{1}{1 - N/N_{cr}} \tag{4.4}$$

It can be also shown that maximum moment may be expressed as:

$$M_{max} = M_0 \frac{1}{1 - N/N_{cr}} \tag{4.5}$$

in which 1 / (1 ? N/N_{cr}) is known as magnification factor. This factor magnifies the first order column moments and thus it becomes possible to take second order moments into account. Magnification factor given by TS500 is sligthly different. It is given as 1 / (1? 1.3N/N_{cr}).

In the examples shown in Fig.4.4 loads produce single curvature bendings and maximum moments and maximum deflections coinside at the same locations. Single curvature bending corresponds to a curve having no inflexion points between the two ends. If end moments of the column shown in Fig.4.4a were not equal or if lateral loading in the column shown in Fig.4.4c were not symmetrical, the curvature would still be single. However, NΔ effect would be smaller though still significant. On the other hand if end moments of a column produce bending in opposite directions this column will bend in double curvature as shown in Fig.4.5.

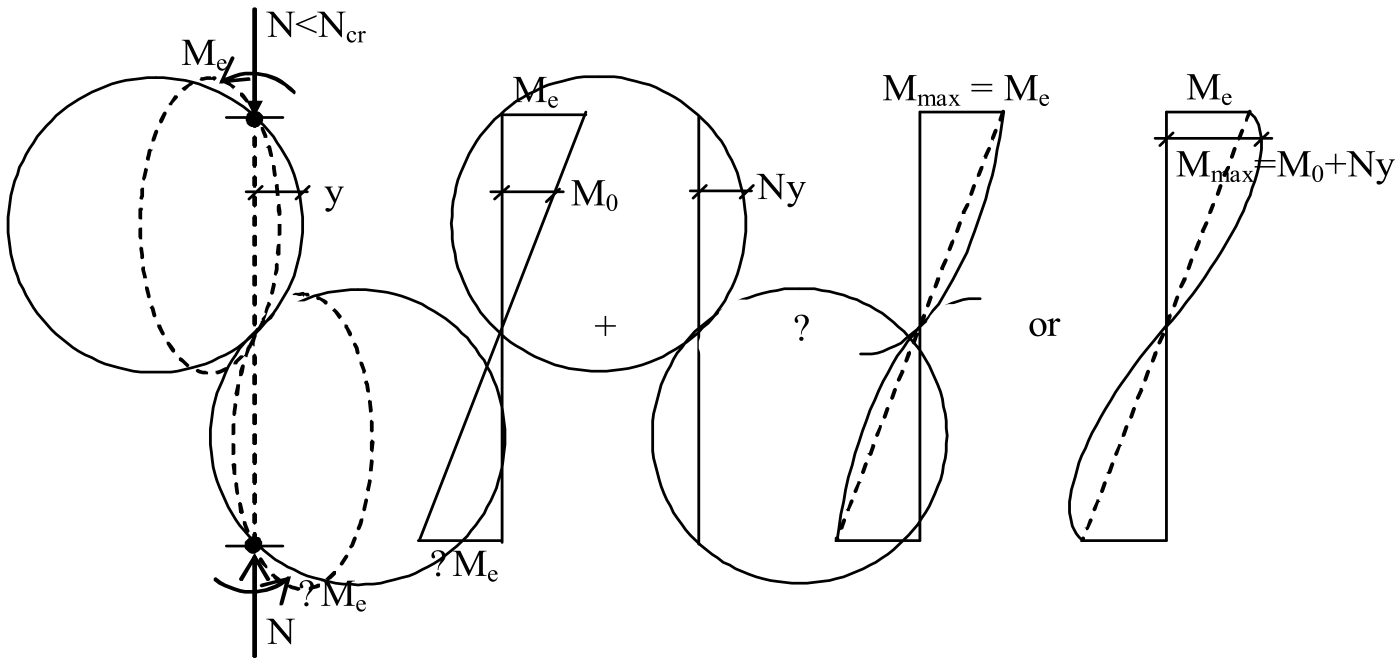


Figure 4.5

In this case deflections either initial or eventual can not be as large as those in the single curvature cases. Deflections here increase also due to axial load but NΔ effect is not too high. As shown in the figure maximum moment is either equal to M_e, that is, no magnification is present, or M_{max} is slightly larger than M_e. For this case deflection equation was expressed by Timoshenko and Gere as:

$$y = y_0 \frac{1}{1 - N/4N_{cr}} \tag{4.6}$$

which also shows that the magnification is smaller in this case. Therefore Eq. (4.5) needs modification for covering all cases. This is done by the introduction of a coefficient c_m into Eq. (4.5). For example TS500 gives the following equation for the calculation of M_{max}:

$$M_{max} = M_0 \frac{c_m}{1 - 1.3N/N_{cr}} \ ? \ M_0 \tag{4.7}$$

where c_m depends on the bracing conditions of the building. For braced columns:

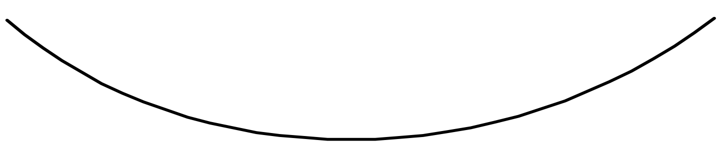
$$c_m = (0.6 + 0.4\frac{M_1}{M_2}) \ ? \ 0.4 \tag{4.8}$$

For unbraced columns:

$$c_m = 1 \tag{4.9}$$

In Eq. (4.8) M_1 is the numerically smaller end moment, M_2 is the numerically larger end moment. Thus by definition $M_0 = M_2$. All these moments are factored end moments. M_1 / M_2 is positive if they produce single curvature, otherwise negative.

Columns in a structure are members of a three dimensional system. They are connected to each other by beams or slabs which are very rigid as far as horizontal motions are concerned. Therefore sidesway of a column is not possible if structure is properly braced. On the other hand if a structure is unbraced, the structure and the columns deflect laterally all together. It shows that the columns of an unbraced building should not be treated individually. A single magnification factor should be determined for all columns of a certain storey. Sidesway of a building can be prevented by using bracing elements. Structural walls (shear walls), rigid service cores, cross-bracings in vertical planes are commonly used bracing elements.



Figs.4.6 and 4.7 show the significant difference between the behaviors of braced and unbraced columns.

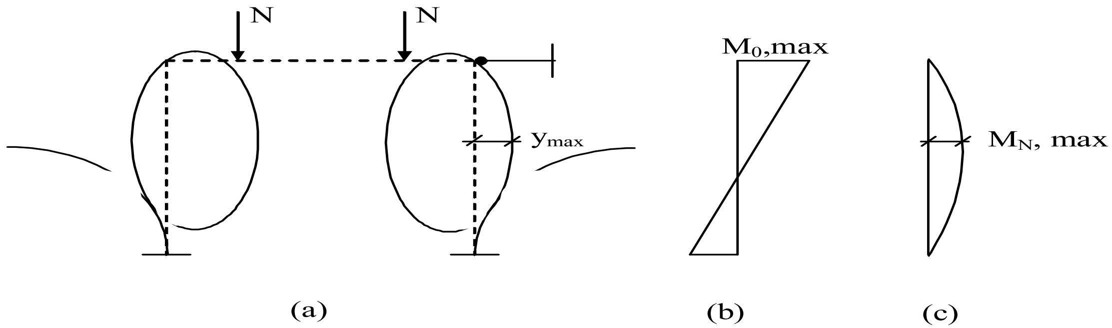


Figure 4.6 Braced Frame

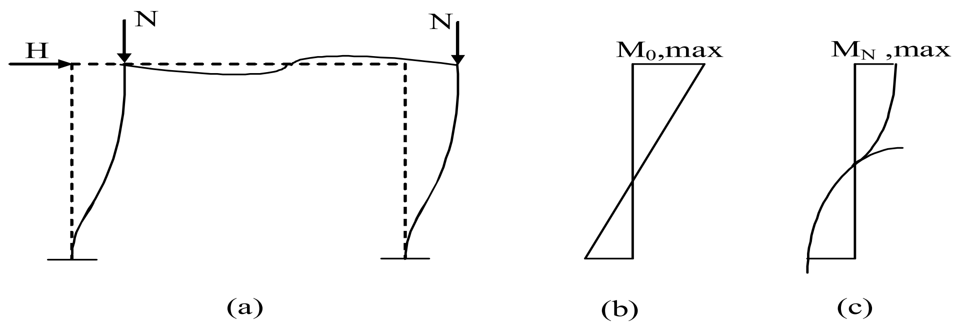


Figure 4.7 Unbraced Frame

In Figs.4.6b and 4.7b first order column moments, in Figs.4.6c and 4.7c second order column moments are shown (In Fig.4.6c second order moments at the ends of the column are ignored). It is clear from the figure that first order maximum moment ($M_{0,max}$) is not increasing in braced column after the superposition of second order moments. There may be moments slightly greater than $M_{o,max}$ in the final moment diagram but even in that case moment magnification is not significant. On the other hand, in unbraced frames maximum values of both first order and second order moments occur at the ends of the columns and resulting increase in the end moments can be rather large.