CIVL471 DESIGN OF RC STRUCTURES

LECTURE NOTE #5

CHAPTER IV

SELENDER COLUMNS (CONT.)

Criteria for Bracing and Slenderness

- Criterion for Bracing:

According to TS500 if the following condition is satisfied at the level i , the columns at this storey (over level i) can be assumed as braced:

$$\varphi = 1.5\Delta_i \frac{\Sigma(N_{di} / L_i)}{V_{fi}} \leq 0.05 \qquad (4.10a)$$

in which V_{fi} is total shear force at level i, Δ_i is relative horizontal displacement over level i (between two stories), N_{di} is design axial load of each column at level i and L_i is the length of this column. If the column lengths are same in the storey, Eq. (4.10a) becomes:

$$\varphi = 1.5 \frac{\Sigma(N_{di})\Delta_i}{V_{fi} L_i} \leq 0.05 \qquad (4.10b)$$

In these calculations it should be assumed that members are not cracked.

- Criterion for slenderness

TS500 gives the following equations for the definition of slenderness:

$$\lambda_{lim} = 34 - 12\frac{M_1}{M_2} \leq 40 \quad \text{(For braced columns)} \qquad (4.11)$$

$$\lambda_{lim} = 22 \quad \text{(For unbraced columns)} \qquad (4.12)$$

As defined earlier M_1 is the numerically smaller moment whereas M_2 is the numerically larger moment. M_1 / M_2 should be taken as (-) in double curvature bendings and 1 if $M_1 = M_2 = 0$. If slenderness ratio (λ) of a column is greater than λ_{lim} this column is designed as a slender column, otherwise as a short column. λ is calculated by using Eq. (4.13)

$$\lambda = \frac{kL_u}{r}$$

K is the effective length factor for the column
L_u is the unsupported length of the column
r is the radius of gyration
r can be calculated as ***0.3h*** for rectangular columns and ***0.25h*** for circular columns

For the braced columns k varies between 0.5 and 1. Therefore using k=1 in above one will be on the safe side.
For unbraced columns k varies from 1 to infinity.

One can use the following procedure to calculate k values:

a) For braced columns:

$$k = 0.7 + 0.05\,(\alpha_1 + \alpha_2) \le (0.85 + 0.05\alpha_1) \le 1 \qquad (4.15)$$

(For fixed ends $\alpha = 0$ should be used).

b) For unbraced columns:

If $\alpha_m < 2$
$$k = (20 - \alpha_m)\,\frac{\sqrt{1+\alpha_m}}{20} \qquad (4.16a)$$

If $\alpha_m \ge 2$
$$k = 0.9\sqrt{1+\alpha_m} \qquad (4.16b)$$

If one end is hinged: $k = 2 + 0.3\alpha_2$ **(4.17)**

where α_2 is the α value calculated at the other end. For the fixed end $\alpha = 0$.

The degree of end restraints at any end may be expressed as:

$$\alpha = \frac{\Sigma(EI / L \text{ of columns})}{\Sigma(EI / L \text{ of beams})} \tag{4.14}$$

In this equation columns and beams (only in the plane of bending) joining at that end should be included. If the same concrete is used for columns and beams E can be canceled in Eq. (4.14). For the calculation of I values of the columns gross concrete section should be used. For the beams the influence of cracking should be considered. Therefore half of the I of the gross section of T beams should be taken. I value of web part can be used as an approximate value. In beamless floors half of the I value of the slab strip used in frame analysis should be considered. Denoting smaller one of the two restraining factors as α_1, the greater one as α_2 and $(\alpha_1 + \alpha_2) / 2 = \alpha_m$

Critical Load N_{cr}

Critical load can be calculated by the following equation:

$$N_{cr} = \frac{\pi^2 EI}{(kL_u)^2} \qquad \textbf{(4.18)}$$

Definitions and equations for the calculations of k and L_u were given above. For the calculation of EI, Eqs. (4.19) and (4.20) may be used.

$$EI = \frac{0.4E_cI_c}{(1+R_m)} \qquad \textbf{(4.19)}$$

If refined calculation is necessary the following relation for ***EI*** can be used

$$EI = \frac{0.2E_cI_c + E_sI_{se}}{(1+R_m)} \qquad \textbf{(4.20)}$$

Where E_c = modulus of elasticity of concrete
I_c = moment of inertia of gross section of column
E_s = modulus of elasticity of steel = $2*10^5$ MPa
I_{se} = moment of inertia of steel about the centroidal axis of column section
$(1 + R_m)$ = reduction factor for creep

Eq. (4.19) is sufficiently accurate for the low steel ratios. If steel ratio is high Eq. (4.20) should be used. However steel ratio is not known before the design is completed. Therefore if refined calculation is not necessary the simpler Eq. (4.19) may be used. Refinement is essential for the cases where $\lambda \geq 60$. In refined calculations I values of transformed sections of the beams should be used. In TS500 the following equations are given for creep coefficient R_m:

For braced columns:

$$R_m = \frac{\text{design axial dead load}}{\text{total design axial load}} \qquad \textbf{(4.21)}$$

For unbraced columns:

$$R_m = \frac{\text{total shear forces in the storey due to sustained loads}}{\text{total design shear forces in the storey}} \qquad \textbf{(4.22)}$$

TS500 requires an explanation for Eq. (4.22) since it normally results in zero.

Moment Magnifier Method for Braced Columns

In a braced column if $\lambda \le \lambda_{lim} = 34 - 12\frac{M_1}{M_2} \le 40$, it is designed as a short column. If $M_1 = M_2 = 0$ in a column $M_1 / M_2 = 1$ should be used in the equation. However, TS500 does not allow the design of concentrically loaded columns. If $M_1 = M_2 = 0$ or eccentricity of the column load is less than e_{min}, M_2 should be adjusted as:

$$M_2 = e_{min} N \tag{4.23}$$

$$e_{min} = 15\ \text{mm} + 0.03h \tag{4.24}$$

in which h is the dimension of the column in the direction of bending. But in small eccentricity cases actual values of M_1 and M_2 should be used for the calculations of λ_{lim}.

Since column dimensions should be selected for structural analysis, they will already be known before the design process. However it is better to check their suitability in an earlier phase of the design. Therefore design of a braced column may be done in the following order:

1- Design the column as a short column after checking eccentricity against e_{min} and adjusting M_2 if necessary. Determine steel ratio. If dimensions are adequate continue.

2- Assuming k = 1 as a safe value calculate λ from Eq. (4.13) and compare with λ_{lim}. If $\lambda \leq \lambda_{lim}$, this column is a short column. Complete the design by calculating and selecting the reinforcement. Otherwise,

3- Refine the calculations. Find the rotational restraint coefficients from Eq. (4.14) and calculate k from Eq. (4.15). Recheck against λ_{lim}. If $\lambda \leq \lambda_{lim}$ complete the design by calculating and selecting the reinforcement. Otherwise,

4- Calculate c_m from Eq. (4.8). If $M_1 = M_2 = 0$, use $M_1 / M_2 = 1$ in the equation. If moments are not zero but $e < e_{min}$ use the real moment values in Eq. (4.8).

4- Calculate EI from Eqs. (4.19) or (4.20) and N_{cr} from Eq. (4.18).

5- Determine β:

$$\beta = \frac{c_m}{1 - 1.3N/N_{cr}} \geq 1 \qquad (4.25)$$

6- Design the column for the pair N and $M_{max} = \beta M_2$

<u>Example 4.1</u>

Design column C102 of the frame shown in Fig.4.8 for vertical load combination. This frame is effectively braced against sidesway by the shear walls suitably located in the building (The beams are symmetrical T beams).

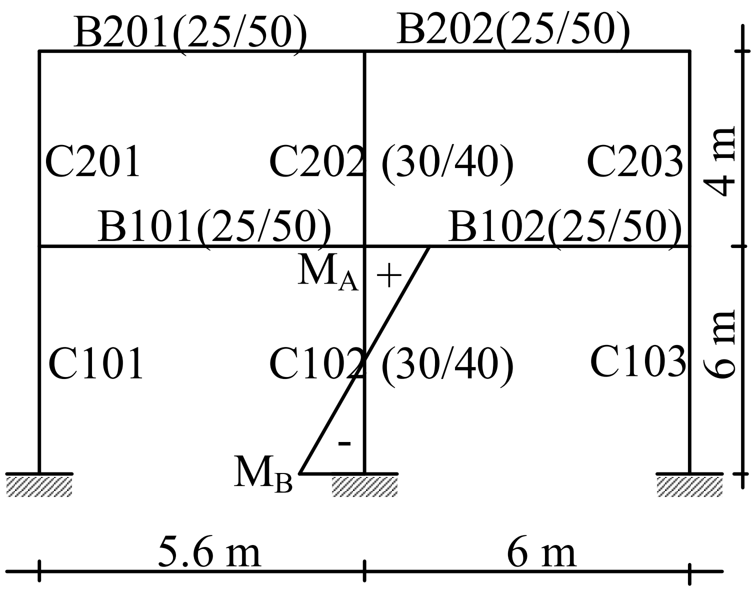


Figure 4.8

Colmn C102	Dead Load	Live Load
N(kN)	700*	500*
M_A(kN-m)	+40	+35
M_B(kN-m)	-18	-16

* Compression

Materials: C25 S420

Solution:

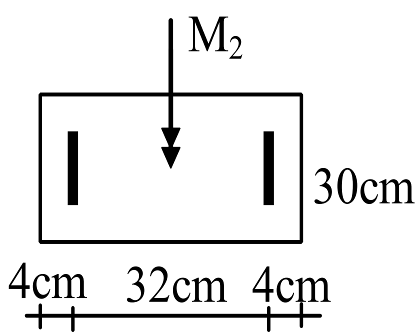
Design loads:

$N = 1.4*700 + 1.6*500 = 980 + 800 = 1780$ kN
$M_A = 1.4*40 + 1.6*35 = 56 + 56 = 112$ kN-m
$M_B = -(1.4*18 + 1.6*16) = -(25.2 + 25.6) = -50.8$ kN-m

$M_2 = M_A = 112$ kN-m $\qquad e = \dfrac{M_2}{N} = \dfrac{112}{1780} = 0.06 \text{ m} = 60 \text{ mm}$

$e_{min} = 15 + 0.03*400 = 15 + 12 = 27 \text{ mm} < 60 \text{ mm}$

Initial design as a short column:



Let us place the reinforcement at the sides parallel to moment vector and assume that d ' = 4 cm
d" / h = 32 / 40 = 0.8. There are no middle bars. Therefore Chart A-3 should be used.

$$\frac{N}{bhf_{cd}} = \frac{1780*10^3}{300*400*17} = 0.87$$

$$\frac{M_2}{bh^2 f_{cd}} = \frac{112*10^6}{300*400^2*17} = 0.14$$

From the chart $\rho_t m = 0.40$ can be read. $m = \frac{f_{yd}}{f_{cd}} = \frac{365}{17} = 21.5$ $\rho_t = \frac{0.4}{21.5} = 0.019$

$\rho_{min} = 0.01 < 0.019 < \rho_{max} = 0.04$ Dimensions are suitable at the moment.

Initial check for slenderness: Assume $k = 1$

Since the hight of the beam is 50 cm, $L_u = 600 - 50 = 550$ cm.

$r = 0.3h = 0.3*40 = 12$ cm $\qquad \lambda = \dfrac{1*550}{12} = 45.8$

Since column bends in double curvature:

$$\lambda_{lim} = 34 - 12\frac{-50.8}{112} = 34 + 5.4 = 39.4 < 40 < \lambda$$

Check the slenderness with the refined k:

I values of columns: $I_c = \dfrac{0.3*0.4^3}{12} = 0.0016\,m^4$

C202: $\dfrac{I_c}{L} = \dfrac{0.0016}{4} = 40*10^{-5}\,m^3$ C102: $\dfrac{I_c}{L} = \dfrac{0.0016}{6} = 27*10^{-5}$

I values of cracked beams: $I_{crack} = \dfrac{0.25*0.5^3}{12} = 0.0026\,m^4$

B101: $\dfrac{I_{crack}}{L} = \dfrac{0.0026}{5.6} = 47*10^{-5}\,m^3$ B102: $\dfrac{I_{crack}}{L} = \dfrac{0.0026}{6} = 43*10^{-5}$

$\alpha_A = \dfrac{(40+27)*10^{-5}}{(47+43)*10^{-5}} = \dfrac{67}{90} = 0.74 = \alpha_2$ $\alpha_B = \alpha_1 = 0$ (fixed end)

$k = 0.7 + 0.05\,(0 + 0.74) = 0.74$ $[< (0.85 + 0.05*0) = 0.85 < 1]$

$\lambda = \dfrac{0.74*550}{12} = 33.92 < \lambda_{lim} = 39.4$ That is C102 is a short column.

Reinforcement:

$A_{st} = \rho_t bh = 0.019*30*40 = 22.8\ cm^2$ Selected: 6Ø22 ($22.81\ cm^2$)

Tie bars: Ø8 Check 8 mm > 22/3 = 7.3 mm OK.

Spacing: $s_{max} = 12Ø = 12*2.2 = 26.4$ cm > 20 cm Select s = 20 cm

Details are shown in Fig.4.9.

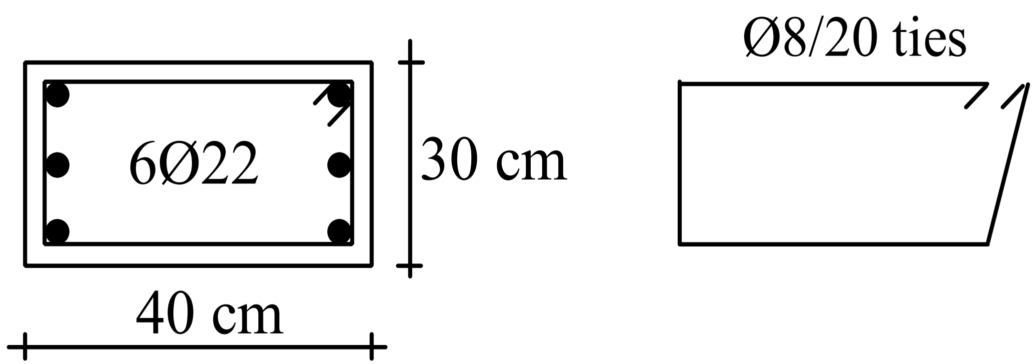


Figure 4.9

Example 4.2

Design the column C102 in Example 4.1 if the loads are as follows:

Colmn C102	Dead Load	Live Load
N(kN)	700*	500*
M_A(kN-m)	+30	+22
M_B(kN-m)	+30	+22

* Compression

Moments are producing single curvature since they are both (+) ve (clasicle sign convention). When these signs are used, correct signs are obtained for M_1 / M_2.

Solution:

Design loads:

$N = 1780$ kN (calculated in Example 1)

$M_A = 1.4*30 + 1.6*22 = +77.2$ kN-m

$M_B = +77.2$ kN-m

$\Big\} \; M_2 = 77.2$ kN-m

$$e = \frac{77.2}{1780} = 0.043\,\text{m} = 43\text{ mm} > e_{min} = 27\text{ mm}$$

Initial design as a short column:

$$\frac{N}{bhf_{cd}} = 0.87 \text{ (calculated in Example 1)}$$

$$\frac{M_2}{bh^2 f_{cd}} = \frac{77.2*10^6}{300*400^2*17} = 0.09$$

If reinforcement is placed as decided in Example 1, Chart A-3 should be used and $\rho_t m = 0.27$ can be found from the chart. Since $m = 21.5$ $\rho_t = 0.27 / 21.5 = 0.0126$

$\rho_{min} = 0.01 < 0.0126 < \rho_{max}$ OK.

Initial check for slenderness:

$$\lambda_{lim} = 34 - 12\frac{77.2}{77.2} = 22 < \lambda = 45.8 \text{ (calculated in Example 1)}$$

Check against the refined value of $\lambda = 33.9$:

$\lambda > \lambda_{lim}$ Therefore slenderness effect will be considered.

Design of the column as a slender column:

$$C_m = 0.6 + 0.4\frac{77.2}{77.2} = 1 > 0.4 \quad \text{OK.}$$

$$R_m = \frac{1.4N_d}{N} = \frac{1.4*700}{1780} = \frac{980}{1780} = 0.55$$

For the calculation of E_c equation given by TS500 will be used:

$$E_c = 3250\sqrt{f_{ck}} + 14000 = 3250\sqrt{25} + 14000 = 30250\,N/mm^2$$
$$= 30250*10^3\ kN/m^2$$

$$I_c = 16*10^{-4}\ m^4 \text{ (calculated in Example 1)}$$

$$EI = \frac{0.4*30250*10^3*16*10^{-4}}{(1+0.55)} = 12490\,kN\text{-}m^2$$

$$N_{cr} = \frac{\pi^2 EI}{(kL_u)^2} = \frac{\pi^2 12490}{(0.74*5.5)^2} = 7442\,kN$$

Magnification factor β:

$$\beta = \frac{c_m}{1\text{-}1.3N/N_{cr}} = \frac{1}{1\text{-}1.3\dfrac{1780}{7442}} = \frac{1}{0.689} = 1.45$$

$$M_{max} = \beta M_2 = 1.45*77.2 = 112 \text{ kN-m}$$

Design:

$$\frac{N}{bhf_{cd}} = 0.87 \qquad \frac{M_{max}}{bh^2 f_{cd}} = \frac{112*10^6}{300*400^2*17} = 0.14$$

Using Chart A-3, $\rho_t m$ = 0.40 can be found. Since m = 21.5 ρ_t = 0.40 / 21.5 = 0.019.

ρ_{min} =0.01 < 0.019 < ρ_{max} =0.04 OK.

Reinforcement:

A_{st} = 0.019*30*40 = 22.8 cm^2 Selected: 6Ø22 (22.81 cm^2)

Tie bars: Ø8   Check: 8 mm > 22 / 3 = 7.3 mm   OK.

Spacing: s_{max} = 12*2.2 =26.4 cm > 20 cm   Selected: s = 20 cm

Details are shown in Fig. 4.10.

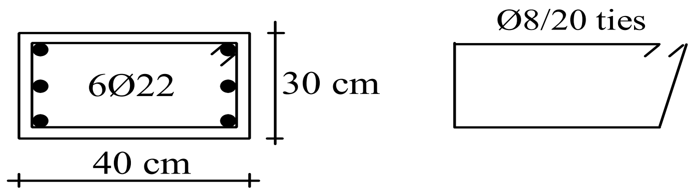


Figure 4.10