# CIVL471 DESIGN OF RC STRUCTURES

LECTURE NOTE #6

CHAPTER IV

SELENDER COLUMNS (CONT.)

## 4.7 MOMENT MAGNIFIER METHOD FOR UNBRACED COLUMNS

For unbraced columns $\lambda_{lim} = 22$. Therefore if $\lambda \leq \lambda_{lim}$ in an unbraced column it is designed as a short column. On the other hand slenderness effect should be considered. That is, the design moment ($M_2$) should be magnified by a factor $\beta$.

As discussed earlier columns of an unbraced structure should not be treated independently. All columns of a storey move laterally at the same time. In other words they are in identical conditions during the sway displacements and slenderness effect related to sidesway is common for all columns of a storey. Therefore TS500 gives the following common magnification factor for all columns of a storey.

$$\beta_s = \frac{c_m}{1 - 1.3\dfrac{\Sigma N}{\Sigma N_{cr}}} = \frac{1}{1 - 1.3\dfrac{\Sigma N}{\Sigma N_{cr}}} \qquad (4.26)$$

where $\Sigma N$ is the sum of the axial design loads and $\Sigma N_{cr}$ is the sum of the critical loads of the columns of the storey. According to TS500 $\Sigma N$ should be equal or less than $0.45\Sigma N_{cr}$. Otherwise dimensions of the column should be increased.

Although all columns of a storey are assumed under the same sway condition there is still the possibility of failure of an individual column. Suppose that a column is overloaded or less stiff than the others. It may buckle without any important relative lateral displacement. In such a case this column behaves like a member of braced frame. Therefore TS500 requires the calculation of a second $\beta$ as if the column is a braced column but $c_m = 1$. Greater of $\beta_s$ and $\beta$ should be used in the design. However if ,

$$\frac{L_u}{r} > \frac{35}{\sqrt{\dfrac{N}{f_{ck} A_c}}} \tag{4.27}$$

$$M_{max} = \beta \beta_s M_2 \tag{4.28}$$

should be used in design.

## Example 4.3 Unbraced Frame

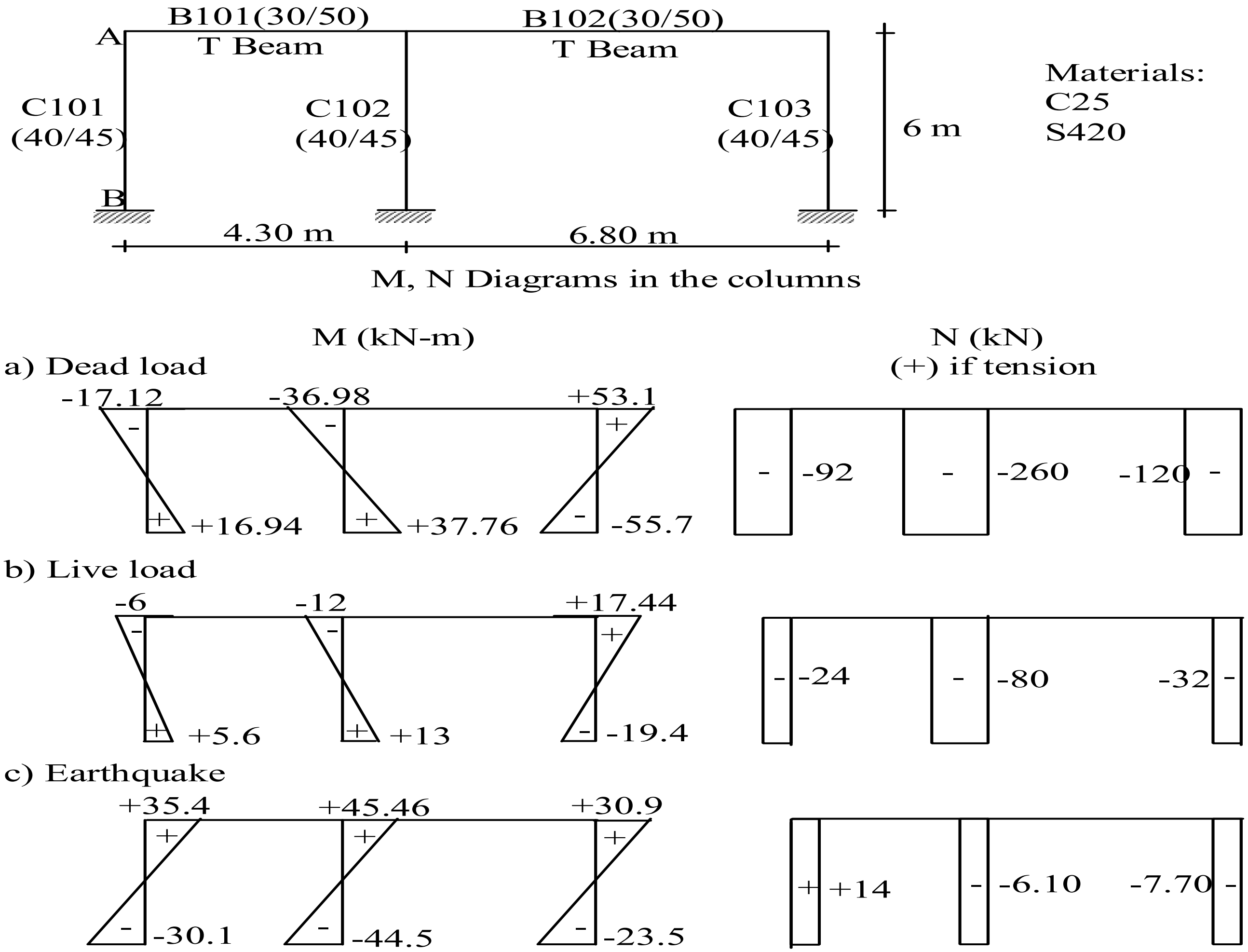



Figure 4.11

Design unbraced column C101 for the following load combinations:

V = 1.4 Dead load + 1.6 Live load
E1 = Dead load + Live load + Earthquake Load
E2 = Dead load + Live load — Earthquake load
E3 = 0.9 Dead load + Earthquake load
E4 = 0.9 Dead load — Earthquake load

Solution:

Design loads:

a) Vertical loads combination V:

$$N = -(1.4*92 + 1.6*24) = -167.2 \text{ kN}$$
$$M_A = -(1.4*17.12 + 1.6*6) = -33.57 \text{ kN-m}$$
$$M_B = +(1.4*19.94 + 1.6*5.6) = +32.68 \text{ kN-m}$$

$M_2 = 33.57$ kN-m $\qquad$ $e = 33.57 / 167.2 = 0.20$ m

b) Earthquake combinations:

1) E1 combination:

$$N = -92 - 24 + 14 = -102 \text{ kN}$$
$$M_A = -17.12 - 6 + 35.4 = +12.28 \text{ kN-m}$$
$$M_B = 16.94 + 5.6 - 30.1 = -7.56 \text{ kN-m}$$

$M_2 = 12.28$ kN-m $\qquad$ $e = 12.28 / 102 = 0.12$ m

2) E2 combination:

$$N = -(92 + 24 + 14) = -130 \text{ kN}$$
$$M_A = -(17.12 + 6 + 35.4) = -58.52 \text{ kN-m}$$
$$M_B = 16.94 + 5.6 + 30.1) = +52.64 \text{ kN-m}$$

$M_2 = 58.52$ kN-m $\qquad$ $e = 58.52 / 130 = 0.45$ m

3) E3 combination:

$$N = -0.9*92 + 14 = -68.8 \text{ kN}$$
$$M_A = -0.9*17.12 + 35.4 = 19.99 \text{ kN-m}$$
$$M_B = 0.9*16.94 - 30.1 = -14.85 \text{ kN-m}$$

$M_2 = 19.99$ kN-m $\qquad$ $e = 19.99 / 68.8 = 0.29$ m

4) E4 combination:

$$N = -0.9*92 - 14 = -96.8 \text{ kN}$$
$$M_A = -0.9*17.12 - 35.4 = -50.81 \text{ kN-m}$$
$$M_B = 0.9*19.94 + 30.1 = 45.35 \text{ kN-m}$$

$M_2 = 50.81$ kN-m $\qquad$ $e = 50.81 / 96.8 = 0.52$ m

$e_{min} = 15 + 0.03*450 = 28.5$ mm $= 0.0285$ m $< e$ in all cases.

Slenderness check:

Rigidity values:

$$\text{C101:} \quad I_c = \frac{0.4 * 0.45^3}{12} = 0.003 \text{ m}^4 \quad L = 6 \text{ m} \quad \frac{I_c}{L} = \frac{0.003}{6} = 5*10^{-4} \text{ m}^3$$

$$\text{B101:} \quad I_{crack} = \frac{0.3 * 0.5^3}{12} = 0.003 \text{ m}^4 \quad L = 4.3 \text{ m} \quad \frac{I_{crack}}{L} = \frac{0.003}{4.3} = 7*10^{-4} \text{m}^3$$

Rotational restraint coefficients:

$$\alpha_A = \alpha_2 = \frac{5*10^{-4}}{7*10^{-4}} = 0.71 \qquad \alpha_B = \alpha_1 = 0 \text{ (fixed end)}$$

$$\alpha_m = (\alpha_1 + \alpha_2) / 2 = (0 + 0.71) / 2 = 0.36 < 2$$

Slenderness ratio:

$$k = \frac{20 - \alpha_m}{20} \sqrt{1 + \alpha_m} = \frac{20 - 0.36}{20} \sqrt{1 + 0.36} = 1.15 \quad L_u = 600 - 50 = 550 \text{ cm}$$

$$\lambda = \frac{1.15 * 550}{0.3 * 45} = 46.85 > \lambda_{lim} = 22 \qquad \text{Slender column}$$

Design:

Columns in a structure should be designed for each load combination and reinforced according to the solution that requires maximum steel. Needless to say column section should be modified if necessary. If computers are used in design these calculations can be done in a very short time without any difficulty. But hand calculations take a long time, therefore in practice some of the load combinations can be eliminated if they are apparently not critical. In this example let us consider vertical load combination where N is maximum and E4 combination where eccentricity is maximum and ignore the other combinations.

A) Design for vertical load combination:

Axial loads:

C101: $N = -167.2$ kN (Already calculated)

C102: $N = -(1.4*260 + 1.6*80) = -492$ kN

C103: $N = -(1.4*120 + 1.6*32) = -219.2$ kN

$$\Sigma N = -(167.2 + 492 + 219.2) = -878.4 \text{ kN}$$

Critical loads:

$$E_c = 3250\sqrt{25} + 14000 = 30250 \text{ N/mm}^2 = 30250*10^3 \text{ kN/m}^2$$

All columns are identical, therefore $I_c = 0.003 \text{ m}^4$ for all columns. In the building sustained loads are vertical loads, for this reason sum of the shear forces in the storey is zero. In other words $R_m = 0$ is obtained by Eq. (4.22). Therefore let us assume $R_m = 0.5$ as a safe and reasonable value. For all columns:

$$EI = \frac{0.4*30250*10^3*3*10^{-3}}{(1+0.5)} = 24200 \text{ kN-m}^2$$

C101:

$$k = 1.15 \text{ (calculated above)} \qquad N_{cr} = \frac{\pi^2 24200}{(1.15*5.5)^2} = 5970 \text{ kN}$$

C102:

$$\text{Column: } \frac{I_c}{L} = 5*10^{-4} \text{ m}^3$$

$$\text{B101: } \frac{I_{crack}}{L} = 7*10^{-4} \text{ m}^3 \qquad \text{B102: } \frac{I_{crack}}{L} = \frac{3*10^{-3}}{6.8} = 4.4*10^{-4} \text{ m}^3$$

$$\alpha_A = \alpha_2 = \frac{5*10^{-4}}{(7+4.4)*10^{-4}} = 0.44 \quad \alpha_B = \alpha_1 = 0 \text{ (fixed support)} \quad \alpha_m = 0.22<2$$

$$k = \frac{20-0.22}{20}\sqrt{1+0.22} = 1.09 \qquad N_{cr} = \frac{\pi^2 24200}{(1.09*5.5)^2} = 6646 \text{ kN}$$

C103:

$$\alpha_A = \alpha_2 = \frac{5*10^{-4}}{4.4*10^{-4}} = 1.14 \qquad \alpha_B = \alpha_1 = 0 \text{ (fixed support)} \quad \alpha_m = 0.57 < 2$$

$$k = \frac{20 - 0.57}{20}\sqrt{1+0.57} = 1.22 \qquad N_{cr} = \frac{\pi^2\, 24200}{(1.22*5.5)^2} = 5305 \text{ kN}$$

$$\Sigma N_{cr} = 5970 + 6646 + 5305 = 17921 \text{ kN}$$

Check: $\Sigma N = 878.4 \text{ kN} < 0.45\Sigma N_{cr} = 0.45*17921 = 8064 \text{ kN}$ OK.

$$\beta_s = \frac{1}{1-1.3\dfrac{878.4}{17921}} = \frac{1}{0.936} = 1.07$$

Individual $\beta$ for vertical load combination:

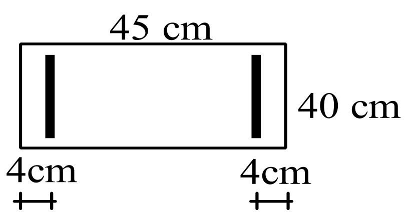
$c_m = 1$ $\quad$ $EI = 24200\ kN\text{-}m^2$ $\quad$ $\alpha_2 = 0.71$ $\quad$ $\alpha_1 = 0$

$k = 0.7 + 0.05\ (0 + 071) = 0.74 < (0.85 + 0) < 1$

$$N_{cr} = \frac{\pi^2\, 24200}{(0.74 * 5.5)^2} = 14419\ kN \qquad \beta = \frac{1}{1 - 1.3\dfrac{167.2}{14419}} = 1.02 < 1.07$$

Design:

$N = 167.2\ kN$ $\qquad$ $M_{max} = 1.07 * 33.57 = 35.92\ kN\text{-}m$




If reinforcement is placed as shown in the figure at the left:
$d'' = 45 - 8 = 37\ cm$ $\quad$ $d''/h = 37/45 \approx 0.8$
$\lambda = 0$ (no middle bars)
Chart A-3 will be used.

$$\frac{N}{bhf_{cd}} = \frac{167200}{400 * 450 * 17} = 0.05 \qquad \frac{M_{max}}{bh^2 f_{cd}} = \frac{35.92 * 10^6}{400 * 450^2 * 17} = 0.03$$

From the chart: $\rho_t m < 0.1$ $\quad = 365 / 17 = 21.5$ $\quad$ $\rho_t < \dfrac{0.1}{21.5} = 0.005 < \rho_{min}$

$\rho_t = \rho_{min} = 0.01$

B) Design for E4 combination:

Axial load:

C101: $N = -96.8$ kN (calculated above)
C102: $N = -0.9*260 + 6.10 = -227.9$ kN
C103: $N = -0.9*120 + 7.70 = -100.3$ kN

$\Sigma N = -(96.8 + 227.9 + 100.3) = -425$ kN

$\Sigma N_{cr} = 17921$ kN (calculated above), $425 \text{ kN} < 0.45*17921 = 8064$ kN

$$\beta_s = \frac{1}{1\text{-}1.3\dfrac{425}{17921}} = \frac{1}{0.969} = 1.03$$

Individual β:

$N_{cr} = 14419$ kN (calculated above)

$$\beta = \frac{1}{1\text{-}1.3\dfrac{96.8}{14419}} = \frac{1}{0.991} = 1.01$$

Design:

$N = 96.8$ kN  $M_2 = 50.81$ kN-m  $M_{max} = 1.03 * 50.81 = 52.33$ kN-m

$$\frac{N}{bhf_{cd}} = \frac{96800}{400*450*17} = 0.03 \qquad \frac{M_{max}}{bh^2 f_{cd}} = \frac{52.33*10^6}{400*450^2*17} = 0.04$$

From Chart A-3:

$\rho_t m < 0.1$  Therefore  $\rho_t < 0.005 < \rho_{min}$

For both load combinations minimum steel ratio is required:

$A_{st} = 0.01*40*45 = 18\ cm^2$  Selected: 6Ø20 (18.84 $cm^2$)

Ties: Ø8  Check: 8 mm > 20 / 3 = 6.67 mm

Spacing: $s_{max} = 12Ø = 12*2 = 24$ cm > 20 cm  Selected s = 20 cm

Details are shown in Fig.4.12.

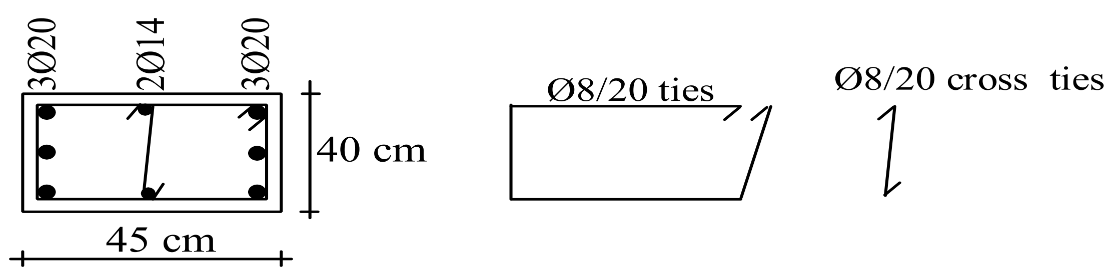



Figure 4.12

2Ø14 bars are placed at the middle since the distance between the corner bars is more than 30 cm.