## CIVL471 DESIGN OF RC STRUCTURES

## LECTURE NOTE \#8

## STRUCTURAL ANALYSIS OF ONE-WAY SLABS

- One-way slabs may be analyzed by any elastic method developed for beams (consider one meter wide strip)
- Twisting moments and twisting rigidities of supporting beams are neglected
- One-way slabs supported by steel beams can be analyzed similarly provided that the distance between the top of the slab and the top of the steel beam is at least 4 cm .
- Design support moments of the slabs supported by beams can be obtained by subtracting $D M=\mathrm{Va} / 3$ from the moment calculated at the center of the support. Here $\mathbf{V}$ is the shear force at the support and $\mathbf{a}$ is the width of the beam. The value of a should not be taken greater than 0.175 times the span length. Also the reduced support moment should not be less than $W_{u} I^{2} / 14$.
- In the slabs supported by masonry walls moments calculated at the centers should be used as design moments.
- Negative support moments due to live loads can be reduced because of the torsional rigidities (generally halved).
- In continuous beam analysis external supports are assumed as freely rotating supports. According to TS 500 top reinforcement equal to at least half of the main reinforcement in adjoining span should be provided in external supports.
- Positive span moments should not be less than $W_{u} I_{u}{ }^{2} / 24$ where $I_{n}$ is the clear span length of the slab.
- Shear stress in slabs are usually very small. In the cases where shear forces are high, it is advised to select slab thickness such that design shear force is less than $0.5^{*} 0.65^{*} f_{c t d} b d$


## AN APPROXIMATE METHOD OF ANALYSIS FOR ONE-WAY SLABS

In continuous one-way slabs if span lengths are equal or nearly equal (the ratio of shortest span to longest span is equal or greater than 0.8 ) and if loads are uniformly distributed and the ratio of live load to dead load is less than 2 bending moments in short direction may be computed by using Eq. (1.1).

$$
\begin{equation*}
M=K W_{u}\left(I_{i}\right)^{2} \tag{1.1}
\end{equation*}
$$

- where $\boldsymbol{W}_{\mathbf{u}}$ is the design load in the corresponding span, $\boldsymbol{I}_{\boldsymbol{i}}$ is the span length and $\boldsymbol{K}$ is a coefficient.
$K$ values:


## For span moment:

$$
\begin{array}{lc}
\text { External spans: } & 1 / 11 \\
\text { Internal spans: } & 1 / 15
\end{array}
$$

For support moments:
a) Slabs with two spans only:

External supports:
Internal supports:

- (1/24)
- (1/8)
b) Slabs with more than two spans:

$$
\begin{array}{ll}
\text { External supports: } & -(1 / 24) \\
\text { First internal supports: } & -(1 / 9) \\
\text { Other internal supports: } & -(1 / 10)
\end{array}
$$

For the calculation of support moments, in Eq. (1.1) average values are used for the loads or the span lengths if their values are different at the spans at two sides of the support.

## Example 1.1




Figure 1.6

Design the slab shown in Fig.1.6 following the provisions of TS500. Select half straight and half bent-up bars.
Live load $=2.5 \mathrm{kN} / \mathrm{m}^{2}, \quad$ Total thickness of finishing material and plaster $=8 \mathrm{~cm}$
Materials: Concrete: C25, Steel: S420

## Solution:

Check: Using center to center distances for spans,

$$
\ln S 102 I_{y} / I_{x}=I_{\text {long }} / I_{\text {short }}=1030 / 510=2.02>2 \text { One-way }
$$

slab

$$
\text { In S101 } \mathrm{I}_{\mathrm{y}} / I_{x}=1030 / 450=2.29>2 \quad \text { One-way slab }
$$

Slab thickness:
Since the span of S102 is the larger, the thickness of
S102 will be chosen and the same thickness will be used for S101 although it is not strictly necessary.

$$
\min . \mathrm{h}=480 / 30=16 \mathrm{~cm}>8 \mathrm{~cm} \quad \text { Selected } \mathrm{h}=16 \mathrm{~cm}
$$

## Loads of the slabs:

In dead load calculations the unit weight of reinforced concrete should be assumed $25 \mathrm{kN} / \mathrm{m}^{3}$ whereas unit weights of mortars, plasters, plain concrete etc. may be taken as $20 \mathrm{kN} / \mathrm{m}^{3}$. Therefore,

> Self weight of slab........................... $0.16 * 25=4.00 \mathrm{kN} / \mathrm{m}^{2}$
> Additional dead load (floor finish+plaster) $. . \quad 0.08 * 20=1.60 \mathrm{kN} / \mathrm{m}^{2}$
> Dead load $\mathrm{W}_{\mathrm{d}}=5.60 \mathrm{kN} / \mathrm{m}^{2}$
> Live load $\quad W_{1}=2.50 \mathrm{kN} / \mathrm{m}^{2}$

Additional dead load can be calculated more precisely by using architectural details. However, the architect or the owner of the building may change them during the construction or later. Therefore many designers use an assumed value which is considered safe enough.

## Structural analysis:

This system can be analyzed by any method developed for continuous beams. However, in this problem we can use the moment coefficients (approximate method) since:

- In short direction $\left.\right|_{\text {shortest }} / I_{\text {longest }}=4.50 / 5.10=0.88>0.8$
- Loads are uniformly distributed
$-\boldsymbol{W}_{\boldsymbol{l}}=2.5 \mathrm{kN} / \mathrm{m}^{2}<2 \boldsymbol{W}_{\boldsymbol{d}}=2 \star 5.60=1120 \mathrm{kN} / \mathrm{m}^{2}$
Coefficients given for two span slabs will be used in moment calculations.

Total factored load (design load):

$$
W_{u}=1.4 W_{d}+1.6 W_{1}=1.4^{*} 5.60+1.6^{*} 2.50=11.84 \mathrm{kN} / \mathrm{m}^{2}
$$

Span moments:

$$
\begin{array}{ll}
\text { S101: } & \frac{1}{11} \mathrm{~W}_{\mathrm{u} 1} 1_{1}^{2}=\frac{1}{11} * 11.84 * 4.5^{2}=21.80 \mathrm{kN}-\mathrm{m} / \mathrm{m} \\
\mathrm{~S} 102: & \frac{1}{11} \mathrm{~W}_{\mathrm{u} 2} l_{2}^{2}=\frac{1}{11} * 11.84 * 5.1^{2}=27.30 \mathrm{kN}-\mathrm{m} / \mathrm{m}
\end{array}
$$

## Support moments:

S101 (external): $-\frac{\mathbf{1}}{\mathbf{2 4}} \boldsymbol{W}_{u 1} l_{1}^{2}=-\frac{\mathbf{1}}{\mathbf{2 4}} * 11.84 * 4.5^{2}=-9.99 \mathrm{kN}-\mathrm{m} / \mathrm{m}$
S101-S102: $\quad \frac{1}{8} W_{u, a v} I_{\mathrm{av}}^{2}=\frac{1}{8} * 11.84 *\left(\left(\frac{4.5+5.1}{2}\right)\right)^{2}=34.10 \mathrm{kN}-\mathrm{m} / \mathrm{m}$
S102 (external): $-\frac{\mathbf{1}}{\mathbf{2 4}} \boldsymbol{W}_{u 2} \boldsymbol{I}_{2}^{2}=-\frac{\mathbf{1}}{\mathbf{2 4}} * \mathbf{1 1 . 8 4} * 5.1^{2}=-12.83 \mathrm{kN}-\mathrm{m} / \mathrm{m}$

This method is an approximate method therefore no adjustment in support moments is necessary. Moment diagram is shown in Fig.1.7. Moment curves are not continuous, because numerically largest positive and negative moments are obtained from different loading patterns.


Figure 1.7

## Design of the slabs:

Cross-sections of the slab strips are rectangle.
Equation $\boldsymbol{M}=\boldsymbol{R b d}^{2}$ will be used in the design. $R$ values are given in the Appendix
$b=100 \mathrm{~cm} \quad d=h-d^{\prime}=16-2=14 \mathrm{~cm}$

Steel area: $A_{s}=\rho b d$

Minimum steel ratio: $\rho_{\text {min }}=0.002$

Maximum spacing: $1.5 \mathrm{~h}=1.5^{*} 16=24 \mathrm{~cm}>20 \mathrm{~cm}$
Therefore $\mathrm{s}_{\max }=20 \mathrm{~cm}$

In slab design if reinforcement is one layer effective depth can be calculated as (slab thickness - 2) cm. Assuming $\varnothing 10$ bars are used and clear concrete cover is $1.5 \mathrm{~cm}, 2 \mathrm{~cm}$ can be easily calculated as the distance between the surface of the concrete cover and the centroid of the bars.

Design for span moments

| Slab | Moment $(\mathrm{kg}-\mathrm{cm}) \mathrm{d}(\mathrm{cm}) \mathrm{R}\left(\mathrm{kg} / \mathrm{cm}^{2}\right)$ | $\rho$ | $\mathrm{A}_{\mathrm{s}}\left(\mathrm{cm}^{2}\right)$ | Selected |  |  |
| :--- | :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| S 101 | $21.80^{*} 10^{4}$ | 14 | 11.12 | 0.0032 | 4.48 | $\varnothing 10 / 17.5(4.49)$ |
| S 102 | $27.30^{*} 10^{4}$ | 14 | 13.92 | 0.0041 | 5.74 | $\varnothing 10 / 13.5(5.82)$ |

## Design for support moments

| Slab | Moment | d | R | $\rho$ | $\mathrm{A}_{\mathrm{s}}$ | Available | Additional |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S101 | $9.99^{*} 10^{4}$ | 14 | 5.10 | $<\rho_{\min }$ | 2.80 | $4.49 / 2=2.245$ | $\varnothing 8 / 70(0.74)$ |
| S101-S102 | $34.1^{*} 10^{4}$ | 14 | 17.4 | 0.0051 | 7.14 | $\frac{4.49+5.82}{2}=5.155$ | $\varnothing 8 / 25(2.01)$ |
| S102 | $12.83^{*} 10^{4}$ | 14 | 6.55 | $<\rho_{\min }$ | 2.80 | $5.82 / 2=2.91$ | - |

Note that at the external supports $\rho<\rho$ min, therefore steel is calculated as As $=\rho \min \mathrm{bd}=0.002 * 100 \star 14=2.8 \mathrm{~cm} 2$. On the other hand since bent-up bars are used available bars at the external supports are equal to at least half of the main steel which is the requirement of TS500. For S101, additional $\varnothing 8 / 90(0.56 \mathrm{~cm} 2)$ is enough but for a uniform placement of steel $\varnothing 8 / 70(0.74 \mathrm{~cm} 2)$ is selected.

## Distribution steel:

S101: required minimum area $\frac{\mathbf{A}_{s}}{\mathbf{5}}=\frac{\mathbf{4 . 4 8}}{\mathbf{5}}=\mathbf{0 . 9 0} \mathrm{cm}^{2}$, selected: $\varnothing 6 / 30\left(0.95 \mathrm{~cm}^{2}\right)$

S102: required minimum area $\frac{\mathbf{A}_{s}}{\mathbf{5}}=\frac{\mathbf{5 . 7 4}}{\mathbf{5}}=\mathbf{1 . 1 5 \mathrm { cm } 2 \text { , selected: } \varnothing 6 / 2 4 ( 1 . 1 8 \mathrm { cm } 2 ) , ~}$
(Spacing should not be higher than 30 cm )
Support bars in the long direction:
S101: required minimum area $0.6 \mathrm{~A}_{\mathrm{s}}=0.6 * 4.48=2.69 \mathrm{~cm}^{2}>1.68 \mathrm{~cm}^{2}(\varnothing 8 / 30)$ Selected: Ø8/18.5 ( $2.72 \mathrm{~cm}^{2}$ )

S102: required minimum area $0.6 \mathrm{~A}_{\mathrm{s}}=0.6 * 5.74=3.44 \mathrm{~cm}^{2}>1.68 \mathrm{~cm}^{2}(\varnothing 8 / 30)$ Selected: Ø8/14.5 (3.47 $\mathrm{cm}^{2}$ )


Figure 1.8

