CIVL471 DESIGN OF RC STRUCTURES

LECTURE NOTE #9

TWO WAY EDGE SUPPORTED SLABS

Behavior of Two-Way Edge Supported Slabs

- Two-way slabs bend actually in all directions. However, in rectangular slabs it is sufficient to study bending in two basic directions. Two-way slabs are reinforced in these two directions. Thus enough resistance is obtained in all directions. It is easy to show that slab loads are carried partly in short direction and partly in long direction. Let us consider two perpendicular beams with the identical cross-sections rigidly jointed at the mid-spans (Fig.1.9) Let the deflection of the joint be Δ due to the concentrated load P acting at the joint.

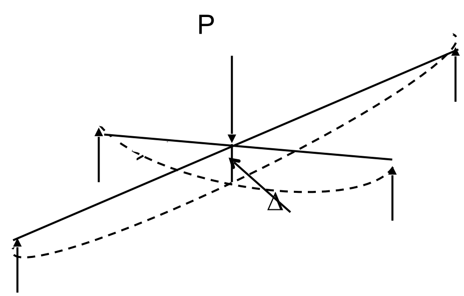


Figure 1.9

- The load may not be carried by only one beam since both of the beams deflect together and thus contribute to load carrying mechanism. If l_s is the span length of the short beam and l_l is the span length of the long beam, the following equations can be written for the common deflection Δ :

$$\Delta = \frac{P_s l_s^3}{48EI} \qquad \Delta = \frac{P_l l_l^3}{48EI} \qquad (1.2)$$

where P_s is the load carried by the short beam, P_l is the load carried by the long beam.

$$\frac{P_s}{P_l} = \left(\frac{l_l}{l_s}\right)^3 \qquad (1.3)$$

On the other hand,

$$P_s + P_l = P \qquad (1.4)$$

Solving Eqs.1.3 and 1.4 simultaneously $\boldsymbol{P_s}$ and $\boldsymbol{P_l}$ can be calculated. For example if $\boldsymbol{l_l / l_s} = 2$,

$$P_s = 8\,P_l \quad , \quad 9\,P_l = P \quad , \quad P_l = \frac{P}{9} \quad \text{and} \quad P_s = \frac{8}{9}P$$

- Note that in above large part of the load is carried by the short beam.
- A slab can be considered as consisting of two sets of parallel strips perpendicular to each other.

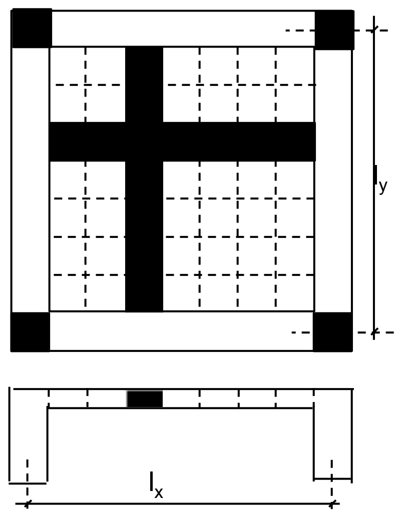


Figure 1.10

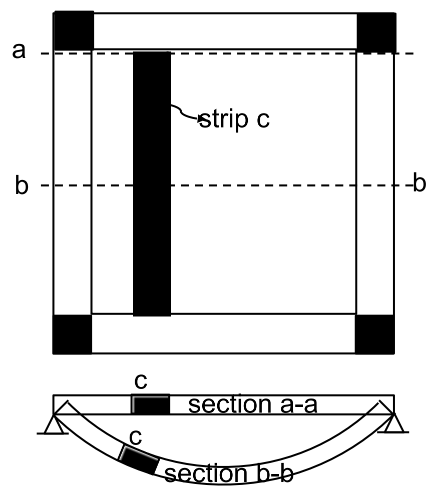


Figure 1.11

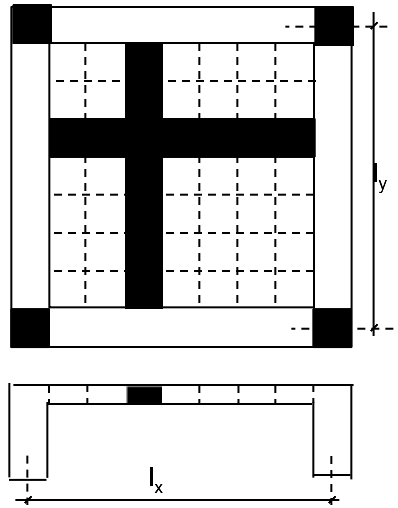


Figure 1.10

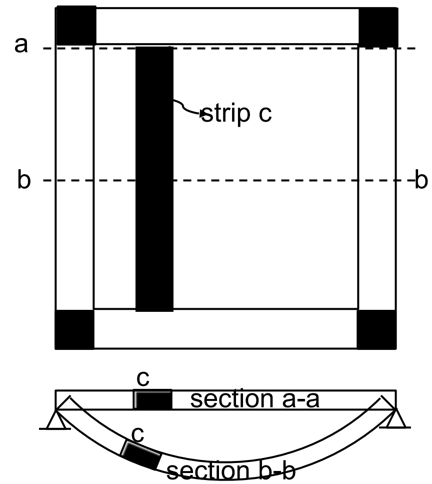


Figure 1.11

- Above strips are very similar to the one considered in figure 1.9.
- However these slabs are not jointed at the mid points and loads on the strips are uniformly distributed.
- It can be shown that with similar calculations

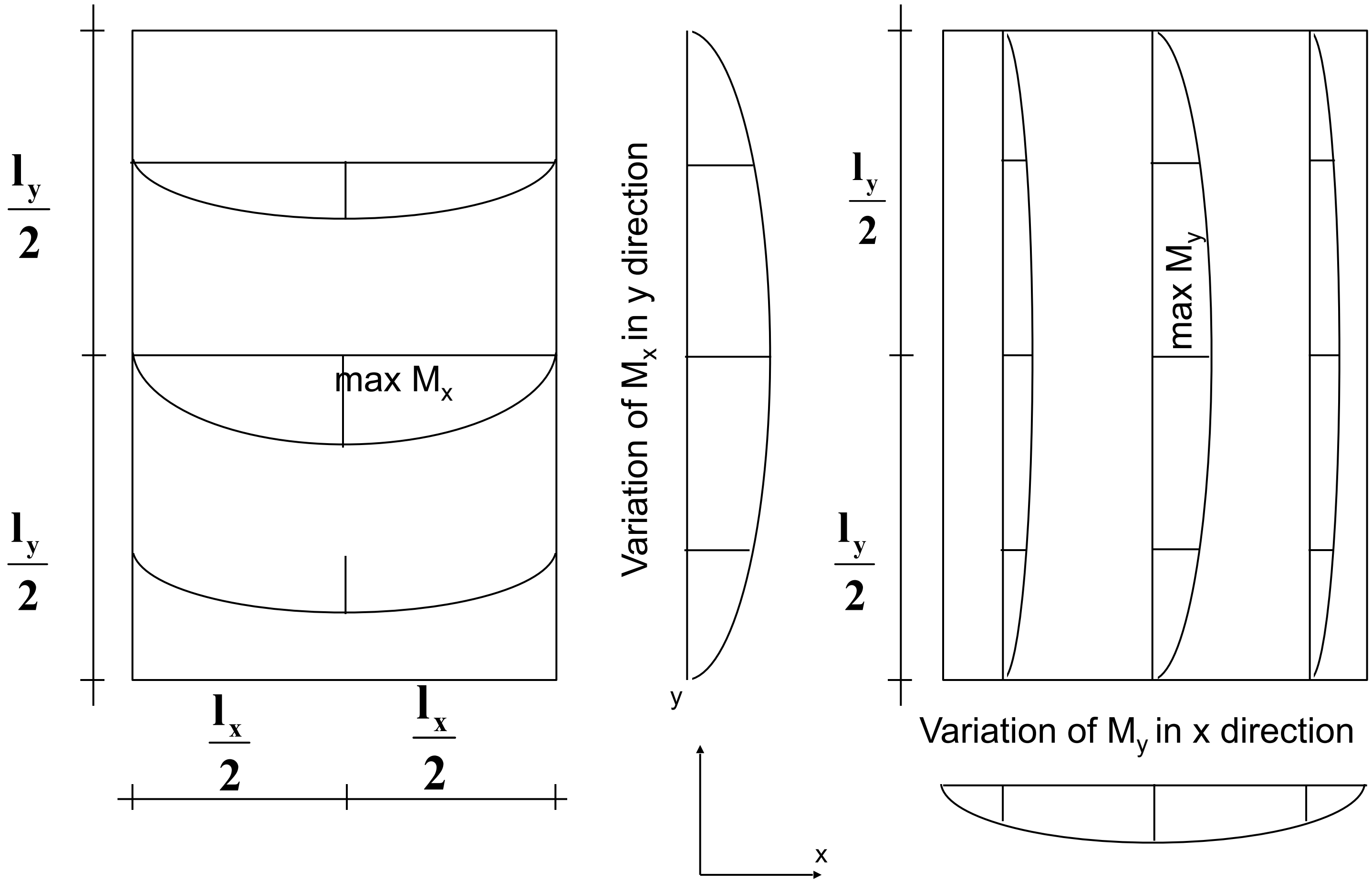
$$W_x / W_y \textbf{ is proportional to } \left(l_y / l_x\right)^4$$

- In reality load carrying mechanism of a slab is not simple. There are many things which influence on the behavior such as:
  - Beams supporting slabs deflects
  - Slab loads are also resisted by twisting moments in addition to bending. Note that twisting action is higher in the strips close to the supports. Twisting effects can be seen in the form of lifting forces at the free corners and if lifting is prevented cracking of the slab at the corners must be controlled by special reinforcements.

- Recalling previous simple model and ignoring twisting moments, in the middle strip of one unit wide strip of a simply supported square slab may be determined as $\boldsymbol{(1/8)(W_u/2)l^2 = 0.0625W_ul^2}$

- However, the actual bending moment is $\boldsymbol{0.048W_ul^2}$. This shows that twisting moments play important role in load carrying system of slabs.

- In a slab subjected to uniformly distributed load maximum moments occur in the middle strips and in the strips closer to the supports moments are smaller as shown in the Fig. 1.12



a) Variation of M_x

b) Variation of M_y

- In slabs if the strip where the maximum moment occurs fails, slab will not fail since the other strips in x and y direction take over loads. The slab fails only after the yielding of steel in both directions and in a rather large area. This is known as redistribution of moments in slabs.

- Test with slabs showed that maximum moment in a simply supported square slab may be computed as $\boldsymbol{0.036W_u l^2}$ which is %25 less than the exact value $\boldsymbol{0.048W_u l^2}$ obtained by elastic analysis.

- For the design of outer quarters of the slabs further reduction in moment can be done in moments since they are smaller than the ones in the middle strips.

ANALYSIS OF TWO-WAY SLABS

- Analysis of slabs can be done by using elasticity theories. However, these methods are complicated and conservative as mentioned above. Therefore various simplified methods have been developed. TS 500 mentions the following methods:
    a) Equivalent frame method
    b) Yield-line method
    c) Approximate method for edge-supported slabs
    d) Approximate method for column-supported (beamless) slabs

- If spans do not differ too much and if more precision is not necessary approximate methods may be used with enough accuracy.
- Equivalent frame method is applicable to the slabs having beams or not.
- However, since it is rather complicated designers usually prefer to use the approximate method for the edge-supported two-way slabs which is known also as coefficient method.

Approximate Method for Two-way Edge-Supported Slabs (Coefficient Method)

- In this Method, Eq. 1.5 is used to calculate positive moments at the middle of the slab and negative moment at faces of supports

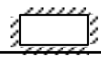
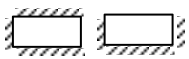
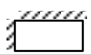
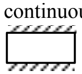
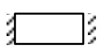
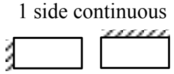
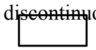
$$M = \alpha W_u l_{xn}^2 \qquad (1.5)$$

Where W_u is the uniformly distributed design load per m^2

l_{xn} is the short clear span of the slab

α is a coefficient

TABLE 1.2 MOMENT COEFFICIENTS FOR EDGE-SUPPORTED TWO-WAY SLABS

TYPE OF SLAB			ε = 1.0	ε = 1.1	ε = 1.2	ε = 1.3	ε = 1.4	ε = 1.5	ε = 1.75	ε = 2.0	LONG DIRECTION
1	4 sides continuous 	-M(SUPP) +M(SPAN)	0.033 0.025	0.040 0.030	0.045 0.034	0.050 0.038	0.054 0.041	0.059 0.045	0.071 0.053	0.083 0.062	0.033 0.025
2	3 sides continuous 	-M(SUP) +M(SPAN)	0.042 0.031	0.047 0.035	0.053 0.040	0.057 0.043	0.061 0.046	0.065 0.049	0.075 0.056	0.085 0.064	0.041 0.031
3	2 adjacent sides continuous 	-M(SUPP) +M(SPAN)	0.049 0.037	0.056 0.042	0.062 0.047	0.066 0.050	0.070 0.053	0.073 0.055	0.082 0.062	0.090 0.068	0.049 0.037
4	2 longer sides continuous 	-M(SUPP) +M(SPAN α	0.056 0.044	0.061 0.046	0.065 0.049	0.069 0.051	0.071 0.053	0.073 0.055	0.077 0.058	0.080 0.060	- 0.044
5	2 shorter sides continuous 	-M(SUP) +M(SPAN)	- 0.044	- 0.053	- 0.060	- 0.065	- 0.068	- 0.071	- 0.077	- 0.080	0.056 0.044
6	1 side continuous 	-M(SUPP) +M(SPAN)	0.058 0.044	0.065 0.049	0.071 0.054	0.077 0.058	0.081 0.061	0.085 0.064	0.092 0.069	0.098 0.074	0.058 0.044
7	All sides discontinuous 	-M(SUPP) +M(SPAN)	- 0.050	- 0.057	- 0.062	- 0.067	- 0.071	- 0.075	- 0.081	- 0.083	- 0.050

ε is equal to l_y/l_x where l_y=long span and l_x=short span

- Two-way slabs are divided into a middle strip and two edge strips as shown below

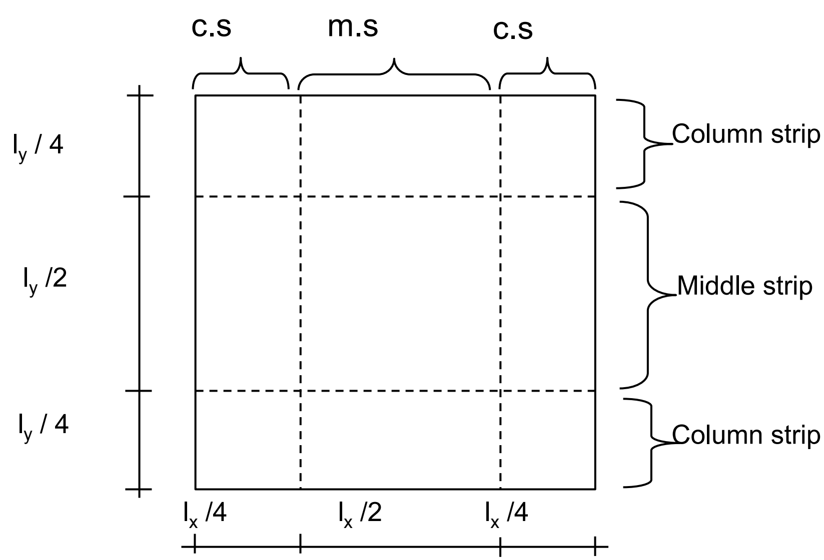


Figure 1.13

- According to TS 500 moments in the middle strips are constant and given by Eq. 1.5. In the edge strip (column strip) 2/3 of the full moment may be used which will be economical for large slabs

- In continuous slabs negative moments develop at continuous edge (at the internal supports). In coefficient method two different negative moment from two slabs will be calculated. If the difference between to moments is less than 20% than the greater moment can be used in design. Otherwise, 2/3 of the difference between two moments may be distributed to two adjacent slabs according to the rigidities of the two adjacent slab strips.

- In this method no moment is calculated at the external supports. If rotation at these supports is completely restricted a negative moment equal to the positive moment should be used at that support. Otherwise, %50 of the positive moment should be assumed.

Thickness and Reinforcement of Two-way Slabs

- According to TS 500 the thickness of an edge-supported slab can not be less than 8 cm and less than $\boldsymbol{h_f}$

$$h_f = \frac{l_{xn}}{15 + \frac{20}{\varepsilon}}\left(1 - \frac{\alpha_s}{4}\right)$$

where α_s is the ratio of the sum of the continuous edges to the perimeter

- Short direction should better be placed below as bottom layer bars crosswise to the layer placed in the long direction.
- According to TS 500 the sum of the steel ratios in two directions should not be less than 0.004 for S220, 0.0035 for S420 and S500.
- Steel ratio can not be less than 0.0015 in each direction.
- Spacing of the bars should not exceed 1.5 times the thickness of the slab and 20 cm in the short direction.
- In the long direction maximum spacing is 25 cm.
- Clear concrete cover protecting the reinforcement should be at least 1.5 cm.

Twisting Reinforcement

- Cracks may develop at the external corners of the external panels due to twisting moments and they can be prevented by the reinforcement as shown in the figure 1.14

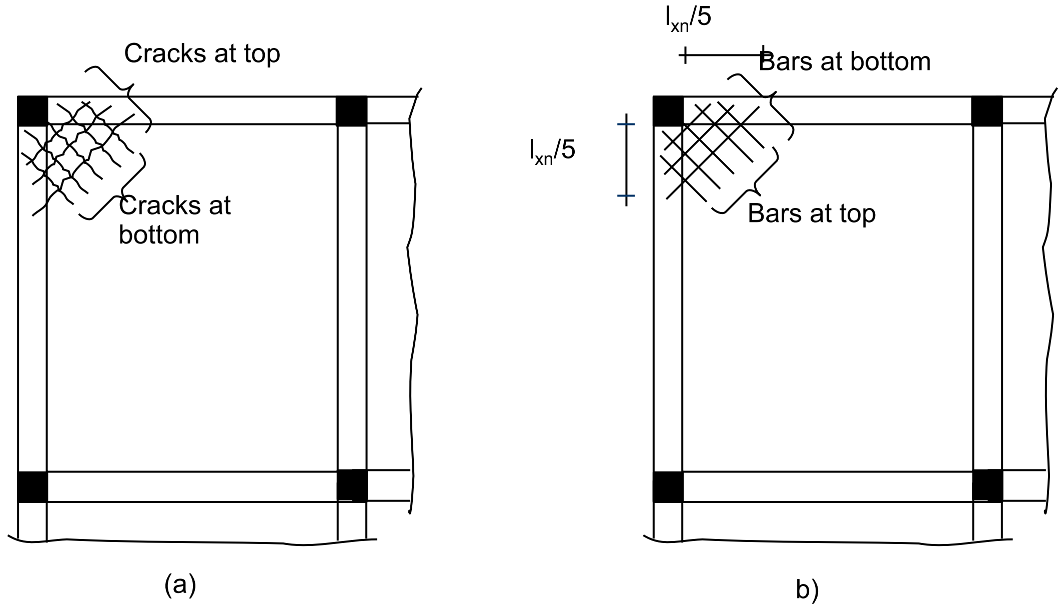


Figure 1.14

- Alternatively twisting bars may be placed parallel to the main steel as shown in Fig.1.15. In that case both top and bottom bars should be two perpendicular layers. The area of each layer either placed as shown in Fig.1.14b or Fig.1.15 should not be less than 3/4 of the area of maximum span reinforcement and they should be located in a square area. Side length of this square should be 1/5 of the short clear span length.

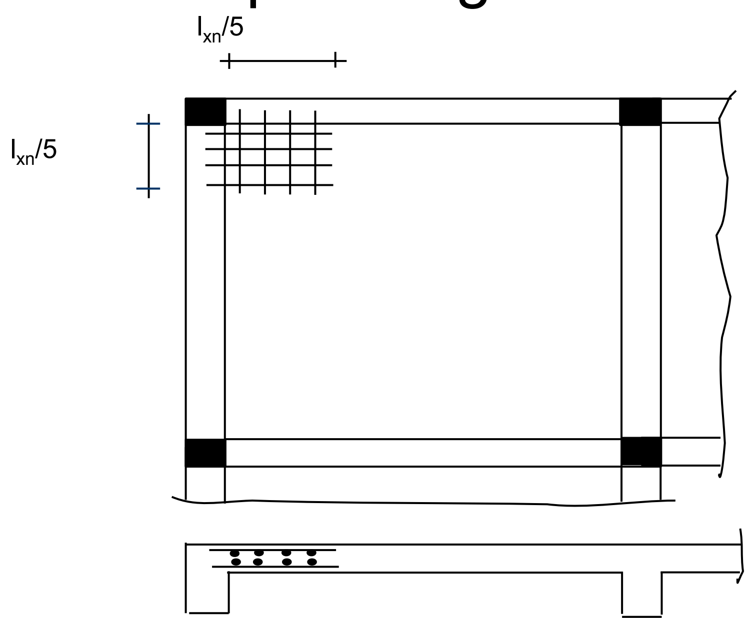


Figure 1.15

Example 1.2a (class notes)

Example 1.2 (page 19 of the text book)

Example 1.2

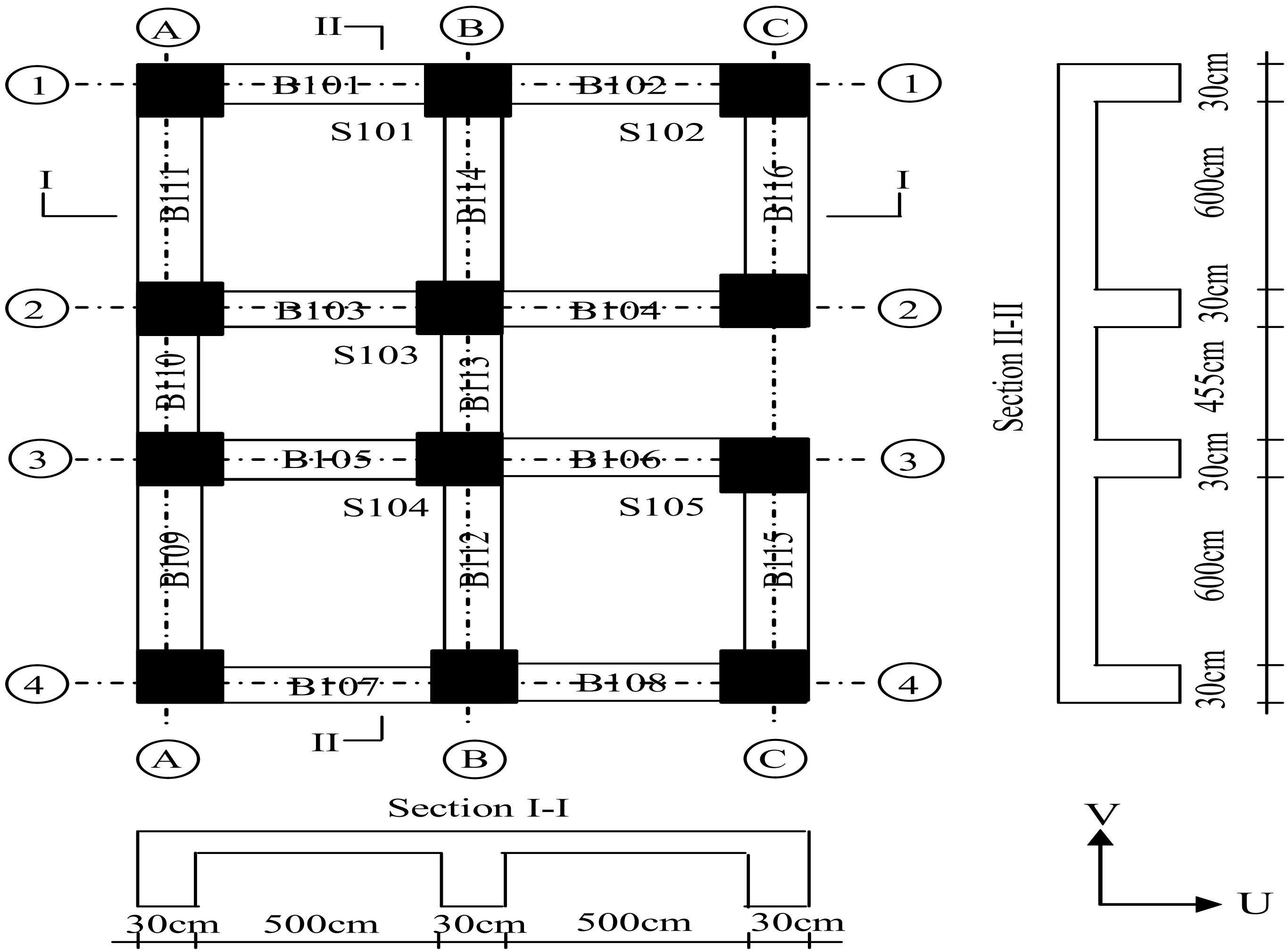


Figure 1.16

Live load: 2.5 kN/m^2
Additional dead load (floor finish, plaster etc.): 1.25 kN/m^2
Materials: C16, S220

Design the slabs shown in Fig.1.16. Use half straight and half bent-up bars.

Solution:

-Determining the types of the slabs:

S101, S102, S104 and S105: $\varepsilon = 630 / 530 = 1.19 < 2$ Two-way slabs

S103: $\varepsilon = 530 / 485 = 1.09 < 2$ Two-way slab

-Selecting the thickness of the slabs:

Minimum thickness calculations:

S101: $\alpha_s = \dfrac{600+500}{2*(600+500)} = 0.5 \qquad h_f = \dfrac{500}{15+\dfrac{20}{1.19}}(1-\dfrac{0.5}{4}) = 13.76 \text{ cm}$

S102: $\alpha_s = \dfrac{600}{2*1100} = 0.27 \qquad h_f = \dfrac{500}{15+\dfrac{20}{1.19}}(1-\dfrac{0.27}{4}) = 14.66 \text{ cm}$

S103: $\alpha_s = \dfrac{500+500}{2*(500+455)} = 0.52 \quad h_f = \dfrac{455}{15+\dfrac{20}{1.09}}(1-\dfrac{0.52}{4}) = 11.87 \text{ cm}$

All these values are greater than 8 cm which is the other minimum thickness specified by TS500. Because of symmetry, S104 is identical to S101 and S105 is identical to S102. From the calculations above it can be seen that each slab can have different thickness. But in practice usually same thickness is chosen for all slabs unless there is a particular reason not to do so. After examining the minimum values h = 15 cm is selected.

- Calculation of the slab loads:

Self-weight of slab: $0.15*25 = 3.75\ kN/m^2$

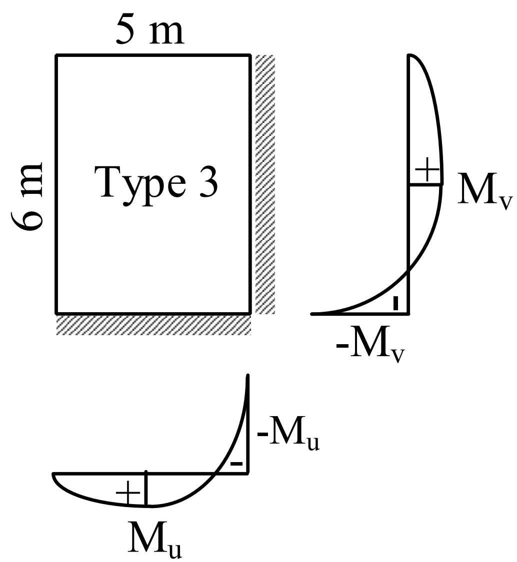
Additional dead load (floor finish, plaster etc.): $1.25\ kN/m^2$

Dead load: $W_d = 5.00\ kN/m^2$

Total factored load: $W_u = 1.4*5 + 1.6*2.5 = 11.00\ kN/m^2$

-Bending moments:

S101:



U direction (short), $\varepsilon = 1.19 \approx 1.20$:

$M_u = 0.047*11.00*5.00^2 = 12.93$ kN-m

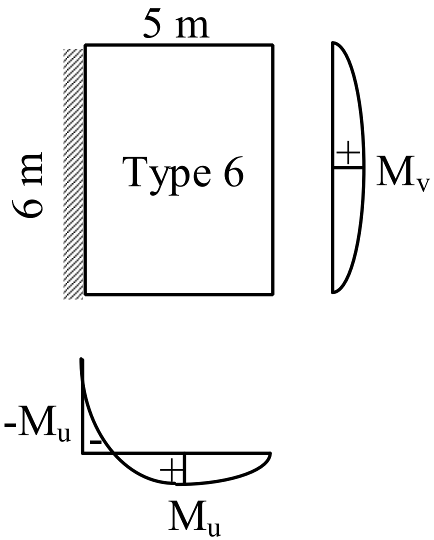
$-M_u = 0.062*11.00*5.00^2 = 17.05$ kN-m

V direction (long) :

$M_v = 0.037*11.00*5.00^2 = 10.18$ kN-m

$-M_v = 0.049*11.00*5.00^2 = 13.48$ kN-m

S102:



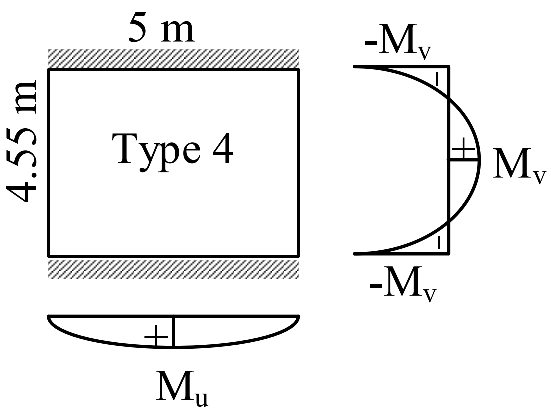
U direction (short), $\varepsilon = 1.19 \approx 1.20$:

$M_u = 0.054*11.00*5.00^2 = 14.85$ kN-m
$-M_u = 0.071*11.00*5.00^2 = 19.53$ kN-m

V direction (long) :

$M_v = 0.044*11.00*5.00^2 = 12.10$ kN-m
$-M_v = 0$

S103:



U direction (long):

$M_u = 0.044*11.00*4.55^2 = 10.02$ kN-m
$-M_u = 0$

V direction (short), $\varepsilon = 1.09 \approx 1.10$:

$M_v = 0.046*11.00*4.55^2 = 10.48$ kN-m
$-M_v = 0.061*11.00*4.55^2 = 13.89$ kN-m

Because of symmetry the moments of S104 are equal to the moments of S101 and the moments of S105 are equal to the moments of S102.

It can be observed that two support moments are calculated at each internal support. For example, for the support between S101 and S102, calculated moment from S101 is -17.05 kN-m whereas -19.53 kN-m from S102. The ratio of these moments is 17.05 / 19.53 = 0.87 > 0.8. Therefore the section at this support will be designed for 19.53 kN-m. Similarly at the support between S101 and S103 the ratio is 13.48 / 13.89 = 0.97 > 0.80, therefore design moment at this support will be 13.89 kN-m.

-Effective depths:

If effective depths are designated by d_x and d_y in short and long directions respectively the following depths can be obtained:

$d_x = 15 - 2 = 13$ cm $\qquad$ $d_y = 13 - 1 = 12$ cm

Here it is assumed that clear concrete cover is 1.5 cm and Ø10 bars are used in both directions. For the top bars at all supports $d = d_x = 13$ cm can be used since there is only one layer bars at a support.

Spacing limits:

Across the short span: $1.5h = 1.5*15 = 22.5$ cm > 20 cm $\quad s_{max} = 20$ cm

Across the long span: $s_{max} = 25$ cm

-Minimum reinforcement:

If ρ_x and ρ_y are the steel ratios in short and long directions respectively the following requirements should be met:

$$\rho_x + \rho_y \geq 0.004 \text{ (steel grade is S220)}, \quad \rho_x \geq 0.0015 \quad \text{and} \quad \rho_y \geq 0.0015$$

Design of the slabs is summarized in the following tables. Because of symmetry only S101, S102 and S103 are included in the table. It is assumed that rotations of external supports are not prevented completely. Therefore 50% of the span moments may be assumed acting as negative moments at these supports. For this reason available bars provided by bent-up bars will be sufficient for external supports.

Design in U direction

Spans

Slab	Moment (Kg-cm)	d (cm)	R (kg/cm^2)	ρ	A_s (cm^2)	Selected
S101	$12.93*10^4$	13	7.65	0.0042	5.46	Ø10/14 (5.61)
S102	$14.84*10^4$	13	8.79	0.0048	6.24	Ø10/12.5 (6.28)
S103	$10.02*10^4$	12	6.96	0.0038	4.56	Ø10/17 (4.62)

Supports
Between S101-S102

Moment	d	R	ρ	A_s	Available	Add
$19.53*10^4$	13	11.56	0.0065	8.45	$\frac{5.61+6.28}{2}=5.95$	Ø8/20 (2.50)

Design in V direction

Spans

Slab	Moment (Kg-cm)	d (cm)	R (kg/cm^2)	ρ	A_s (cm^2)	Selected
S101	$10.18*10^4$	12	7.07	0.0039	4.68	Ø10/16.5 (4.76)
S102	$12.10*10^4$	12	8.40	0.0046	5.52	Ø10/14 (5.61)
S103	$10.48*10^4$	13	6.20	0.0034	4.42	Ø10/17.5 (4.49)

Supports
Between S101-S103

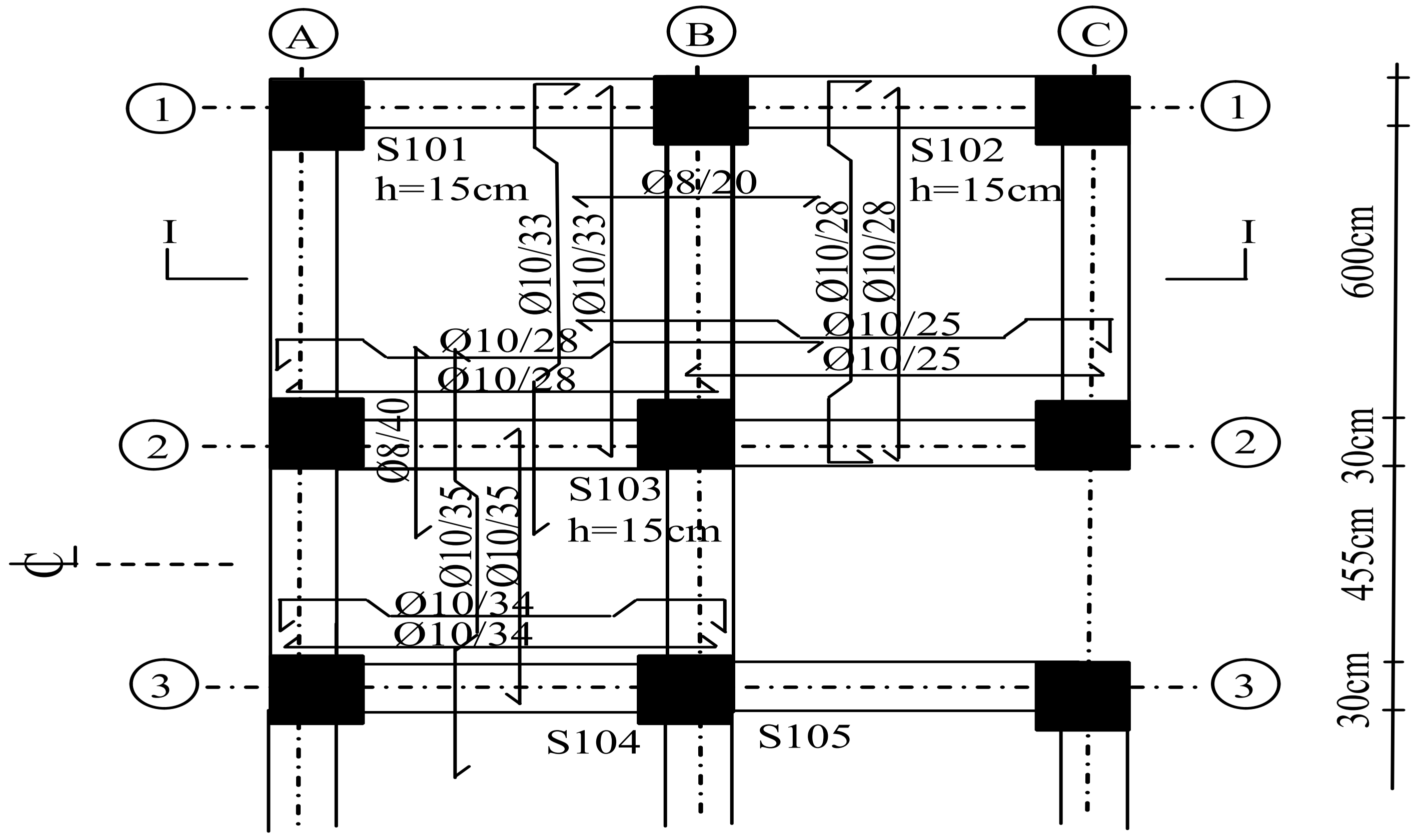
Moment	d	R	ρ	A_s	Available	Add
$13.89*10^4$	13	8.22	0.0045	5.85	$\frac{4.76+4.49}{2}=4.63$	Ø8/40 (1.26)

From the tables it can be seen that all steel ratios are higher than 0.0015 and in all slabs $\rho_x + \rho_y > 0.004$.

Twisting reinforcement:

In the corner of S101: $3*5.61/4 = 4.21\ cm^2$ selected: Ø10/18.5 (4.25)
In the corner of S102: $3*6.28/4 = 4.71\ cm^2$ selected: Ø10/16.5 (4.76)

All details are shown in Figs.1.17 and 1.18.


A
B
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2
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S101
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h=15cm
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S104
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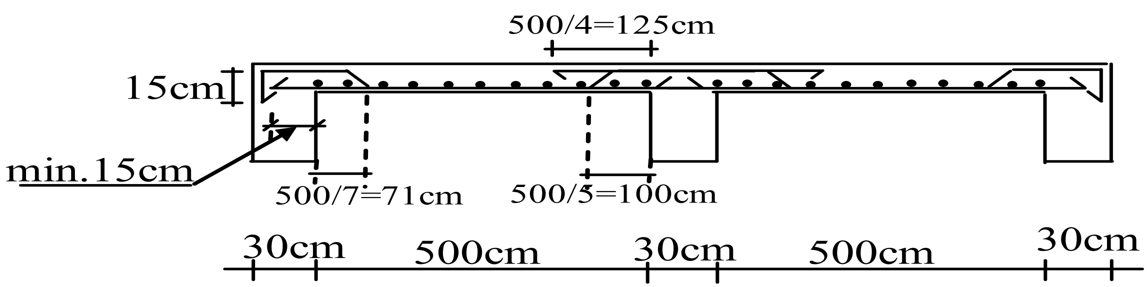
Section I-I


500/4=125cm
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min.15cm
500/7=71cm
500/5=100cm
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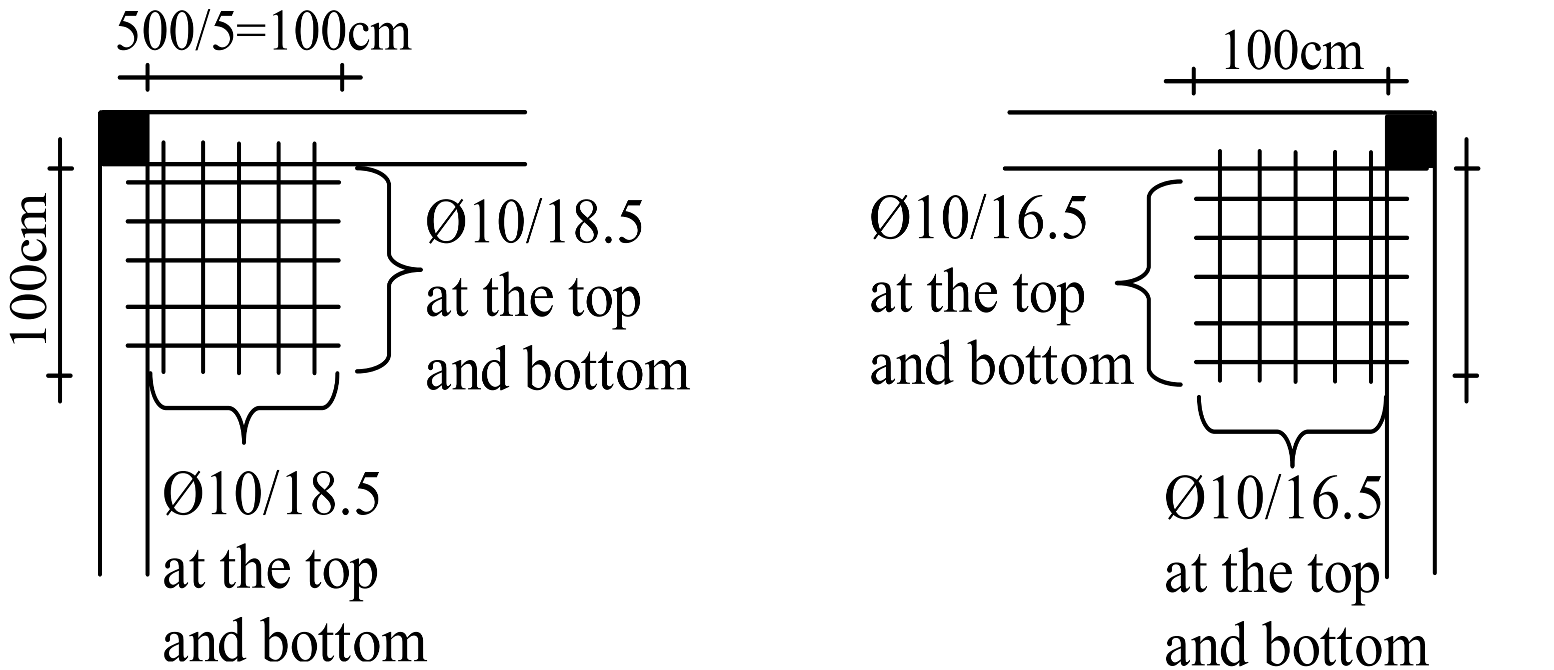
500cm
30cm
500cm
30cm



a) Twisting reinforcement at the corner of S101

b) Twisting reinforcement at the corner of S102

Figure 1.18