# Chapter 5: Present Worth Analysis

One of the methods used in evaluating the project alternatives is the Present Worth Analysis. To help formulate alternatives, projects are categorized as:

- **Mutually exclusive –** Only one of the viable projects can be selected.

- **Independent –** More than one viable project may be selected.

Alternatives to be evaluated are also two types:

**Revenue –** Each alternative generates cost (or disbursement) and revenue (or receipt) cash flow estimates, and possibly savings.

**Service –** Each alternative has only cost cash flow estimates.

In identifying alternatives, the **Do Nothing (DN)** is an alternative to be considered in the evaluation process unless it is absolutely required that one of the defined alternatives be selected.

It is to be noted that **DN is an alternative only when the alternatives are revenue type**. It should be emphasized here that **salvage value is not considered revenue** when determining the type of alternatives. Salvage value may be viewed as a cost reduction item.

It is further noted that when the alternatives are **service type**, the selection **can only be mutually exclusive** with only one selected and with no consideration for DN.

In evaluating alternatives, the following criteria are used:

**Equal-Life Alternatives**

**One alternative –** Present worth, PW, is calculated at the MARR. If PW ≥ 0, which indicates that the MARR is met or exceeded, the alternative is acceptable. If PW < 0, alternative is not acceptable and we opt for DN.
It should be obvious from this that one alternative evaluation is relevant only for the revenue type alternatives. (PW of service type alternatives are less than zero as they only include costs).

**Two or more alternatives –** Calculate the PW values of all the alternatives at the MARR.
For mutually exclusive alternatives, whether service or revenue type, **select the alternative with the PW value that is numerically largest.**
For **independent** alternatives (applicable only for revenue types), **select all** the alternatives **that have PW ≥ 0**.

**Examples:**

**1**. Mehmet plans to invest in new equipment that will improve the efficiency of his production. As a result, he expects to increase his annual net income by $12000. If the new equipment costs $60000 and has a life of 10 years, with its salvage value $5000 at that time, should he make this investment if MARR is 15% per year?

$$\begin{aligned} PW &= -60000 + 12000(P/A,15\%,10) + 5000(P/F,15\%,10) \\ &= -60000 + 12000(5.0188) + 5000(0.2472) \\ &= 1461.6 \end{aligned}$$

Since PW > 0, Mehmet should invest in the new equipment.

**2**. We plan to buy a van for delivery of our products. We can buy a European model that will have a first cost of $22000, an operating cost of $2000 per year, and a salvage value of $12000 after 3 years. Alternatively, we can buy a Japanese model that will have a first cost of $26000, an operating cost of $1200 per year, and a $15000 resale value after 3 years. At an interest rate of 15% per year, which model should we buy?

European Model:

$$\begin{aligned} PW_E &= -22,000 – 2000(P/A,15\%,3) + 12,000(P/F,15\%,3) \\ &= -22,000 – 2000(2.2832) + 12,000(0.6575) \\ &= \$-18,676 \end{aligned}$$

Japanese Model:

$$\begin{aligned} PW_J &= -26,000 – 1200(P/A,15\%,3) + 15,000(P/F,15\%,3) \\ &= -26,000 – 1200(2.2832) + 15,000(0.6575) \\ &= \$-18,877 \end{aligned}$$

Since the PW of the European model is numerically larger, we should buy the European model.

**Different-Life Alternatives**

The PW of the mutually exclusive alternatives must be compared over the same number of years. This requirement can be satisfied by either of two approaches:

- Compare the alternatives over a period of time equal to the **least common multiple (LCM)** of their lives. (The assumptions are that the service provided by the alternatives will be needed for the LCM of years and the selected alternative will be repeated over each life cycle of the LCM in exactly the same manner).

- Compare the alternatives using a **study period** of length **n** years, which does not necessarily take into consideration the useful lives of the alternatives.

Once the cash flow for each alternative is established over LCM of years, the process of evaluation is as outlined for the equal-life alternatives.

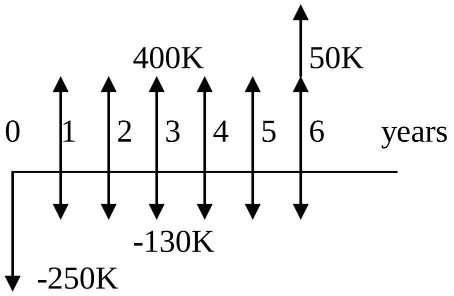
It is worth to remember that we evaluate each alternative one by one when dealing with independent alternatives and, therefore, we do not have to extend the analysis over LCM of years.

**Examples:**

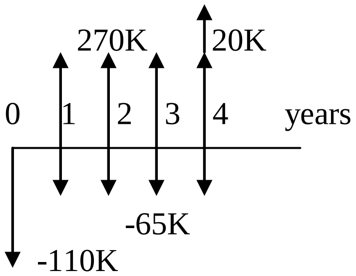
**3**. a) Which of the following alternatives should be chosen if MARR is %18 per year.

| | A ($) | B($) |
|---|---|---|
| First cost | - 250,000 | - 110,000 |
| Annual operating cost | - 130,000 | - 65,000 |
| Annual revenues | 400,000 | 270,000 |
| Salvage value | 50,000 | 20,000 |
| Life | 6 | 4 |

b) If the projects are independent, what will be your decision?



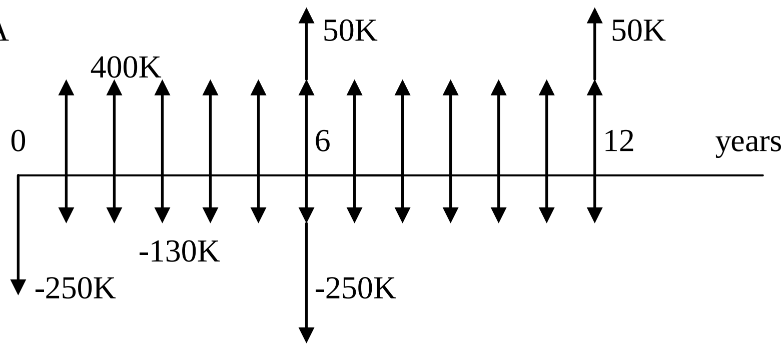

Cash flow diagram for alternative A



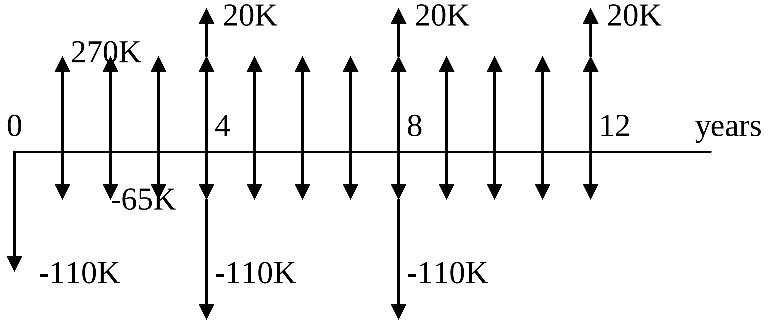

Cash flow diagram for alternative B

We have different lives for the alternatives. Comparison must be made over the same period, i.e. LCM of years since a study period is not stated. For these alternatives LCM = 12. Therefore, cash flows must be extended to 12 years by assuming that costs and revenues of Project A is repeated for another cycle (total of two cycles) and of Project B, another two cycles (total of three cycles). The resultant cash flows are as follows:

Cash flow for A



Cash flow for B




Then,

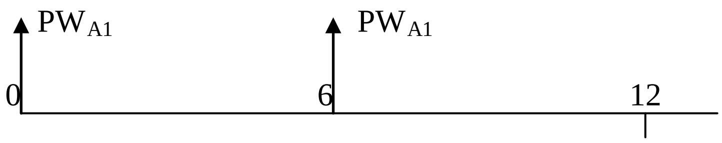
$PW_A$ = -250000 – 250000(P/F,18%,6) + (400000 – 130000)(P/A,18%,12)
+ 50000(P/F,18%,6) + 50000(P/F,18%,12)
= 976944

$PW_B$ = -110000 – 110000(P/F,18%,4) – 110000(P/F,18%,8)
+ (270000 – 65000)(P/A,18%,12) + 20000(P/F,18%,4) + 20000(P/F,18%,8)
+ 20000(P/F,18%,12)
= 804988

We select A as its PW is larger.

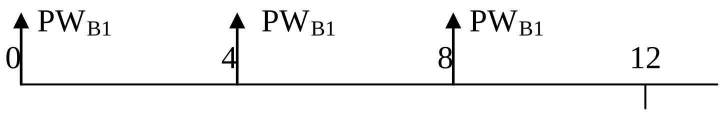
The solution can also be reached by the following method:

If we calculate the PW of each project for one cycle only, the cash flows reduce to:




Cash flow diagram for A where $PW_{A1}$ is the PW of one cycle only given by,

$PW_{A1}$ = -250000 + (400000 – 130000)(P/A,18%,6) + 50000(P/F,18%,6) = 712873.5




Cash flow diagram for B where $PW_{B1}$ is the PW of one cycle only given by,

$PW_{B1}$ = -110000 + (270000 – 65000)(P/A,18%,4) + 50000(P/F,18%,4) = 451786

Then,
$PW_A$ = $PW_{A1}$[1 + (P/F,18%,6)] = 712873.5(1 + 0.37043)
= 976943

$PW_B$ = $PW_{B1}$[1 + (P/F,18%,4) + (P/F,18%,8)] = 451786(1 + 0.5158 + 0.2660)
= 804992

We have the same result.

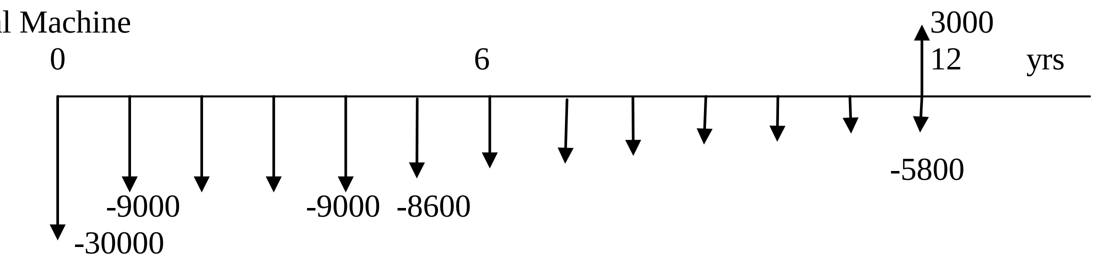
(b) If projects are independent, we select both as their PW > 0.

It is to be **noted that** the PW of one cycle is enough to make a decision. However, if the PW for the LCM of years is already obtained as in the initial solution of (a), this value can also be used for making a decision. (If PW > 0 for one cycle, this is true for more than one cycle also).

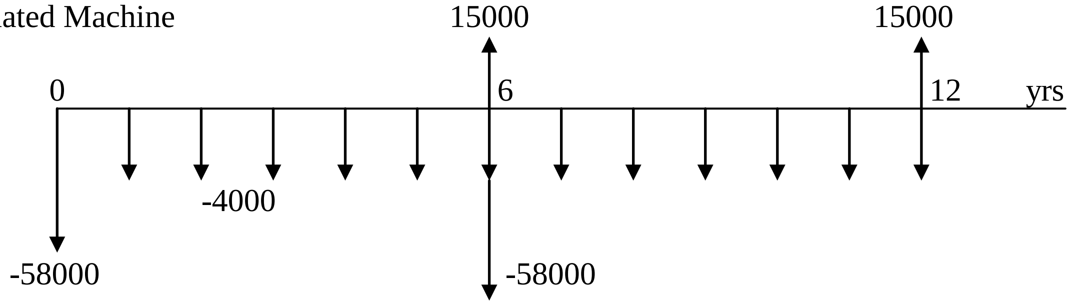
**4**. A mining company is considering purchasing a machine which costs $30 000 and is expected to last 12 years, with a $3 000 salvage value. The annual operating expenses are expected to be $9 000 for the first 4 years, but owing to decreased use, the operating costs will decrease by $400 per year for the next 8 years. Alternatively, the company can purchase a highly automated machine at a cost of $58 000. This machine will last only 6 years, and its salvage value will be $15 000. Its operating cost will be $4 000 per year. If MARR is 20% per year, which machine should be selected on the basis of a present-worth analysis?

The lives are 12 and 6 years. Therefore, comparison has to be made over LCM (=12) years since a study period is not stated. The cash flow diagrams for the two alternatives over 12 years are as follows:

Normal Machine




Automated Machine




Let A : Normal machine and B : Automated machine

For Machine A:

$PW_A$ = -30000 – 9000.(P/A,20%,12) + 400.(P/G,20%,9).(P/F,20%,3) + 3000.(P/F,20%,12)
= - 66968.4

or,

$PW_A$ = -30000 – 9000.(P/A,20%,4) – [8600.(P/A,20%,8) - 400.(P/G,20%,8)].(P/F,20%,4) + 3000.(P/F,20%,12)
= - 66968.4

For Machine B:

$PW_B$ = - 58000 – 58000.(P/F,20%,6) + 15000.(P/F,20%,6) + 15000.(P/F,20%,12) – 4000.(P/A,20%,12)
= - 88473.7

Select machine A.

5. Company X is considering the Projects that have the following costs:

| | Project A | Project B |
|---|---|---|
| First cost | $15000 | $18000 |
| Annual Operating Costs | 3500 | 3100 |
| Salvage Value | 1000 | 2000 |
| Life, years | 6 | 9 |

(a) Using a MARR value of 15% per year, determine which alternative should be selected on the basis of a present-worth analysis.
(b) Company X has a standard practice of evaluating all projects over a 5-year period. If a study period of 5 years is used and the salvage values are expected to be $1500 and $4000 for A and B respectively, which project should be selected?

(a) Evaluation is to be carried out over 18 (LCM) years since a study period is not given.

$PW_A$ = -15000[1 + (P/F,15%,6) + (P/F,15%,12)] + 1000[(P/F,15%,6) + (P/F,15%,12) + (P/F,15%,18)] – 3500(P/A,15%,18)
= -45036

$PW_B$ = -18000[1 + (P/F,15%,9)] + 2000[(P/F,15%,9) + (P/F,15%,18)] –3100(P/A,15%,18)
= -41384

Select B as its PW is numerically larger.

(b) The study period is now stated to be 5 years. We, therefore, have to limit the evaluation to the study period, i.e. 5 years. (We need not evaluate over LCM of years).

Then,

$PW_A$ = -15000 – 3500(P/A,15%,5) + 1500(P/F,15%,5)
= -25987

$PW_B$ = -18000 – 3100(P/A,15%,5) + 4000(P/F,15%,5)
= -26403

The selection is now A.

**Future Worth**

Evaluations are carried out in a similar manner to the PW calculations.

**Capitalized Cost**

Capitalized Cost (CC) is the present worth of an alternative that will last forever. If we consider the formulation for the P/A factor and let n go to infinity, we get:

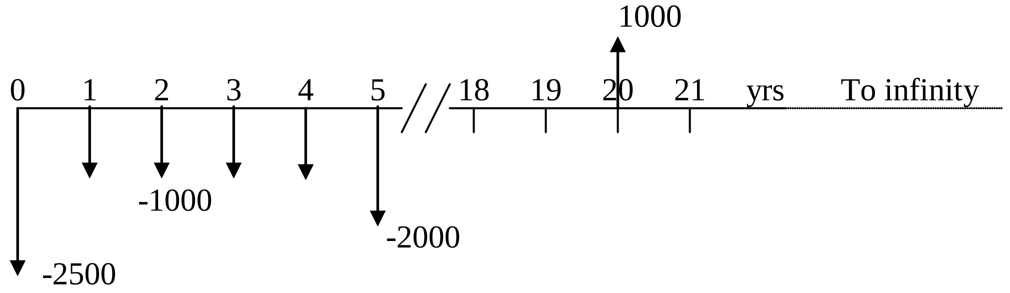
Present worth, **P = CC = A/i**

**Note: In this formulation, the period of A and the interest period must be the same, i.e. if payment period for A is monthly, then i has to be the rate per month, etc.**

In calculations of capitalized costs we have to deal with recurring or periodic and nonrecurring types of cash flows. The following shows the capitalized cost calculations for such cash flows.

**Nonrecurring type of cash flows in a project that lasts forever:**

Supposing interest rate is 10%/year and the cash flows are as shown.



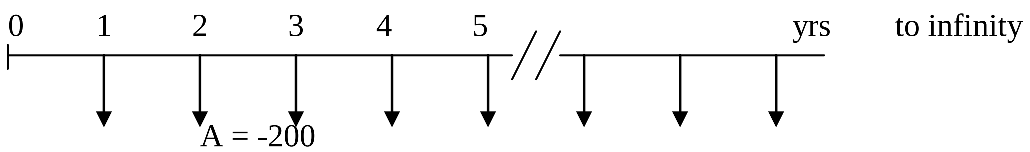

Since the life extends to infinity, the PW of these amounts are their capitalized costs.

CC = -2500 – 2000(P/F,10%,5) – 1000(P/A,10%,4) + 1000(P/F,10%,20)

**Recurring type of cash flows – infinite life**

Considering the following equal amount annual amounts with the interest rate at 10% per year.

i = 10% per year

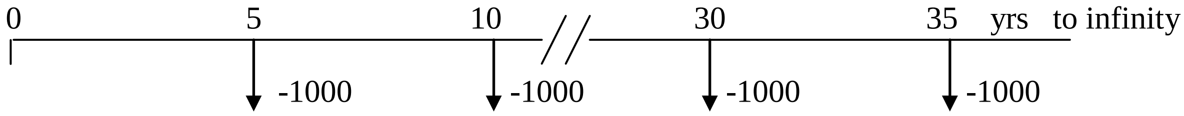



CC = A/i = -200/0.1 = -2000

**Note: The period of A and the interest period are the same, i.e. year in this case.**

Considering now the following:

i = 10% per year




This shows a periodic cash flow. The relationship, CC = A/i, gives the capitalized cost for such cash flows. However, in using this relationship we have to **ensure that A and i have the same period**, i.e. if A is yearly, interest rate has to be per year, or if A is monthly, then i has to be the rate per month, etc. In this case the period for A is 5 years and the interest rate is per year. Therefore, to be able to use the relationship CC = A/i, we either (i) have to use the interest rate per 5-year or, (ii) we have to convert the 5-yearly cash flow into a yearly periodic cash flow.

(i) The interest rate/5-year = $(1 + 0.5/5)^5 - 1 = 0.6105$ or 61.05% per 5-year

Then, using the 5-yearly values for A and i,

CC = -1000/0.6105 = -1638

(ii) If we consider the first cycle (year zero to year 5) in the above cash flow, we can annualize it by,

A = -1000(A/F,10%,5) = -1000(0.1638) = - 163.8

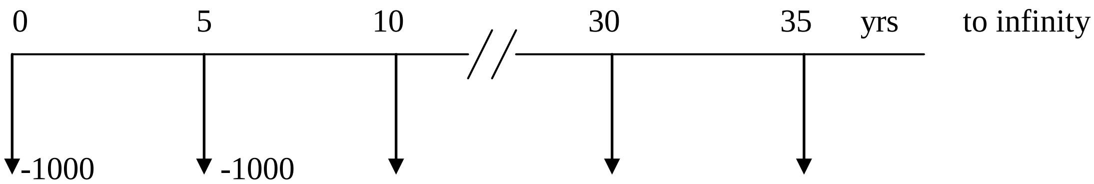
This now represents annual equivalent (from year 1 to year 5) of –1000 at year 5.

If we now annualize –1000 at year 10 between the years 5 and 10, and -1000 at year 15 between the years 10 and 15, and continue, we can see that –163.8 is the annual equivalent of the above 5-yearly cash flow from year 1 to infinity. Then, using yearly values for A and i,

CC = A/i = -163.8/0.1 = -1638

If the cash flow starts at year zero as shown below, then the procedure is similar to the above:

i = 10% per year




The only difference between this cash flow and the above is the amount at year zero. The CC of the amount at year zero is –1000. Therefore, following the above procedure, we obtain

CC = -1000 –1000/0.6105 or = -1000 – 1000(A/F,10%,5)/0.1
= - 2638

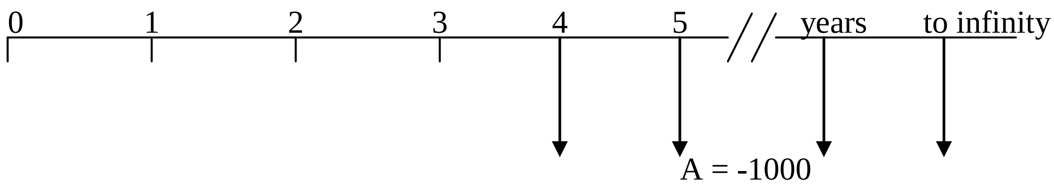
Alternatively, we can annualize the –1000 amounts by using the (A/P) factor since the amount is always at the start of the cycle:

$$A = -1000(A/P,10\%,5) = -1000(0.2638) = -263.8 \quad \text{(every year from year 1 to infinity)}$$

Then, $CC = -263.8/0.1 = -2638$

Finally, we may have periodic amounts that are shifted:

i = 12% per year



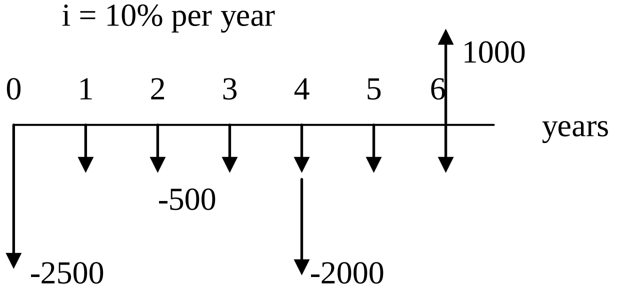

Capitalized cost of the periodic amounts, CC1 = -1000/0.12
However, this value is at year3. Then, at year zero,

$$CC = -1000(P/F,12\%,3)/0.12$$
$$= -5931.67$$

**Capitalized cost of a project that has finite life**

Considering, as an example, a project that has the following cash flow with the interest rate at 10% per year.

i = 10% per year




The life is given as 6 years. If we wish to find the capitalized cost of this project, then it has to be assumed that the project cash flow extends into infinity, i.e. the cycle of the cash flow is repeated into infinity. In such a configuration the cash flow becomes recurrent type (repeated every six years). Based on the above analysis, if we determine the annual equivalent of one cycle, we effectively determine the periodic cash flow into infinity. Then,

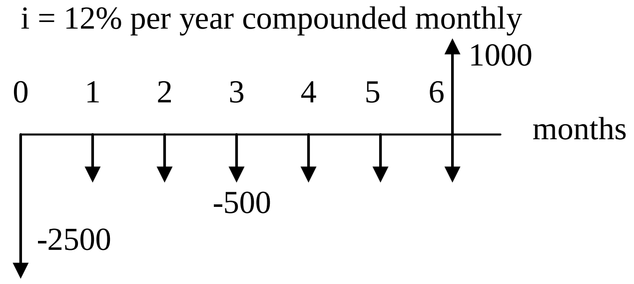
$$A = [-2500 - 2000(P/F,10\%,4)](A/P,10\%,6) + 1000(A/F,10\%,6) - 500$$

**or**

$$= -2500(A/P,10\%,6) + [1000 - 2000(F/P,10\%,2)](A/F,10\%,6) - 500$$
$$= -1258.06$$

Then, $CC = -1258.06/0.1 = -12580.6$

We now consider the following example:




In this example the payment period is now monthly. We, therefore, have to use the monthly interest rate in the capitalized cost calculations. Here, monthly rate is 1% per month. Then,

A = -2500(A/P,1%,6) + 1000(A/F,1%,6) – 500
= - 768.825

and, CC = -768.825/0.01 = -76882.5

We can compare alternatives using Capitalized Cost method. The following are some examples dealing with CC and comparisons using CC method.

**Examples:**

**6**. Determine the capitalized cost of $100000 at time 0, $25000 in years 1 through 5, and $50000 per year from year 6 on. Interest rate = 10% per year.

CC = -100,000 – 25,000(P/A,10%,5) – 50,000(P/F,10%,5)/0.10

= -100,000 – 25,000(3.7908) – 500,000(0.6209)

= $-505,220

**7**. An investor claims he can earn 15% per year. If he invests $10000 now, $30000 three years from now, and $8000 per year for 5 years starting 4 years from now, how much money can he withdraw every year forever, beginning 12 years from now?

We first determine the future value, F11, of the investment in year 11.

F11 = 10,000(F/P,15%,11) + 30,000(F/P,15%,8) + 8000(F/A,15%,5) (F/P,15%,3)

= 10,000(4.6524) + 30,000(3.0590) + 8000(6.7424)(1.5209)

= $220,330

From year 12 onwards, he can withdraw forever the amount:

A = P i = 220,330(0.15)

= $33,050 per year

**8**. We have two options to consider for providing water supply to a city. One involves a pipeline at a cost of $200 million and operating costs of $6 million per year, and the other involves construction of a canal at a cost of $325 million and maintenance costs of $1 million per year. If both facilities are expected to last forever, which should be built at an interest rate of 10% per year?

$$CCp = -200 - 6/0.10$$
$$= \$-260 \text{ million}$$

$$CCc = -325 - 1/0.10$$
$$= \$-335 \text{ million}$$

Select the pipeline.

**9**. Compare the alternatives shown below, using an interest rate of 15% per year.

| | New Machine | Used Machine |
|---|---|---|
| First cost, $ | 44000 | 23000 |
| AOC, $ | 7000 | 9000 |
| Overhaul every 2 years, $ | - | 1900 |
| Overhaul every 5 years, $ | 2500 | - |
| Salvage value, $ | 100000 | 3000 |
| Life, years | ∞ | 7 |

New Machine: Since life is infinite, we can calculate CC directly,

$$CC_N = -44000 - 7000/0.15 - 2500(A/F,15\%,7)/0.15 + 100000(0.0)$$
$$= -93138.67$$

Used Machine: Since life is finite, we need to calculate the AW of one cycle cash flow:

$$AW_U = -23000(A/P,15\%,7) - 9000 + 3000(A/F,15\%,7)$$
$$- 1900[(P/F,15\%,2) + (P/F,15\%,4) + (P/F,15\%,6)](A/P,15\%,7)$$
$$= -15061.06$$

Then,

$$CC_U = -15061.06/0.15$$
$$= -100407.03$$

We select the New Machine.

**10**. Compare the alternatives below on the basis of their capitalized costs using an interest rate of %10 per year compounded semiannually.

| | A ($) | B($) | C($) |
|---|---|---|---|
| First cost | - 50,000 | - 300,000 | - 900,000 |
| Maintenance cost per Semiannual period | - 30,000 | - 10,000 | - 3,000 |
| S.V | 5000 | 70,000 | 200,000 |
| Life (years) | 2 | 4 | $\infty$ |

The payment period for the periodic amounts is six months. We, therefore, have to measure **n** in terms of six-months and also use the interest rate applicable to six-month period (= 5% per six-month in this case).

As the alternatives have finite lives, we determine their AW's for one cycle prior to calculating their CC.

Alternative A with n = 4 (4 six-months in 2 years):

$$AW_A = -50{,}000(A/P,5\%,4) - 30{,}000 + 5000(A/F,5\%,4)$$
$$= -50{,}000(0.282) - 30{,}000 + 5000(0.232)$$
$$= \$-42940$$

Then, $CC_A = -42940/0.05 = \$-858800$

Alternative B with n = 8:

$$AW_B = -300{,}000(A/P,5\%,8) - 10{,}000 + 70{,}000(A/F,5\%,8)$$
$$= -300{,}000(0.1547) - 10{,}000 + 70{,}000(0.1047)$$
$$= \$-49081$$

and $CC_B = -49081/0.05 = \$-981620$

Alternative C:

The CC of the salvage value for C is zero, since n is infinity.

$$CC_C = -900{,}000 - 3{,}000/0.05$$
$$= \$-960000$$

Select Alternative A.

**11**. Use the capitalized costs method to choose the best alternative. Interest rate is %9 per year compounded monthly.

| **ALTERNATIVE** | **A** | **B** | **C** |
|---|---|---|---|
| first cost $ -------------- | -65000 | -325000 | -975000 |
| monthly maintenance cost $------------------------ | -35000 | -25000 | -20000 |
| recurring overhaul every 4 years $ -------------------- | ------ | ------- | -70000 |
| salvage value------------------ | 10000 | 30000 | 300000 |
| life, years | 6 | 30 | ∞ |

The time period to use now is month. Accordingly, n and interest rate are monthly bases.

%9 per year compounded monthly = %0.75 per month

$A_A$= - 6500(A/P,%0.75,72) – 35000 +10000(A/F, %0.75,72) = - 36067 $

$CC_A$= -36067/0.0075 = - 4,808,887 $

$A_B$= -325000(A/P, %0.75,360) – 25000 + 30000(A/F, %0.75,360) = -27600 $

$CC_B$= -27600/0.0075= -3,680,000 $

$CC_C$= - 975000 – 20000/0.0075 – 70000(A/F, %0.75,48) (1/0.0075)= - 3,803,973 $

Choose lowest cost which is alternative B.