Chapter 6: Annual Worth Analysis

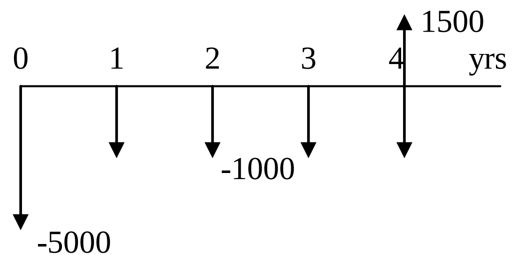
Annual Worth (AW) analysis is another method of comparing alternatives. It is often the preferred approach.

The AW value, which has the same economic interpretation as A used thus far, is related to PW and FW values by,

AW = PW.(A/P,i,n) = FW.(A/F,i,n)

where **n** is the number of years for equal-service comparison, i.e.**LCM** or the stated study period of the PW analysis.

Let us now consider the following cash flow for a project that is to be compared against another project that has a life of 6 years:



To compare this against the alternative with a 6-year life, the analysis needs to be extended over 12 years (=LCM) for equal-service comparison. Therefore, we have to extend the cash flow of this alternative to 12 years, i.e. cycle has to be repeated two more times (total of 3 cycles). As also outlined before in capitalized cost calculations, whether we use one cycle, or two cycles, or three or more cycles of this cash flow in calculating the AW value, we get the same value. That is, the **AW value obtained by using only one cycle of cash flow applies for every year of the life cycle, and for each additional life cycle. Then,**

The AW value has to be calculated for <u>only one life cycle</u>. Therefore, it is not necessary to use the LCM of lives, as it is for PW and FW analyses.

The procedure in evaluating the alternative(s) is then similar to those for the PW method. The major difference being that AW value will now be calculated by considering the cash flow for one cycle only.

Examples:

1. Company ABC plans to purchase new equipment to improve productivity. The equipment cost is $25000 and is expected to have a market value of $5000 at the end of its 5-year life. If the expected improvement in productivity will net $8000 per year and Company's MARR is 20% per year, should the Company purchase this equipment?

AW = -25000(A/P,20%,5) + 8000 + 5000(A/F,20%,5)
= 312.40

Since AW > 0, then the equipment should be purchased.

2. Projects that have the following costs are under consideration:

	Project A	Project B
First cost	$62000	$77000
Annual Operating Costs	15000	21000
Salvage Value	8000	10000
Life, years	4	6

Using an interest rate of 15% per year, determine which alternative should be selected on the basis of annual worth analysis.

We only need to consider one cycle of cash flow to calculate AW values:

$$AW_A = -62000(A/P,15\%,4) - 15000 + 8000(A/F,15\%,4)$$
$$= -35114.58$$

$$AW_B = -77000(A/P,15\%,6) - 21000 + 10000(A/F,15\%,6)$$
$$= -40204.08$$

We select A because its AW value is numerically larger.

3. A firm has available three investment proposals A, B, and C having cash flow profiles below. The firm has a MARR of 1.53095% per month.

	A	B	C
Initial investment	$200 000	$300 000	$150 000
Annual receipts (income)	160 000	190 000	200 000
Annual costs	100 000	110 000	150 000
Salvage value	50 000	100 000	50 000
Life, years	8	11	7

Using the annual worth analysis, (a) determine the preferred proposal. (b) if the projects are independent, which of them should be selected?

Annual rate for MARR = $(1 + 0.0153095)^{12} - 1 = 0.2$ or 20% per year.

(a) Considering only one cycle of cash flow for each alternative:

$$AW_A = -200000.(A/P,20\%,8) + (160000 - 100000) + 50000.(A/F,20\%,8)$$
$$= -200000 \times 0.2606 + 60000 + 50000 \times 0.0606$$
$$= 10910$$

$$AW_B = -300000.(A/P,20\%,11) + (190000 - 110000) + 100000.(A/F,20\%,11)$$
$$= 13780$$

and

$$AW_C = -150000.(A/P,20\%,7) + (200000 - 150000) + 50000.(A/F,20\%,7)$$
$$= 12258$$

Select B since its AW is numerically largest and also, since it is better than DN.

(b) Select all three since AW > 0 for each one.

4. Alternative methods I and II are proposed for a plant operation. The following is comparative information:

	Method I	Method II
Initial investment, $	10,000	40,000
Annual Disbursements:		
- Labor, $	12,000	4,000
- Power, $	250	300
- Rent, $	1,000	500
- Maintenance, $	500	200
- Property Taxes and Insurance, $	400	2000
Salvage Value, $	1,000	5,000
Life, years	5	10

All other expenses are equal for the two methods, and income from the operation is not affected by the choice. If the MARR is 15% per year, which alternative is the better choice?

Using AW method is easiest:

$AW_I = - 10000.(A/P,15\%,5) – (12000 + 250 + 1000 + 500 + 400) + 1000.(A/F,15\%,5)$
$= - 16984.88$

$AW_{II} = - 40000.(A/P,15\%,10) – (4000 + 300 + 500 + 200 + 2000) + 5000.(A/F,15\%,10)$
$= - 14723.75$

Select II.

5. Ajax Company is designing a processing facility. Two options are being considered with the following cash flows:

	A	B
Capital investment	$33200	$47600
Annual expenses:		
Electrical energy	$2165	$1720
Maintenance	$1100 in year 1, and increasing $500/yr thereafter	$500 in year 4, and increasing $100/yr thereafter
Salvage value	0	$5000
Useful life, years	5	9

The MARR is 20% per year. (a) Using annual worth method, which option should be selected? (b) What will be the present worth of each option?

(a) $AW_A = -33200.(A/P,20\%,5) –2165 –[1100 + 500.(A/G,20\%,5)]$
$= - 15187$

$AW_B = -47600(A/P,20\%,9) + 5000.(A/F,20\%,9) – 1720$
$– [500.(P/A,20\%,6) + 100.(P/G,20\%,6)].(P/F,20\%,3).(A/P,20\%,9)$
$= -13622$

Select option B.

(b) For the PW comparison we have to use LCM of years, which is 45 here. Then, as indicated earlier,

$$PW = AW(P/A,i,n) = AW(P/A,20\%,45)$$

and, applying to A and B:

$$PW_A = -15187.(P/A,20\%,45) = -15187x4.9986 = -75913.74$$
$$PW_B = -13622.(P/A,20\%,45) = -68090.93$$

Select option B.

Note: It is worth remembering this method of calculating PW of the alternatives. It is the quickest and easiest way of calculating PW of the alternatives when lives are different.

6. Compare the alternatives below on the basis of present worth using an interest rate of 14.224% per year compounded quarterly.

	A	B
First cost, $	45000	24000
Annual operating cost, $/year	31000	35000
Overhaul (cost) in years 2 and 4,	-	6000
Overhaul (cost) in year 5, $	12000	-
Salvage Value (receipt), $	10000	8000
Life, years	8	6

$$i/year = (1 + 0.14224/4)^4 – 1 = 0.15 \quad \text{or 15\% per year.}$$

AW_A = -45,000(A/P,15%,8) – 31,000 – 12,000(P/F,15%,5)(A/P,15%,8) +10,000(A/F,15%,8)
= -45,000(0.22285) – 31,000 – 12,000(0.4972)(0.22285) + 10,000(0.07285)
= $-41,629

Since LCM of lives = 24,
PW_A = AW_A(P/A,15%,24) = -41629(6.4338)
= $-267832.7

AW_B = -24,000(A/P,15%,6) – 35,000 – 6000[(P/F,15%,2) + (P/F,15%,4)](A/P,15%,6) + 8000(A/F,15%,6)
= -24,000(0.26424) – 35,000 – 6,000[0.7561 + 0.5718](0.26424) + 8000(0.11424)
= $-42,533

PW_B = AW_B(P/A,15%,24) = -42533(6.4338) = $-273648.8

Select A.

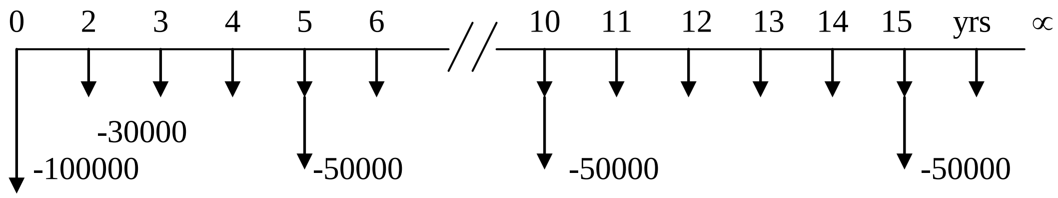
AW of a Permanent Investment

This is the annual worth equivalent of the capitalized cost. For this type of analysis, the AW of the initial investment of a project of infinite life is the perpetual annual interest earned on the initial investment, i.e. A = Pi.

Cash flows recurring at regular or irregular intervals are handled exactly as in conventional AW computations; they are converted to equivalent uniform annual amounts 'A' for one cycle. This automatically annualizes them for each succeeding life cycle.

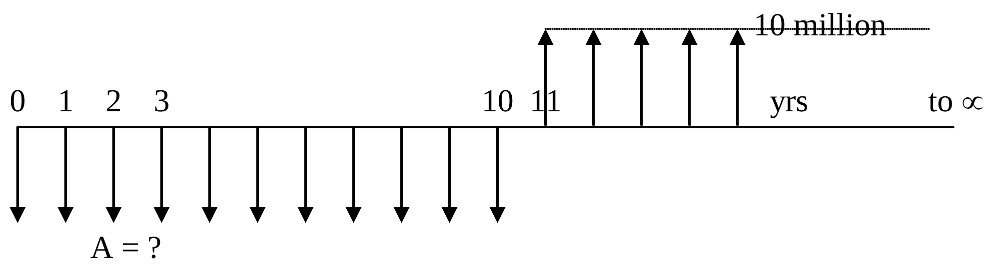
Examples:

7. The cash outflow for a project having infinite life is as follows: $100000 now, $30000 each year, and an additional $50000 every 5 years. Determine its perpetual equivalent annual worth at an interest rate of 20% per year.



AW = -100,000(0.20) – 30,000 – 50,000(A/F,20%,5)
= -20,000 – 30,000 – 50,000(0.13438)
= $-56,719 per year

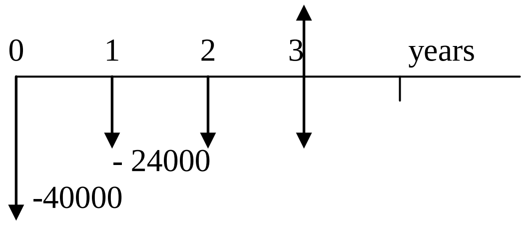
8. We wish to establish a research fund by making equal annual deposits, starting now and for 10 more years, so that $10 million per year will be available for research. If the first research grant is to be awarded 11 years from now, how much should we deposit every year if the fund will generate income at a rate of 15% per year?



First find P in year 10 for the $10 million annual amounts and then use the A/F factor to find A ($ in million):

P10 = -10/0.15 = $-66.667
A = -66.667(A/F,15%,11) = -66.667(0.04107)
= $-2,738,000 per deposit

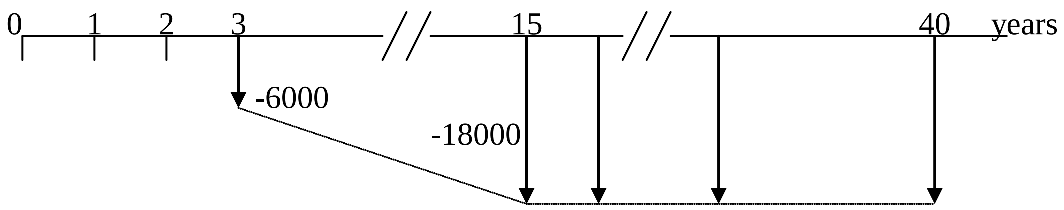
9. We have $40000 initial investment, $24000 per year annual costs, and a $6000 salvage value for a project having 3 years life. Determine the perpetual equivalent annual worth at an interest rate of 20% per year.



The perpetual uniform annual worth is the AW on one life cycle:

AW = -40,000(A/P,20%,3) – 24,000 + 6000(A/F,20%,3)
= -40,000(0.47473) – 24,000 + 6000(0.27473)
= $-41,341

10. The cost of maintaining a road started 3 years after it was built with a cost of $6000. From then on it increased by $1000 per year through year 15, after which they remained constant through the road's 40-year life. Assuming it is replaced with a similar road, what is the perpetual equivalent cost, at an interest rate of 10% per year?



First find PW and then convert to AW:

PW = -[6000(P/A,10%,13) + 1000(P/G,10%,13)](P/F,10%, 2)
– 18,000(P/A,10%,25) (P/F,10%,15)
= -[6000(7.1034) + 1000(33.3772)](0.8264) – 18,000(9.0770)(0.2394)
= $-101,919

AW = -101,919(A/P,10%,40) = -101,919(0.10226)
= $-10,422

11. Company X has two options, A and B, for the manufacture of switches. The estimated cash flows for each option are given below. (a) Use an annual worth comparison to determine which option is preferable at an interest rate of 19.09% per year compounded semiannually. (b) If the options are independent, determine which are economically acceptable. (All dollar values are in millions).

	A	B
First cost	30	10
Annual cost, $/year	5	1.2
Annual income, $/year	14	6
Upgrade costs every 5 years	-	2
Salvage value	7	5
Life, years	10	Infinity

$i/year = (1 + 0.1909/2)^2 – 1 = 0.2$ or 20% per year

(a) AW_A = -30(A/P,20%,10) – 5 + 14 + 7(A/F,20%,10)
= -30(0.23852) + 9 + 7(0.03852)
= 2.114M ($2114000)

AW_B = -10(0.20) – 1.2 + 6 – 2(A/F,20%,5) + 5(0.0)
= - 2.0 + 4.8 – 2(0.13438)
= 2.5312M ($2531200)

Select B.

(b) Both options have AW > 0; therefore, both are acceptable at i = 20% per year.

12. For the cash flow below, use an annual worth comparison to determine which alternative is best at an interest rate of %10 per year compounded semiannually.

	A ($)	B($)	C($)
First cost	- 50,000	- 300,000	- 900,000
Maintenance cost per Semiannual period	- 30,000	- 10,000	- 3,000
S.V	5000	70,000	200,000
Life (years)	2	4	∞

The payment period for the periodic amounts is six months. We, therefore, have to measure **n** in terms of six-months and also use the interest rate applicable to six-month period (= 5% per six-month in this case).

Alternative A with n = 4 (4 six-months in 2 years):

AW_A = -50,000(A/P,5%,4) – 30,000 + 5000(A/F,5%,4)
= -50,000(0.282) – 30,000 + 5000(0.232)
= $-42940

Alternative B with n = 8:

AW_B = -300,000(A/P,5%,8) – 10,000 + 70,000(A/F,5%,8)
= -300,000(0.1547) – 10,000 + 70,000(0.1047)
= $-49081

Alternative C:

AW_C = -900,000(0.05) - 3000
= $-48000

Select Alternative A.