Chapter 1: Engineering Economy

Fundamentally, engineering economy involves formulating, estimating, and evaluating the economic outcomes when alternatives to accomplish a defined purpose are available.

In carrying out engineering economy analyses, the selection criterion is based on 'measure of worth' of the alternatives. Some measures of worth are, Present Worth (PW), Annual Worth (AW), Future Worth (FW), Rate of Return (ROR), Benefit/Cost ratio (B/C), etc.

When determining a measure of worth, the fact that money today is worth a different amount in the future is considered; that is, the time value of money is accounted for.

The change in the amount of money over a given time period is called the time value of money; it is the most important concept in engineering economy.

Interest Rate and Rate of Return

Interest is the manifestation of the time value of money. It is the difference between an ending amount of money and the beginning amount. For example, if I had borrowed $1000 a year ago and I paid back $1100 today, then the interest is $100.

Interest = amount to be paid back – original amount

and, Interest Rate = (interest accrued per time unit/original amount)x100%

The Rate of Return also has the same definition:

Rate of Return = (interest accrued per time unit/original amount)x100%

The time unit of the rate is called the interest period.

The term interest rate paid is more appropriate for the borrower's perspective, and the rate of return earned is better for the investor's perspective.

Examples:

1. Ali borrowed $10000 today to purchase a car. He must repay $10700 a year from now. Determine the interest amount and the interest rate paid.

Interest = 10700 – 10000 = $700

Percent interest rate = (700/10000)x100% = 7% per year.

2. Company X has just completed a project. In the project, the company realized a rate of return of 32% per year. If the company investment in the project was $6 million, what was the amount of profit the company made in the first year?

Profit = 6,000,000 (0.32)
= $1,920,000

3. Company Y has just paid off a loan that it received a year ago. If the total amount of money paid was \$1.6 million and the interest rate on the loan was 12% per year, how much money had the company borrowed?

$$1{,}600{,}000 = P + P\,(0.12) \text{ where P is the amount the company borrowed.}$$
$$1.12P = 1{,}600{,}000$$
$$P = \$1{,}428{,}571$$

Equivalence

Economic equivalence means that different sums of money at different times are equal in economic value. It is related to the time value of money and the interest rate. For example, if the interest rate is 10% per year, \$100 today is equivalent to \$110 one year from today. It is derived from:

$$100 + 0.1(100) = 110$$

Examples:

4. Company Z has to decide whether they should replace their office furniture now or a year from now. If they replace now, it will cost them \$14500, and if they do it next year, it will be \$16000. At an interest rate of 12% per year, should they do it now or next year?

$$\text{Equivalent future cost} = 14{,}500(1.12)$$
$$= \$16{,}240$$

The company should buy next year. (the future cost of \$14500 is greater than \$16000).

5. An investment of \$40000 one year ago and \$50000 one year from now are equivalent at what interest rate?

Value of \$40000 today = $40000(1 + i)$
Value of \$50000 today = $50000/(1 + i)$
Then, $40000(1 + i) = 50000/(1 + i)$

Alternatively, value of \$40,000 two years ahead will be equal to \$50,000:

$$40{,}000\,(1 + i)\,(1 + i) = 50{,}000$$

Then,

$$(1 + i)^2 = 50{,}000/40{,}000$$

$$i = 0.118 \quad \text{or} \quad = 11.8\% \text{ per year}$$

Simple and Compound Interest

Simple interest is calculated using the principal only, ignoring any interest accrued in preceding interest periods:

Interest = (principal)(number of periods)(interest rate)

As an example, we borrow $1000 for 3 years at 10% per year simple interest. How much money do we have to pay back at the end of 3 years?

Total interest = (1000)(3)(0.1) = 300
Amount due after 3 years = 1000 + 300 = $1300

For **compound interest**, the interest accrued for each interest period is calculated on the principal plus the total amount of interest accumulated in all previous periods:

Interest = (principal + all accrued interest)(interest rate)

As an example, we now borrow $1000 for 3 years at 10% per year compound interest. How much do we pay at the end of 3 years?

Year 1 interest = 1000(0.1) = 100

Total amount due after year 1 = 1000 + 100 = 1100

Total amount due after year 2 = 1100 + 1100(0.1) = 1210

Total amount due after year 3 = 1210 + 1210(0.1) = 1331

Examples:

6. Bank X is offering compound interest of 5% per year on savings. Bank Y is offering 6% per year simple interest on a 3-year deposit. Which offer is more attractive if we wish to keep our savings in the bank for 3 years?

Assume a principal amount of $1,000:

Compound Amount in 3 years = $1,000\ (1 + 0.05)^3$
= $1157.63

Simple Amount in 3 years = 1000 + 1000 (0.06)(3)
= $1180

Simple interest offer is better.

7. An investment that was made 16 years ago is now worth $300000. How much was the initial investment at an interest rate of 10% per year (a) simple interest and (b) compound interest?

(a) P + P (0.10)(16) = 300,000
2.6 P = 300,000
P = $115,385

(b) $P\ (1 + 0.10)^{16}$ = 300,000
P = $65,289

Terminology and Symbols

The equations and procedures of engineering economy utilize the following terms and symbols.

P = value or amount of money at present or time 0. Also referred to as present worth (PW), present value (PV), etc.
F = value or amount of money at some future time. Also referred to as future worth (FW) or future value (FV).
A = series of consecutive, equal, end-of-period amounts of money. Also, A is called annual worth (AW), equivalent uniform annual worth (EUAW).
n = number of interest periods; years, months, days.
i = interest rate or rate of return per time period; % per year, % per month, etc.
t = time, stated in periods; years, months, days.

Examples:

8. I plan to borrow $10000 now to buy a car. I plan to pay the entire principal plus 8% per year interest after 5 years. Identify the engineering economy symbols involved and their values.

P = $10000   i = 8% per year   n = 5 years   F = ?

Future amount F is unknown.

9. I just borrowed $2000 at 7% per year for 10 years and must repay the loan in equal yearly payments. Determine the symbols involved and their values?

P = $2000   A = ? per year   i = 7% per year   n = 10 years

Minimum Attractive Rate of Return

Engineering alternatives are evaluated on the basis that a reasonable ROR can be achieved. This reasonable rate is called the Minimum Attractive Rate of Return (MARR).

Cash Flows: Their Estimation and Diagramming

Cash flows are described as the inflows and outflows of money.

Samples of Inflow: revenues, operating cost reductions, asset salvage value, income tax savings, receipts from stock and bond sales, etc.

Samples of Outflow: first cost of assets, operating costs, maintenance costs, loan interest and principal payments, income taxes, upgrade costs, etc.

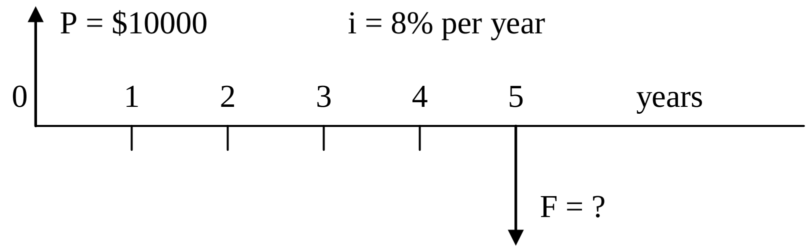
Net cash flow = receipts – disbursements
= cash inflows – cash outflows

The end-of-period convention means that all cash flows are assumed to occur at the end of an interest period. When several receipts and disbursements occur within a given interest period, the net cash flow is assumed to occur at the end of the interest period.

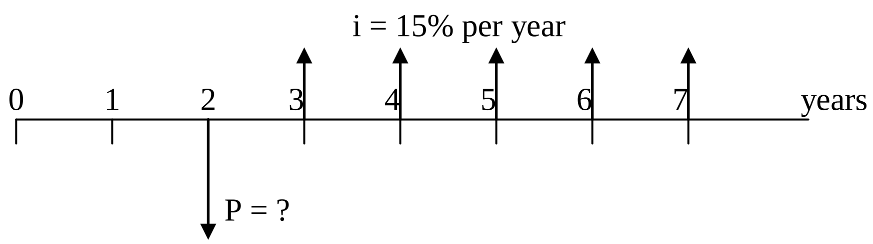
Cash flow diagram is a very useful tool in economic analysis, especially when the cash flow series is complex. It is a graphical representation of cash flows drawn on a time scale.

Examples:

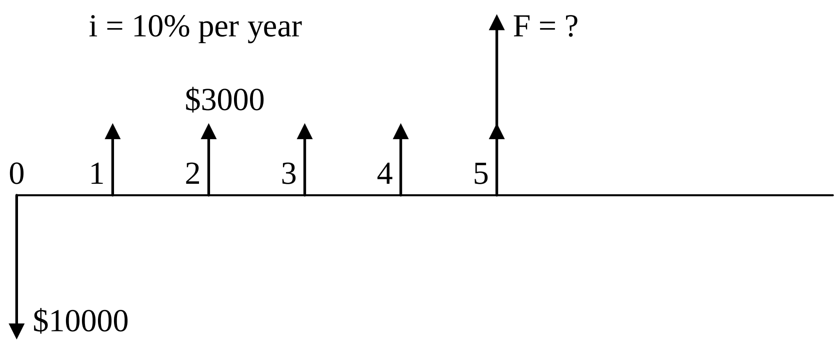
10. I borrow $10000 at 8% per year for 5 years. How much do I pay back at the end of 5 years? Construct the cash flow diagram.



11. I want to deposit an unknown lump-sum amount into an investment fund 2 years from now that is large enough to withdraw $4000 per year for 5 years starting 3 years from now. If the rate of return is estimated to be 15% per year, construct the cash flow diagram.



12. Construct a cash flow diagram for the following cash flows: $10000 outflow at time zero, $3000 per year inflow in years 1 through 5 at an interest rate of 10% per year, and an unknown future amount in year 5.



Rule of 72: Estimating Doubling Time and Interest Rate

Sometimes it is helpful to estimate the number of years n or the rate of return i required for a single cash flow amount to double in size.

Estimated n = 72/i

For example, at a rate of 5% per year, it would take approximately 72/5 = 14.4 years for a current amount to double. (Actual time = 14.3 years).

Alternatively, **Estimated i = 72/n**

For example, in order for money to double in a period of 12 years, a compound rate of return of approximately 72/12 = 6% per year would be required. (Exact answer = 5.946% per year).