IENG/MANE 332 lecture notes

Reference: PRODUCTION, Planning, Control, and Integration by SIPPER & BULFIN

Chapter 6- Part 1: EOQ, EPQ

INVENTORY: INDEPENDENT DEMAND SYSTEMS

Inventory is used in most manufacturing, service, wholesale, and retail activities, and because it can enhance profitability and competitiveness, it is widely discussed within the manufacturing sector. What is inventory? What issues, problems, and complexities are associated with it? To understand these questions, we discuss a simple manufacturing-distribution system- a doughnut bakery store. Most people are familiar with this type of operation.

When you enter the store, you notice trays loaded with all kinds of doughnuts, which are the **finished goods** inventory of the store. Doughnuts are baked and put on trays so that when you enter the store, you can immediately be served. This inventory exists because of a temporary lull between two activities- in this case supply (the baking process) and demand (you, the customer). Another type of inventory in this system is **raw material**- the flour and ingredients needed to prepare a doughnut. It too represents a lull between supply (obtaining the new material) and demand (cooking the doughnuts).

Let's look at the inventory decisions the shop owner has to make. The first decision is quantity- how many doughnuts of each type to prepare, or how much flour and other ingredients to order. The second decision concerns timing, i.e., when to place an order for the given quantity. Should doughnuts be made when a doughnut tray is empty, or when there are 10 left? Should flour be ordered once a week, or when it reaches a certain minimum quantity?

These two decisions are influenced by the demand for the finished product- how many doughnuts will be sold in the next few hours or days. This demand is uncertain. We do not know in advance when and how many customers will come into the store and how many doughnuts of each type they will purchase. At best, we can forecast this demand.

To account for the uncertainty, the owner may keep a large quantity of each doughnut available, thus being able to respond to any future demand. The penalty for doing so might be getting stuck with many unsold doughnuts, which are thrown away when they become state. On the other hand, if customers want a certain doughnut that is unavailable, a different penalty, at least the revenue lost because of being short, will be incurred.

What is gradually unfolding is that even for this simple example, the inventory issue is not simple. It is a key element in the profitability of the doughnut store. In manufacturing organizations with hundreds of products, the inventory problem is even more complex and difficult to solve.

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1 INVENTORY CONCEPTS

The doughnut example shows that inventory is an important and complex system and those we must understand its nature before analyzing it. We use this section to do just that. First, we expand the decision of the role of inventory. Inventory systems have specific terminology, which we present next. We then identify inventory costs and present some measures of effectiveness for inventory systems. We conclude by discussing common inventory policies and the relevance of inventory models.

1.1. The Role of Inventory

So far, we have described inventory but have not defined it. From the many available definitions, we have selected the following one:

A quantity of commodity in the control of an expertise, held for some time to satisfy some future demand.

For the manufacturing sector, the commodity is principally materials: raw material, purchased items, semi-finished products, spare parts, and supplies.

This definition reiterates what we noticed in the doughnut example. Inventory is a "buffer" between two processes-supply and demand. The supply process contributes commodity to the inventory, whereas demand depletes the same inventory. Inventory id necessary because of differences in rates and timing between supply and demand, and this difference can be attributed to both internal and exogenous factors. Internal factors are a matter of policy, but exogenous factors are uncontrollable. Among the internal factors are economies of scale, operation smoothing, and customer service. The most important exogenous factor is uncertainty.

Economies of Scale may make inventory desirable, even if it is possible to balance supply and demand. There are certain fixed costs associated with production and purchasing; these are set-up cost and ordering cost. To recover this fixed cost and reduce the average unit cost, many units of an item may be purchased or produced. These large lot sizes will be ordered infrequently and placed in inventory to satisfy future demand.

Operation Smoothing is used when demand varies over time. An example would be antifreeze or jet skis. Inventory, accumulated in periods of low demand, is used to satisfy higher demand in other periods, which enables the production facility to be operated at a relatively constant production rate, a desirable feature in manufacturing.

Customer Services is another reason to carry an inventory. Inventory is built up so that customer demand can be met immediately from stock, yielding customer satisfaction.

Uncertainty was discussed in the doughnut store example. One way to hedge against uncertainty is to hold more inventory than the forecast demand, which avoids to prospect of running out of stock if actual demand exceeds the forecast. This extra inventory is called safety stock. The resupply process is another source of uncertainty that may justify holding safety stock. The lead time is the time between issuing an order and receiving it. When the lead time is uncertain, we may not receive an order on the date we planned. The safety stock gives some protection from a production stoppage due o lead time uncertainty.

The roles of inventory described so far are operational. Often the vagaries of the market create an economic advantage of maintain inventory. Price fluctuations in the market may justify acquiring more raw material than is required for estimated future demand. We emphasize that this is highly speculative and should be left to the financial function in the organization rather than operations management. Accordingly, this role of inventory will not be discussed further.

1.2 Inventory Terminology

In our doughnut store example, we identified the demand as being uncertain and mentioned two types of inventory-raw material and finished goods. We formally define the different demand environments and the various inventory types.

The demand environment can be classified

The **demand environment** can be classified into two major categories: deterministic or stochastic and independent or dependent.

Deterministic or stochastic: Deterministic means that future demand for an inventory item is known with certainty; random future demand is called stochastic. Each of these cases requires different analysis. The stochastic case is more realistic but also more difficult to handle.

Independent or dependent demand: Demand for an item not related to any other item and primarily influenced by market conditions is called independent demand. Examples include retail sales or finished goods in manufacturing. Dependent demand is very typical to manufacturing- the demand for one item is derived from the demand of another item. An example would be a car, wheels, and bolts. Each car requires four wheels, and each wheel requires bolts. The demand for cars is independent; wheels and bolts have dependent demand. There is a three level hierarchy here, called product structure. Thus, one car generates demand for four wheels (excluding the spare) and 16 bolts. In our doughnut store example, the demand for doughnuts is independent, and the demand for flour is dependent. This chapter focuses on independent demand systems.

Inventory types in production systems are classified according to the value added during the manufacturing process. The classifications are raw material, work in process (WIP), and finished goods; each type is defined below.

Raw Materials include all items required for the manufacturing and assembly process. Typically, they are as follows:

- Material needing further processing (flour, wood, steel bars)
- Components that go into the product as is (computer chips, bolts)
- Supplies (welding electrodes, glue, screws)

Work in Process (WIP) is inventory in the production system waiting to be processed or assembled and may include semi-finished products (a bolt that has been threaded but not coated) or subassemblies (TV picture tubes).

Finished Goods are the outputs of production process, sometimes called end items-anything from cars to shirts to soft drink bottles. The demand for finished goods is usually independent. Also, finished goods of one manufacturing organization may be raw materials for another one.

1.3 Inventory Costs

We defined inventory as a “quantity of commodity”; as such it incurs costs. Purchasing cost is the obvious one. Other types of costs are ordering (setup) cost, holding cost, shortage cost, and system operating cost. We elaborate on each of these.

Purchasing Cost is the per-item cost paid to the supplier (sometimes called material cost). Let c be unit cost and Q be number of units purchased (lot size). Then the total purchasing cost is cQ-a linear function of Q. In some cases the supplier has a price schedule based on the quantity purchased. This unit cost is a function of Q, and the purchase cost is a more complex function.

If we manufacture the unit, c includes both material cost and the variable cost to manufacture a unit. The total manufacturing cost for a production lot is cQ.

Ordering Cost is the cost of preparing and monitoring the order-is incurred each time an order is placed with a supplier. It is independent of the lot size purchased, and therefore it is a fixed cost denoted by A. However, the annual ordering cost, which we discuss later, depends on the lot size. For a manufactured lot, the fixed cost is determined by the set-up cost, which includes the cost of preparing the machine for the production run and possibly some material start-up costs for rejects early in the run. The same notion, A, is used for the setup cost.

The total cost for purchasing or producing a lot is

$$A + cQ$$

It consists of a fixed component A and a variable component cQ

Inventory ties up capital, consumes space, and require maintenance, which all cost money. This is called **holding cost** or carrying cost and includes the following costs:

- Opportunity cost
- Storage and handling cost
- Taxes and insurance
- Pilferage, damage, spoilage, obsolescence, etc.

The costs of carrying inventory begin with the investment inventory. Money tied up in inventory cannot earn a return elsewhere. This cost is an opportunity cost, usually expressed as a percentage of the investment. The lowest value of this opportunity cost is the interest the money would earn in a savings account. Most companies have better opportunities than savings accounts, and most have a minimum rate of return used for evaluating investments, usually called the cost of capital. The same rate can be used as part of the inventory carrying cost.

We calculate costs as a percentage of the investment in inventory and add them to the opportunity cost, generating a total inventory holding cost. Thus, if the cost of capital is 25 percent annually and the other types of costs amount to an additional 10 percent, the total inventory holding cost is 35 percent. That is, for every dollar invested in inventory for one year, we pay 35 cents. Define

i = total inventory holing cost (expressed as a percentage)

It is the cost of carrying $1 of inventory for one unit of time. Because we usually measure inventory in units rather than dollars, and recalling that the cost of one unit is c, we obtain

$$h = ic$$

In which h is the cost to carry one **unit** of inventory for one unit of time-expressed in dollars. Typical annual values of i are 25 percent to 40 percent, but i can be as high as 60 percent.

In the doughnut store example we introduced the concept of **shortage cost**. A shortage occurs when there is demand for an out-of-stock item. A shortage can either be backlogged or lost; demand for durable goods is often backlogged. Thus, if the store does not have the TV you want, you may be willing to wait until they get it. On the other hand, demand may be lost if the doughnut store does not have the kind of doughnut you want. If you go elsewhere, it is called a lost sale.

In both cases we pay a penalty. If the demand is lost, the major penalty is lost profit and loss of goodwill. If the demand is backlogged, there are additional costs to expedite, costs to keep records and a reputation for bad customer service. Material shortage for production is usually backlogged, and the penalty is production stoppage, expedition, and possible late delivery of the product to the customer.

Two types of shortage costs are possible. One results just because a unit is short; the other considers the length of time the unit is short.

We define:

π= shortage cost per unit short

$\hat{\pi}$= shortage cost per unit short per unit of time

Typically, π is used for lost sales; backorders use both. Note that $\hat{\pi}$ are used by backorders what h is to inventory. Shortage cot is often hard to estimate and may be an educational guess.

Finally, there are costs related to operating and controlling the inventory system, called **system operating cost.** This cost may be large; for example, there is the cost of computer hardware and software for inventory control. Ironically, most inventory models were developed before or in the early days of the computer era, and therefore this cost was often overlooked.

1.4. Measures of Effectiveness

Inventory is basically a service entity. If inventory satisfies demand as it occurs, then the service is perfect; otherwise there are problems with the service. Providing a high level of service is not free. The study of inventory systems is a **trade-of analysis** between the benefits and costs of carrying inventory. The goal is to maximize the benefits while minimizing the cost, a difficult mission. It is even more complex when there are many different items in inventory.

First we focus on costs, with benefits viewed as opportunity cost. Later we examine models that address service benefits. There are two approaches to measures of effectiveness- a modeling approach or a managerial approach.

The study of inventory systems is a trade-off analysis between the benefits and costs carrying inventory. The goal is to maximize the benefits while minimizing the cost, a difficult mission.
The **Modeling Approach** optimizes the inventory system. Cost minimization is the criterion used in most inventory models; profit maximization could also be used. For most inventory systems these criteria are equivalent, because profit is the difference between price and cost. We focus on models because they are simpler to handle. Another reason is that, cost is a fact, but price is a policy. Costs are known, but prices may differ due to management policy or market pressure.
A common measure of effectiveness for inventory models is *minimum average total cost per unit time*. A unit of time may be a day, week, month, or year. The total cost includes the cost elements previously discussed. We use the average because holding and shortage costs are proportional to the inventory level, which may vary during the period. To compute the average total cost we average the inventory or shortage over time and multiply it by h or $\hat{\pi}$.
The **managerial approach** is generally used for multi-item inventory systems. The immediate goal is to report the size of the inventory to management. One measure of inventory size is the total inventory investment at the reporting date. Multiply the on-hand quantity of each item by its cost, and sum the result for all items. To get a relative measure of whatever we have "too much" or "too little" inventory or to compare our performance with our competitors and "industry standards" two other measures are used:

$$\text{Months of supply} = \frac{\text{Total inventory investment}}{\text{Average forecast demand (\$/month)}}$$

$$\text{Annual inventory turnover} = \frac{12\left[\text{Average forecasted demand (\$/month)}\right]}{\text{Total inventory investment}}$$

The first measure indicates how long future demand can be met from on-hand inventory. The second measure indicates how fast the inventory is turned; the *higher* this value is, the *lower* the investment in inventory will be. These measures are changed slightly for different purposes and types of inventories (raw material, finished goods). To check future performance, use the forecast for demand, but we use the actual demand to evaluate past performance. A quick way to calculate the inventory turnover from a company's balance sheet is

$$\text{Inventory turnover} = \frac{\text{Value of sales}}{\text{Value of inventory}}$$

Comparing this future to turnover of other companies or industry standards gives an indication of the performance of the inventory operation.

1.5. Inventory Policies

The major element impacting inventory is demand. From the production control standpoint, we assume demand is an uncontrolled variable. There are three important factors in an inventory system – called decision variables- that can be controlled:

- What to order? -Variety decision
- When to order? -Timing decision
- How much/many to order?-Quantity decision

To gain an understanding of these inventory decisions, we examine a single-item system. The variety decision is irrelevant, and the other two decisions are made using two different inventories policies-known as periodic review.
Periodic Review Policy. At fixed time intervals – say a week, a month or any time T), check inventory level I, and issue an order if I is below a certain predetermined level R, called the

reorder point (timing decision). The size of the order Q is the amount required to bring the inventory to a predetermined level S (quantity decision).

The size of Q varies from period to period. Figure 6-1 presents this policy assuming one unit is demanded at a time and orders are delivered instantaneously. At t_1 the inventory level is above the reorder point R, so no order is needed. At the next review time, t_2, T time period after t_1, $I_{t_2} < R$ and an order for $Q = S - I_{t_2}$ units is issued. This policy is often referred to as a periodic policy or fixed order interval policy.

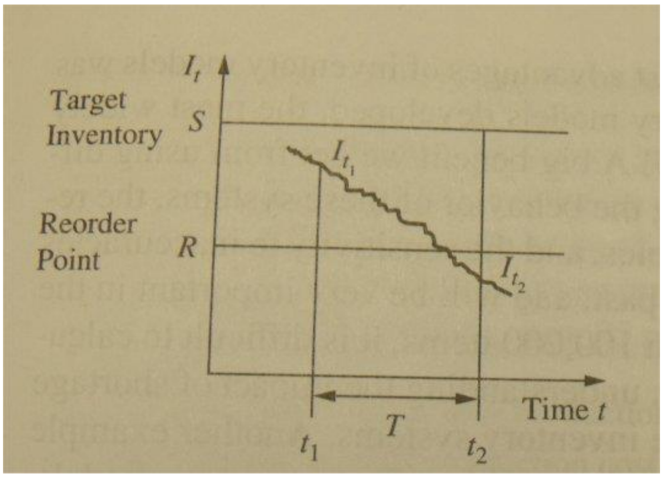

Figure 6.1. Periodic Review Policy

Continuous Review Policy

In this policy, the level of inventory is continuously monitored. When the inventory level reaches the recorder point R(timing decision), a fixed quantity Q is ordered (quantity decision). This is a continuous (Q, R) policy, or fixed reorder quantity policy. Figure 6-2 presents this policy, assuming instantaneous delivery of the order, and demand of one unit at a time.

Before the computer era, periodic review systems were more popular, because they were easier to implement manually. With computers readily available, implementing a continuous review policy is easy. Continuous review has certain merits over periodic review; as we discuss later under control decisions.

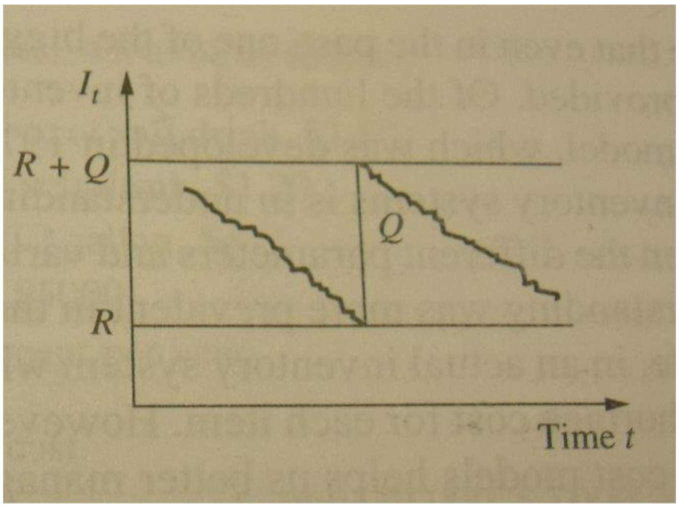

Figure 6.2. Continuous Review Policy

Example 1: Let's assume a Company has received an order for 5,000 widgets for a total sales price of $5,000 and wants to determine the gross profit that will be generated by completing the order. First, the variable costs per widget must be determined.
Let's assume the following:
Annual Widgets Produced: 100,000
Raw Materials Costs: $10,000
Direct Labor Costs: $50,000
From this information, we can conclude that each widget costs 10 cents ($10,000 / 100,000 widgets) in raw materials and 50 cents ($50,000 / 100,000 widgets) in direct labor costs.
Using the formula above, we can calculate that XYZ Company's total variable cost on the order is:
5,000 x ($0.10 + $0.50) = $3,000
Therefore, the company can reasonably expect to earn a $2,000 gross profit ($5,000 - $3,000) from the order.

Example 2: The total cost formula is used to derive the combined variable and fixed costs of a batch of goods or services. The formula is the average fixed cost per unit plus the average variable cost per unit, multiplied by the number of units. The calculation is:

(Average fixed cost + Average variable cost) x Number of units = Total cost
For example, a company is incurring $10,000 of fixed costs to produce 1,000 units (for an average fixed cost per unit of $10), and its variable cost per unit is $3. At the 1,000-unit production level, the total cost of the production is:
($10 Average fixed cost + $3 Average variable cost) x 1,000 Units = $13,000 Total cost

Example 3: let's assume it costs Company XYZ $10,000 to purchase 5,000 widgets that it will resell in its retail outlets. Company XYZ's cost per unit is:
$10,000 / 5,000 = $2 per unit
Often, calculating the cost per unit isn't so simple, especially in manufacturing situations. Usually, costs per unit involve variable costs (costs that vary with the number of units made) and fixed costs (costs that don't vary with the number of units made).
Example 4: For example, at XYZ Restaurant, which sells only pepperoni pizza, the variable expenses per pizza might be:

Flour: $0.50
Yeast: $0.05
Water: $0.01
Cheese: $3.00
Pepperoni: $2.00
Total: $5.56 per pizza
Its fixed expenses per month might be:
Labor: $1,500
Rent: $3,000
Insurance: $200
Advertising: $500

Utilities: \$450
Total: \$5,650
If company XYZ sells 10,000 pizzas, then its cost per unit would be:
Cost per unit = \$5.56 + (\$5,650/10,000) = \$6.125

2.1. Static Lot Sizing Models

A constant and uniform demand environment is not common in the real world. However, it is a convenient starting point for developing inventory models and gaining an understanding of the relationships within an inventory system.

There are four models:

- Economic Order Quantity (EOQ)
- Economic Production Quantity (EPQ)
- Resource Constraints
- Fixed Order Quantity

2.1.1. Economic Order Quantity (EOQ). This model is the most fundamental of all inventory models and was introduced in 1913 by Harris. It is also known as Wilson formula, because Wilson advocated its use. The importance of this model is that it is still one of the most widely used inventory models in industry, and it serves as a basis for more sophisticated inventory models.

We assume the following decision environment:

- There is a single inventory system.
- Demand is uniform and deterministic amounts to D units per unit time – day, week, month, or year. We will use annual demand, but any other unit of time can be used, as long as the rest of the parameters are calculated for the same time unit.
- No shortages are allowed.
- There is no order lead time (time from ordering to receipt).
- All the quantity ordered arrives at the same time, which is called infinite replenishment rate.
- Instantaneous receipt of all quantities ordered.

This model is suitable for raw material purchase in production or for a retail environment. The decision variable for this model is Q. the number of units to be ordered, a positive real number. The cost parameters are all known with certainty and they are as follows:

c = unit cost (\$/unit)
i = total annual inventory holding cost (% per year)
$h = ic$ = total annual inventory handling cost (dollars per unit per year)
A = ordering cost (\$/order)

In addition, we define

D = demand per unit time
T = cycle length, the length of time between placement (or receipt) of replenishment
$K(Q)$ = total average annual cost as a function of the lot size Q
I_t = on-hand inventory at time t (quantity of material actually in stock)

The basic concept of this model is to create a balance between two opposing costs-ordering costs and holding costs. Ordering cost is a fixed cost; the more we order; the less the cost per unit will be. Holding cost is a variable cost that is lower the less inventory we have. This balance is achieved through minimizing $K(Q)$, the total average annual cost.

A helpful tool in analyzing inventory systems is the inventory geometry-a graphical description of I_t , shown in figure 6-4.

We assume inventory level is Q at time zero. As time moves, the inventory is depleted at a rate of D units per year (i.e. the slope of the inventory line is $-D$). When the inventory level reaches zero, we order Q units. Because we assume that lead time is zero and the replenishment rate is infinite, the inventory level will immediately rise to Q, and the process will repeat. Because of the shape of the inventory geometry, this model is sometimes called the sawtooth model.

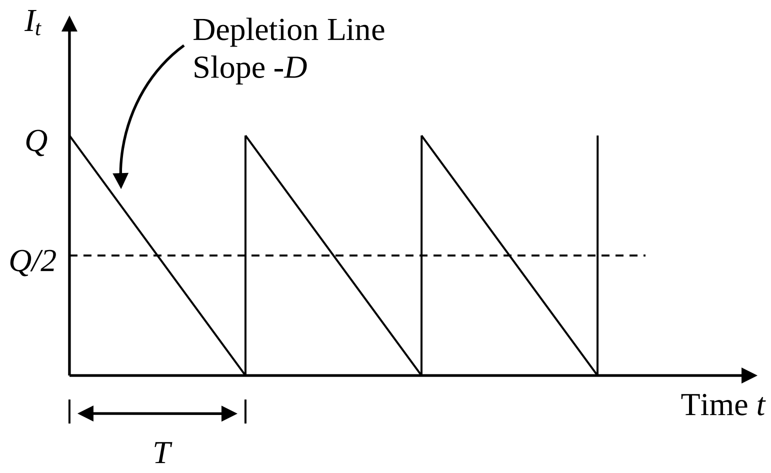

Figure 6.4. EOQ inventory geometry

This pattern is called a cycle, and during a year, there may be several cycles. Let T be the inventory cycle length. From the inventory geometry we note that

$$T = \frac{Q}{D}$$

Let $\bar{I}$ be the average inventory. From figure 6-4 we see

$$\bar{I} = \frac{\text{Area under the inventory curve}}{T} = \frac{\text{Inventory triangle area}}{T} = \frac{1}{T}\frac{QT}{2} = \frac{Q}{2}$$

This result can be obtained intuitively, as the inventory level fluctuates between 0 and Q, so the average is $Q/_2$. The maximum inventory level is

The maximum inventory level is

$$I_{max} = Q$$

There are three types of costs-purchasing cost, ordering cost, and inventory holding cost. For each cycle, the costs are

CQ= purchasing cost

A= ordering (or set-up) cost

$icT\frac{Q}{2} = hT\frac{Q}{2}$ = average inventory holding cost

Thus, the average cost per cycle is

$$cQ + A + hT\frac{Q}{2}$$

Note that in the above, hT is the cost of carrying one unit in inventory for T time units.

To obtain the average annual cost $K(Q)$, we multiply the average cost per cycle by the number of cycles, which is $1/_T$, we get

$$K(Q) = \frac{cQ}{T} + \frac{A}{T} + h\frac{Q}{2}$$

Because $\frac{1}{T} = \frac{D}{Q}$, the average total annual cost is

$$K(Q) = cD + \frac{AD}{Q} + h\frac{Q}{2}$$

We wish to find the value of decision variable Q that minimizes $K(Q)$. This is achieved by solving the equation

$$K'(Q) = \frac{dK(Q)}{dQ} = -\frac{AD}{Q^2} + \frac{h}{2} = 0$$

Because the second derivative of $K(Q)$ is positive, $K(Q)$ is a convex function and achieves its minimum at the point where the derivative is zero, solving the above equation yields Q^* is known as economic order quantity (EOQ).

$$K''(Q) = \frac{AD}{2Q^3} \geq 0$$

- always positive,
- K(Q) is a convex function,
- minimum is achieved at $K''(Q) = 0$

$$Q^* = \sqrt{\frac{2AD}{h}}$$

Minimum total average annual cost;

$$K(Q^*) = cD + \sqrt{2ADh}$$

Annual ordering (set-up) cost = $\frac{AD}{Q^*}$

Annual holding cost = $h\left(\frac{Q}{2}\right)$

Example 6-1 Demand for the Child Cycle at Best Buy is 500 units per month. Best Buy incurs a fixed order placement, transportation, and receiving cost of Rs. 4,000 each time an order is placed. Each cycle costs Rs. 500 and the retailer has a holding cost of 20 percent. Evaluate the number of computers that the store manager should order in each replenishment lot?

$$Demand = D = 500 * 12 = 6{,}000$$
$$Ordering\ cost = A = \$4{,}000$$
$$Holding\ cost = h = 0.2 * 500 = \$100$$
$$Economic\ order\ quanitity = Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(4{,}000)(6{,}000)}{100}} = 693\ units$$

Example 6-2 ABC Ltd. uses EOQ logic to determine the order quantity for its various components and is planning its orders. The Annual consumption is 80,000 units, Cost to place one order is Rs. 1,200, Cost per unit is Rs. 50 and carrying cost is 6% of Unit cost. Find EOQ, No. of order per year, Ordering Cost and Carrying Cost and Total Cost of Inventory.

$$Demand = D = 80{,}000$$
$$Ordering\ cost = A = \$1{,}200$$
$$Holding\ cost = h = 0.6 * 50 = \$3$$
$$Economic\ order\ quanitity = Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(1{,}200)(80{,}000)}{3}} = 8000\ units$$
$$Number\ of\ order\ per\ year = T = \frac{D}{Q^*} = \frac{80{,}000}{8000} = 10\ orders\ per\ year$$
$$Total\ ordering\ cost = AT = 1{,}200 * 10 = \$\ 12000$$
$$Total\ holding\ cost = h\frac{Q^*}{2} = 3 * \frac{8000}{2} = \$\ 12000$$

$$Total\ inventory\ cost = Total\ ordering\ cost + Total\ holding\ cost = 12{,}000 + 12{,}000 = \$24{,}000$$

Example 6-3 Midwest Precision Control Corporation is trying to decide between two alternate Order Plans for its inventory of a certain item. Irrespective of the plan to be followed, demand for the item is expected to be 1,000 units annually. Under Plan 1st, Midwest would use a teletype for ordering; order costs would be Rs. 40 per order. Inventory holding costs (carrying cost) would be 200 per unit. Under Plan 2nd order costs would be 30 per order. And holding costs would 20% of unit Cost (unit cost=480). Find out EOQ and Total Inventory Cost than decide which Plan would result in the lowest total inventory cost?

Plan 1:

$$Demand = D = 1{,}000$$
$$Ordering\ cost = A = \$40$$
$$Holding\ cost = h = \$200$$
$$Economic\ order\ quanitity = Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(40)(1{,}000)}{200}} = 20\ units$$
$$Number\ of\ order\ per\ year = T = \frac{D}{Q^*} = \frac{1{,}000}{20} = 50\ orders\ per\ year$$
$$Total\ ordering\ cost = AT = 40 * 50 = \$\ 2{,}000$$
$$Total\ holding\ cost = h\frac{Q^*}{2} = 200 * \frac{20}{2} = \$\ 2{,}000$$
$$Total\ inventory\ cost = Total\ ordering\ cost + Total\ holding = 2{,}000 + 2{,}000 = \$4{,}000$$

Plan 2:

$$Demand = D = 1{,}000$$
$$Ordering\ cost = A = \$30$$
$$Holding\ cost = h = 0.2 * 480 = \$96$$
$$Economic\ order\ quanitity = Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(30)(1{,}000)}{96}} = 25\ units$$
$$Number\ of\ order\ per\ year = T = \frac{D}{Q^*} = \frac{1{,}000}{25} = 40\ orders\ per\ year$$
$$Total\ ordering\ cost = AT = 30 * \frac{1{,}000}{25} = \$\ 1{,}200$$
$$Total\ holding\ cost = h\frac{Q^*}{2} = 96 * \frac{25}{2} = \$\ 1{,}200$$
$$Total\ inventory\ cost = Total\ ordering\ cost + Total\ holding\ cost = 1{,}200 + 1{,}200 = \$2{,}400$$

Plan 2 is better than plan 1 (because of lower total inventory cost)

Example 6-4 A small welding shop uses welding rods at a uniform rate. Marwin, the owner, orders the rods from a local supplier. Marwin estimates the annual demand is about 1000 pounds. To place each order, he has to spend about $3.60 for the phone call and paper work. Marwin pays $2 per pound of rods, and holding costs are based on a 25 percent annual rate.
Find EOQ, No. of order per year, Ordering Cost and Carrying Cost and Total Cost of Inventory.

$$Demand = D = 1{,}000$$
$$Ordering\ cost = A = \$3.6$$
$$Holding\ cost = h = 0.2 * 2 = \$0.5$$
$$Economic\ order\ quanitity = Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(3.6)(1{,}000)}{0.5}} = 120\ units$$
$$Number\ of\ order\ per\ year = T = \frac{D}{Q^*} = \frac{1{,}000}{120} = 8.33\ orders\ per\ year$$
$$Total\ ordering\ cost = AT = 3.6(\frac{1{,}000}{120}) = \$\ 30$$
$$Total\ holding\ cost = h\frac{Q^*}{2} = 0.5 * \frac{120}{2} = \$\ 30$$
$$Total\ inventory\ cost = Total\ ordering\ cost + Total\ holding\ cost = 30 + 30 = \$60$$

The fact that annual inventory holding cost and annual ordering cost are equal should not be surprising. We showed that the optimum is at the intersection of the two curves. This problem could also be solved based on monthly or weekly quantities.

Sensitivity of $K(Q^*)$. In the real world it is sometimes impractical to order exactly Q^*units. Assume, for example, that $Q^* = 1357$ and the item of interest comes in boxes of 1000 units each. Should we order one or two boxes? This question leads to examining the sensitivity of the function $K(Q)$ to deviation of Q from the optimum Q^*. This sensitivity is measured by the ratio is measured by $\frac{K(Q)}{K(Q^*)}$

When there is no deviation($Q = Q^*$), the value of this ratio is 1. For ease of computation, we ignore the purchasing cost Cd in this ratio, because it does not change the general shape of the cost curve but simply moves it up by an amount cD. We obtain

$$\frac{K(Q)}{K(Q^*)} = \left(\frac{\frac{AD}{Q} + h\frac{Q}{2}}{\sqrt{2ADh}}\right) = \frac{1}{2Q}\sqrt{\frac{2AD}{h}} + \frac{Q}{2}\sqrt{\frac{h}{2AD}} = \frac{Q^*}{2Q} + \frac{Q}{2Q^*} = \frac{1}{2}(\frac{Q^*}{Q} + \frac{Q}{Q^*})$$

Example 6-5 Sensitivity of EOQ. Suppose the welding rods from example 6-1 are ordered in packages of 75 pounds each. How many packages should Marwin order?
Solution: in example 6-4, the economic order quantity is 120 pounds, and the new order quantity should be either one package (75 pounds) or two packages (150 pounds). We apply sensitivity

$$\frac{K(Q)}{K(Q^*)} = \frac{1}{2}(\frac{Q^*}{Q} + \frac{Q}{Q^*})$$

1. Set $Q = 75$;

$$\frac{K(75)}{K(120)} = \frac{1}{2}\left[\frac{120}{75} + \frac{75}{120}\right] = 1.1125$$

2. Set $Q = 150$;

$$\frac{K(150)}{K(120)} = \frac{1}{2}\left[\frac{120}{150} + \frac{150}{120}\right] = 1.025$$

Marwin will better off by ordering two packages each time.

2.1.2 ECONOMIC PRODUCTION QUANTITY (EPQ) WITH EXTENTIONS.

This extension of the EOQ model relaxes the assumption of infinite replenishment rate. Instead, there is a finite replenishment rate, which is typical of a manufactured item in which the lot is typical of a manufactured item in which the lot is delivered overtime according to the production rate.

We also allow shortages to occur and be backlogged, assuming there is a maximum level of backlog that management is willing to tolerate. Backlogs occur in production systems because of either lack of material, lack of capacity, or both. Recall that shortage has two associated costs, π and $\hat{\pi}$. Because $\hat{\pi}$ is to shortage what h is to inventory, we evaluate it the same way- by considering the average shortage. Because π is the shortage (penalty) cost per unit short, we need to maximum shortage to evaluate it. Let

ψ = replenishment production rate, measured in the same units as the demand
Q = size of the production lot
A = ordering (set-up) cost
c = unit production cost
B_t = shortage (backorder) level at time t
$\overline{B}$ = average shortage level
b = max B_t

The inventory geometry for this case is shown in figure 6-7.

We assume that at time zero the inventory level is –b. at this point we issue a production order for Q units and because the lead time is zero, production starts immediately. The production rate is ψ, but because there is demand at the same time, the net replenishment rate is $\psi - D$ and the replenishment line has a positive slope. When Q units has been manufactured, inventory reaches its maximum value, I_{max}, and production stops. The inventory is depleted at the demand rate D. when the inventory level reaches –b, production resumes and the cycle repeats.

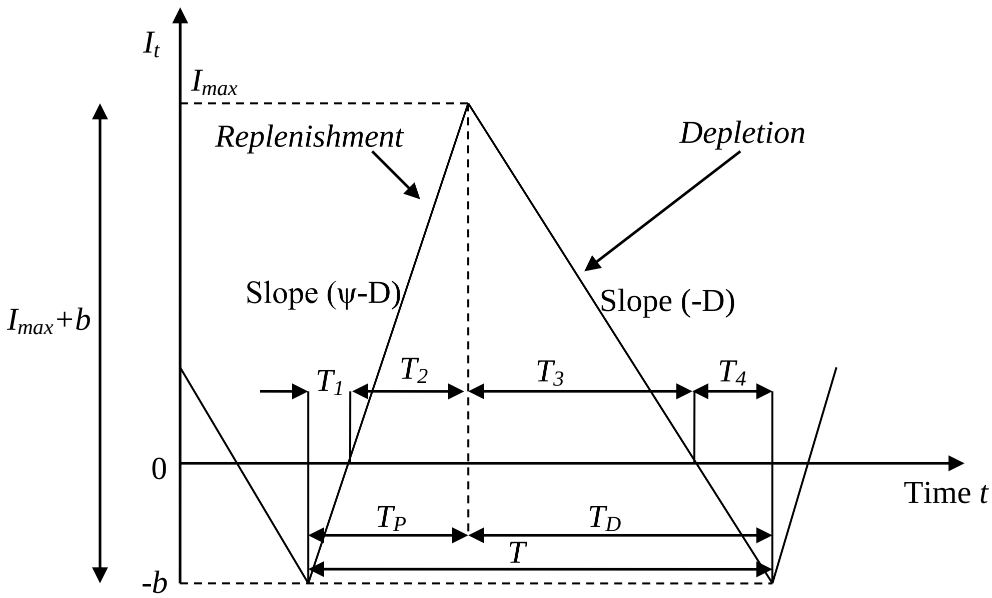

Figure 6-7 Inventory geometry: EPQ with backlog

Following a basically similar procedure as in the EOQ case:

Cycle time: $T = \frac{Q}{D}$

Time to produce Q units: $T_P = \frac{Q}{\psi}$

Time to deplete maximum inventory: $T_D = \frac{I_{\max}}{D}$

From the inventory geometry:

$$I_{max} + b = T_P(\psi - D) = \frac{Q}{\psi}(\psi - D) = Q(1 - \frac{D}{\psi})$$

$$I_{max} = Q\left(1 - \frac{D}{\psi}\right) - b$$

The on-hand inventory is positive during$T_2 + T_3$, whereas shortages are backlogged during T_1 and T_4. Production takes place during$T_P = T_1 + T_2$, whereas inventory depletion occurs during$T_D = T_3 + T_4$. From the inventory geometry we get

Time to recover from the backlog: $T_1 = \frac{b}{\psi - D}$

Time to generate I_{max}: $T_2 = \frac{I_{\max}}{\psi - D}$

Time to deplete I_{max}: $T_3 = \frac{I_{\max}}{D}$

Time to generate backlog of b: $T_4 = \frac{b}{D}$

To get the equation for $K(Q, b)$, we need $\bar{I}$ (average of inventory) and $\bar{B}$ (average of backlog)

$$\bar{I} = \frac{1}{2T} I_{\max} \left(T_2 + T_3\right)$$

$$\bar{I} = \frac{{I_{\max}}^2}{2T}\left(\frac{\psi}{D(\psi - D)}\right)$$

The total shortage cost per cycle is: $\pi b + \hat{\pi} T\bar{B}$

The total annual inventory holding cost is:

$$\frac{1}{T}\left(hT\bar{I}\right) = h\bar{I} = \frac{h\left[Q\left(1 - \frac{D}{\psi}\right) - b\right]^2}{2Q\left(1 - \frac{D}{\psi}\right)}$$

The average annual shortage cost:

$$\frac{1}{T}\left[\pi b + \hat{\pi} T\bar{B}\right] = \frac{\pi b D}{Q} + \frac{\hat{\pi} b^2}{2Q\left(1 - \frac{D}{\psi}\right)}$$

The total annual cost is:

$$K(Q,b) = cD + \frac{AD}{Q} + \frac{h\left[Q\left(1-\frac{D}{\psi}\right)-b\right]^2}{2Q\left(1-\frac{D}{\psi}\right)} + \frac{\pi bD}{Q} + \frac{\hat{\pi} b^2}{2Q\left(1-\frac{D}{\psi}\right)}$$

$K(Q,b)$ = purchasing cost + ordering cost + holding cost + fixed storage + backlog

To find Q^* and b^*, solve:

$$\frac{\partial K}{\partial Q} = 0 \quad \text{and} \quad \frac{\partial K}{\partial b} = 0 \qquad (\text{for } \hat{\pi} \neq 0)$$

$$Q^* = \sqrt{\frac{2AD}{h\left(1-\frac{D}{\psi}\right)} - \frac{(\pi D)^2}{h(h+\hat{\pi})}}\sqrt{\frac{h+\hat{\pi}}{\hat{\pi}}}$$

$$b^* = \frac{(hQ^* - \pi D)\left(1-\frac{D}{\psi}\right)}{(h+\hat{\pi})}$$

Example 6-6 SuperSauce produces a certain salad dressing. The demand for this dressing is about 400 pounds per month, and SuperSauce can manufacture it at the rate of 2000 pounds per month. To initiate production, the machines have to be thoroughly checked and cleaned, and it costs the company \$120 per set-up. The cost to produce this dressing is \$3 a pound, and the inventory holding cost is estimated as 20 percent annually. If the demand for this dressing exceeds the available inventory, it is backlogged. Management estimates that a backlog accrues two types of cost – loss of goodwill and shortage penalty. The loss of goodwill is estimated to be \$0.1 per pound short, and the shortage penalty is estimated to be \$1.2 per pound short per month. Analyze the problem.

<u>*Solution:*</u>

A = \$120 per set-up
i = 20% annually — c = \$3 per pound
$h = 0.2 \times \$3 = \$0.6$ per pound
π = \$0.1 per pound
$\hat{\pi}$ = \$1.2 per pound per month = \$14.4 per pound per year
D = 400/ month = 4800/ year
ψ = 2000/ month = 24,000/ year

Economic production quantity

$$Q^* = \sqrt{\frac{2AD}{h\left(1-\frac{D}{\psi}\right)} - \frac{(\pi D)^2}{h(h+\hat{\pi})}}\sqrt{\frac{h+\hat{\pi}}{\hat{\pi}}} = \sqrt{\frac{(2)(120)(4800)}{0.6\left(1-\frac{4800}{24000}\right)} - \frac{[(0.1)(4800)]^2}{(0.6)(0.6+14.4)}}\sqrt{\frac{0.6+14.4}{14.4}}$$

Q^* = 1605 pounds/lot

The optimal maximum backorder level is

$$b^* = \frac{(hQ^* - \pi D)(1-\frac{D}{\psi})}{(h+\hat{\pi})} = \frac{(0.6)(1605)-(0.1)(4800)(1-\frac{4800}{24000})}{(0.6+14.4)} = 25.76 = 26$$

The economic order quantity is 1605 pounds, the maximum backorder level is 26 pounds, and production takes $4800/_{24} = 20$ percent of the time. The annual total cost of inventory is

$$K(Q^*, b^*) = cD + \frac{AD}{Q} + \frac{h\left[Q\left(1-\frac{D}{\psi}\right)-b\right]^2}{2Q\left(1-\frac{D}{\psi}\right)} + \frac{\pi bD}{Q} + \frac{\hat{\pi}b^2}{2Q\left(1-\frac{D}{\psi}\right)}$$

$$K(1573, 25) = (3)(4800) + \frac{(120)(4800)}{1573} + \frac{(0.6)\left[1573\left(1-\frac{4800}{24000}\right)-25\right]^2}{2(1573)\left(1-\frac{4800}{24000}\right)}$$
$$+ \frac{(0.1)(25)(4800)}{1573} + \frac{(14.4)(25)^2}{2(1573)\left(1-\frac{4800}{24000}\right)} = \$15{,}136$$

From the EPQ with backlog model we obtain two special cases- EPQ without backlog and EOQ with backlog.

Economic Production Quantity (EPQ) In this case, we prohibit shortages by setting the shortage cost to be infinite. Obviously, no backlog is planned for this case, so *b*=0. The cost equation becomes

$$K(Q) = cD + \frac{AD}{Q} + h\frac{Q}{2}\left(1-\frac{D}{\psi}\right),$$

By setting b=0 in the previous cost equation. In the same way we obtain

$$Q^* = \sqrt{\frac{2AD}{h\left(1-\frac{D}{\psi}\right)}}$$

In this case the value of Q^* higher than in the EOQ case, because $(1 - D)/_{\psi} < 1$. However, the value of $\bar{I}$ is lower than before, because of a period of combined replenishment and depletion. The term $(1 - D)/_{\psi}$ is the effective replenishment rate. Note that when $\psi \to \infty$, we obtain the EOQ

Example 6-7 The Rainbow Paint Manufacturing Company has a varied product line. One of their products is latex paint. Rainbow can manufacture the paint at an annual rate of 8000 gallons. The unit cost to produce one gallon of paint is \$0.25, and the annual inventory holding cost is 40 percent. Prior to each production run, clean-up and check-out operations are performed, at a cost of \$25. Analyze the problem.

Solution: the basic information for Rainbow's Latex production is

- A = \$25 per set-up
- i = 40% annually
- c = \$0.25 per gallon
- h = 0.40 × \$0.25 = \$0.10 per gallon per year
- D = 4000 gallons per year
- ψ = 8000 gallons per year

The annual average total inventory cost is given by

$$K(Q) = cD + \frac{AD}{Q} + \frac{hQ}{2}(1 - \frac{D}{\psi})$$

And the economic production quantity is

$$\text{EPQ} = Q^* = \sqrt{\frac{2AD}{h\left(1 - \frac{D}{\psi}\right)}} = \sqrt{\frac{(2)(25)(4000)}{(0.1)\left(1 - \frac{4000}{8000}\right)}} = 2000$$

Calculating,

$$T_P = \frac{Q}{\psi} = \frac{2000}{8000} = 0.25 \text{ Year} = 3 \text{ months}$$

$$T = \frac{Q}{D} = \frac{2000}{4000} = 0.5 \text{ Year} = 6 \text{ months}$$

i.e., there are two cycles per year.