IENG/MANE 332 lecture notes

Reference: PRODUCTION, Planning, Control, and Integration by SIPPER & BULFIN

Chapter 6- Part 2

INVENTORY: INDEPENDENT DEMAND SYSTEMS

EOQ with backlog

This case is one of infinite replenishment rate in which shortages are allowed and backlogged. By setting $\psi \to \infty$ we obtain

$$K(Q,b) = cD + \frac{AD}{Q} + \frac{h[Q-b]^2}{2Q} + \frac{2\pi bD + \hat{\pi}b^2}{2Q}$$

Yielding for $\hat{\pi} = 0$

$$Q^* = \sqrt{\frac{2AD}{h} - \frac{(\pi D)^2}{h(h+\hat{\pi})}}\sqrt{\frac{h+\hat{\pi}}{\hat{\pi}}}$$

$$b^* = \frac{(hQ^* - \pi D)}{(h+\hat{\pi})}$$

Example 6-8 EOQ with backlog Jane sells, among other items, solvents. The demand is very steady at 500 gallons per year. The cost for placing and order is \$50, and each gallon costs Jane \$2. Inventory holding cost is 20 percent annually. If the demand exceeds the inventory, Jane estimates that there will be two types of penalty costs associated with the backorder. The loss of goodwill is \$0.2 per unit short, and a "bookkeeping" cost of \$0.2 per unit short per year. Find economic order quantity, maximum acceptable inventory, cycle time, and minimal total annual average cost

Solution: the various parameters are

A = \$50

D = 500 gallons/year

i = 20%

c = \$2/unit

$h = 0.20 \times \$2 = \$0.40$ unit-year

π = \$0.2 per gallon

$\hat{\pi}$ = \$0.2 per gallon per year

Because Jane allows backorders, the average annual inventory cost is

$$K(Q,b) = cD + \frac{AD}{Q} + \frac{h[Q-b]^2}{2Q} + \frac{2\pi bD + \hat{\pi}b^2}{2Q}$$

And the economic order quantity and optimal maximum backorder will be

$$Q^* = \sqrt{\frac{2AD}{h} - \frac{(\pi D)^2}{h(h+\hat{\pi})}}\sqrt{\frac{h+\hat{\pi}}{\hat{\pi}}} = \sqrt{\frac{(2)(50)(500)}{0.4} - \frac{[(0.2)(500)]^2}{0.4(0.4+0.2)}}\sqrt{\frac{0.4+0.2}{0.2}}$$

$Q^* =$ 500 gallons

$$b^* = \frac{(hQ^* - \pi D)}{(h + \hat{\pi})} = \frac{(04)(500) - (0.2)(500)}{(0.4 + 0.2)} = 166.7 \approx 167$$

The minimal total annual average cost is

$$K(500,167) = (2)(500) + \frac{(50)(500)}{500} + \frac{(0.4)[500 - 167]^2}{(2)(500)} + \frac{(2)(0.2)(500)(167) + (0.2)(167)^2}{(2)(500)}$$

$K(500,167) =$ \$1133.33

Because $Q^* = 500$ is equal to D, the reorder cycle is one year.

2.1.3 QUANTITY DISCOUNTS. The EOQ model assumes that the unit cost is constant, no matter what quantity is purchased. In reality, supplier may induce their customers to place larger orders by offering them quantity discounts. If the quantity purchased is greater than a specified "price break" quantity, the cost per unit is reduced. It is common practice to include this discount policy in the published price schedule.

The tendency of the buyer is to take advantage of this situation, especially if the item purchased is one that is used continuously. However, purchasing larger quantities means larger inventory, with higher inventory cost. So, the savings gained by purchasing at a lower unit cost may be lost by accruing inventory holding cost. We again see a need to balance opposing costs. Do we purchase more to take advantage of the cost breaks or purchase less to keep low inventory, resulting in lower inventory holding cost? This balance is obtained by modifying the basic EOQ model.

Two price break schedules are common. The **all-units discount** applies the discounted price to all units beginning with the first unit, if the quantity purchased exceeds the price break quantity. The other schedule applies the discounted price only to those units over the price break quantity-an **incremental discount** schedule. We introduce notation for quantity discounts. Unless otherwise stated, the notation is the same as the EOQ notation. Let

m = the number of price breaks

q_j = the upper limit of the jth price break interval

c_j = the cost of a unit in the jth price break interval $[q_{j-1}, q_j]$

Q_j = the EOQ quantity, calculated using c_j

Q_j^* = the best order quantity in interval j

Q^* = the optimal order quantity over all prices

$K_j(Q)$ = the cost of Q units in interval j

$K_j(Q_j)$ = the cost of EOQ units in interval j

$K_j(Q_j^*)$ = the minimum cost in interval j

$K^*(Q^*)$ = the minimum cost over all prices

$C_j(Q)$ = the cost to purchase Q units in interval j

Example 6-9_Coldpoint is a home appliance manufacturer. The company purchases a certain component for their products. Southern Electronics and ElectroTech are two companies that make the component, and their products and services are equal, so the component will be bought based solely on cost. Both companies offer a discount policy based on order quantity. However, these two companies post different price schedules. For Southern Electronics, if the order quantity is less than 500 (q_1) units, the price is \$0.60 per unit; if the quantity is 500 or more but less than 1000 (q_2), the unit price is \$0.58; any quantity 1000 units or over has a unit price of \$0.56. ElectroTech offers the same quantity range and prices. However, the discount rare applies only to the excess amount ordered. That is, if the order quantity is 500 units, the first 499 units cost \$0.60 and the 500th unit cost \$0.58. If 1000 units are ordered, the first 499 units cost \$0.60 and the next 500 [500, 999] cost \$0.58. Any unit beyond the 1000th costs \$0.56. Table below describes the two price schedules.

TABLE 6-1 $C(Q)$ for two price schedules

Quantity (Q)	**Southern Electronics**	**ElectroTech**
$0 \le Q < 500$	$0.60Q$	$0.6Q$
$500 \le Q < 1000$	$0.58Q$	$(0.6\times500) + 0.58(Q\text{-}500)$
$1000 \le Q < \infty$	$0.56Q$	$(0.6\times500) + 0.58(500) + 0.56(Q\text{-}1000)$

The graphical description of the two different schedules is shown in figure 6-8 and 6-9. The average cost per unit, $(C_j(Q))/Q$, is equal to C_j in the all-units discount and is larger than C_j in the case of incremental discounts.

All-units discounts. As before, our objective is to find Q that minimizes average total annual cost. Let

$$Q_j = \sqrt{\frac{2AD}{ic_j}}$$

$$K_j(Q_j) = c_jD + \frac{AD}{Q} + h\frac{ic_j}{2}$$

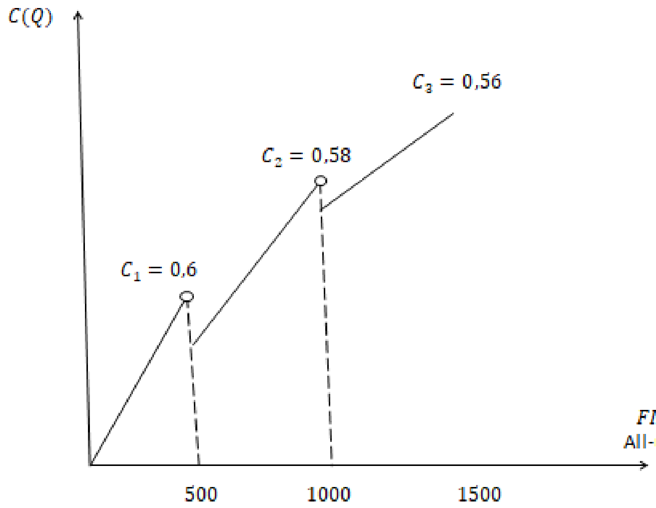

FIGURE 6 − 8
All-units discount

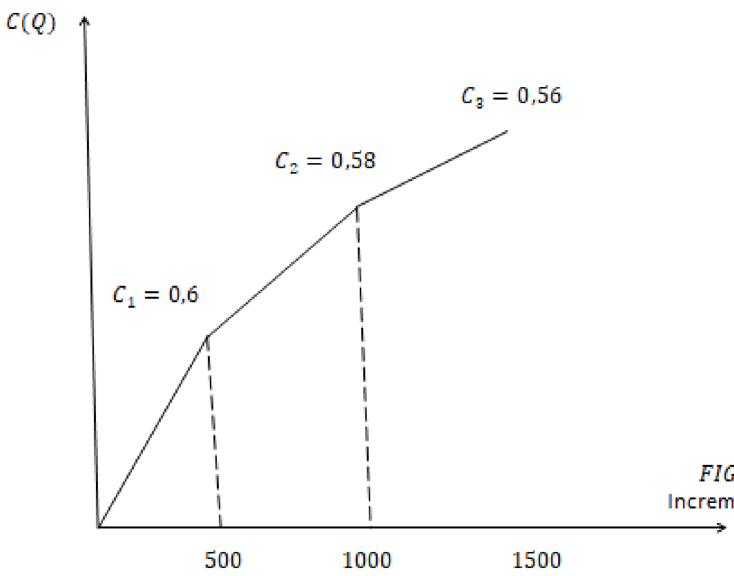

FIGURE 6 − 9
Incremental discount

This is the basis for formulating the procedure for finding the optimum solution for the all-units discount policy. It is;

Step 0: Set $Q^* = 0$, $K^*(Q^*) = \infty$, and $j = m$.

Step 1: Compute Q_j, if $q_{j-1} \leq Q_j \leq q_j$, go to step 3. Otherwise, set $Q_j^* = q_j$ and $K^*(Q^*) = K_j(q_j)$.

Step 2: If $K_j(Q_j^*) < K^*(Q^*)$, set $Q^* = Q_j$ and $K^*(Q^*) = K_j(Q_j^*)$. Set $j=j-1$ and go to step 1.

Step 3: Set $K_j(Q_j^*) = c_j D + \sqrt{2ADic_j}$. If $K_j(Q_j^*) < K^*(Q^*)$, set $Q^* = Q_j^*$ and $K^*(Q^*) = K_j(Q_j^*)$. Stop; the optimal order quantity Q^* with total cost $K^*(Q^*)$.

The following example demonstrates this procedure.

Example 6-10. All units discount. We continue the Coldpoint example. The company estimates the cost of placing an order to be \$20, and the uniform annual demand for this subcomponent is 800 units. Inventory carrying cost is estimated to be 20 percent annually. We want to find the best purchasing policy if the subcomponent were to be ordered from Southern Electronics.

Solution: We note the basic parameters of this problem

A = \$20
D = 800 units per year
i = 20 percent annually
m = 3

Step 0: Set $Q^* = 0$, $K^*(Q^*) = \infty$, and $j = m = 3$.

Step 1: Calculate the Q_3 with $c_3 = 0.56$

$$Q_j = \sqrt{\frac{2AD}{ic_j}} \quad \Rightarrow \quad Q_3 = \sqrt{\frac{2(20)(800)}{(0.20)(0.56)}} = 535$$

Because $Q_3 < 1000$, set $Q_3^* = 1000 = q_2$ and calculate

$$K_3 q_3 = c_3 D + \frac{AD}{q_2} + ic_3 \frac{q_2}{2}$$

$$K_3(q_3) = (0.56)(800) + \frac{(20)(800)}{1000} + (0.20)(0.56)\left(\frac{1000}{2}\right) = 520$$

Step 2: $K_3(1000) < K^*(Q^*)$, so set $Q^* = 1000$ and $K^*(1000) = 520$. $j = 3-1 = 2$ and go to step 1.

Step 1: Calculate Q_2 with $c_2 = 0.58$

$$Q_2 = \sqrt{\frac{2(20)(800)}{(0.20)(0.58)}} = 525$$

Because 500 < 525 < 1000, this is a feasible order quantity at the given price, so go to step 3.

Step 3: Calculate $K_2(Q_2^*) = c_2 D + \sqrt{2ADic_2} = (0.58)(800) + \sqrt{2(20)(800)(0.20)(0.58)} \cong 525$

Because 525 > 520, the economic order quantity is 100 units and the annual average inventory cost is $520 at the unit price of $0.58.

Incremental Discount. Now we examine the incremental discount option for ElectroTech presented in example 6-9. As shown in table 6-2, we can evaluate the average unit cost for each price break region. The unit cost used for evaluating the total average annual cost is the average unit cost in interval j, i.e., $C_j(Q) / Q$.

TABLE 6-2 $C(Q)$ Average unit cost

Quantity (Q)	Southern Electronics	ElectroTech
$0 \le Q < 500$	0.60	0.6
$500 \le Q < 1000$	0.58	$10/Q + 0.58$
$1000 \le Q < \infty$	0.56	$30/Q + 0.56$

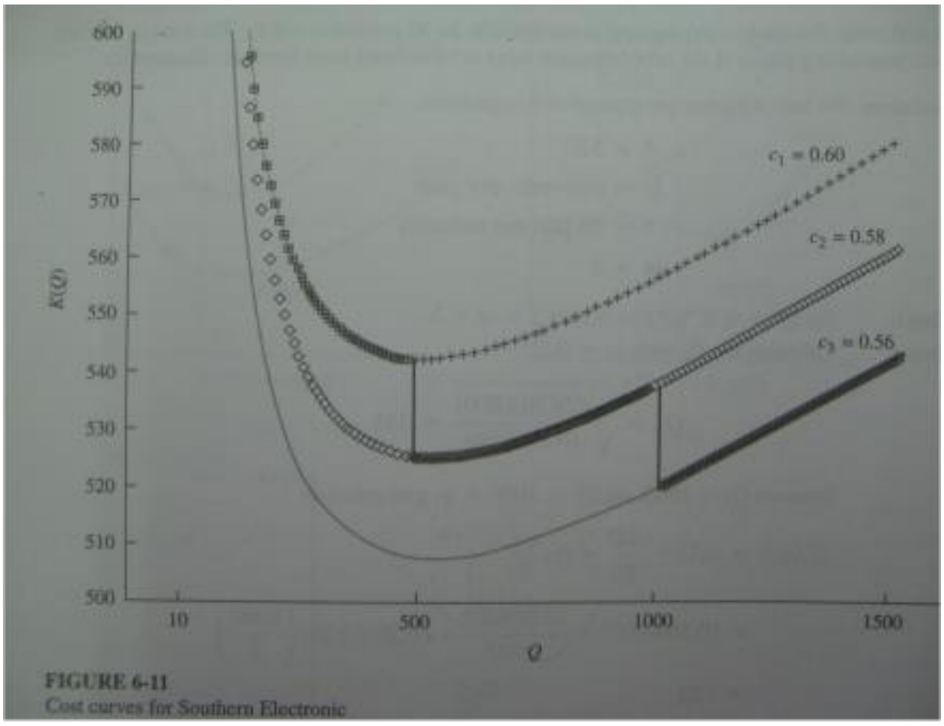

Figure 6.11. Cost curves for Southern Electronic

The average annual cost function for $q_{j-1} < Q < q_j$ is

$$K_j(Q) = \frac{C_j(Q)}{q} D + \frac{AD}{Q} + i\left(\frac{C_j(Q)}{Q}\right)\left(\frac{Q}{2}\right)$$

$K_j(Q)$ is valid only between price break points $[q_{j-1}, q_j]$. it can be shown that the minimum cost point will never occur at a price break point. Further, if the optimal Q for an interval is in the interval, there is no guarantee that it is the best overall; we must compute the best Q for each price break, calculate the cost for any Q falling its proper region, and choose the smallest cost. Differentiating $K_j(Q)$ and setting the result equal to zero gives the optimal Q for interval j as

$$Q_j = \sqrt{\frac{2D\left[A + C(q_{j-1}) - c_j q_{j-1}\right]}{ic_j}}$$

Where $C(q_{j-1})$ is the total cost at break point $j-1$.

The algorithm for the incremental discount problem is

Step 0: Set $Q^* = 0$, $K^*(Q^*) = \infty$, and $j = 1$.

Step 1: Compute Q_j, if $q_{j-1} \le Q_j \le q_j$, compute $K_j(Q_j)$. If Q_j is not in the interval, Set $K_j(Q_j) = \infty$.

Step 2: Set $j = j + 1$. If $j \le m$, go to step 1.

Step 3: Let $K_l(Q_l) = \text{Min}_{j=1,m}\ K_j(Q_j)$; $Q^* = Q_l$ and $K^*(Q^*) = K_l(Q_l)$.

We demonstrate this procedure in example 6-11.

Example 6-11. **Incremental discount**. If gold point considers purchasing from ElectroTech, what is the best purchasing policy?

Solution. Recall that A = \$20, D = 800 units/year, i = 0.20 annually. Other relevant data and calculations are given in table 6-3. We follow the procedure for incremental discount to find Q^*.

Step 0: Set $Q^* = 0$, $K^*(Q^*) = \infty$, and $j = 1$.

Step 1: Compute

$$Q_1 = \sqrt{\frac{2D\left[A + C(q_0) - c_1 q_0\right]}{ic_1}} = \sqrt{\frac{2(800)[20 + 0 - 0]}{(0.20)(0.60)}} \cong 516$$

Because $Q_1 = 516 > 500$, it is not in the interval, so set $K_1(Q_1) = \infty$.

Step 2: Set $j = 1 + 1 = 2 < 3 = m$, so go to step 1.

Step 1: $$Q_2 = \sqrt{\frac{2D\left[A + C(q_1) - c_2 q_1\right]}{ic_2}} \cong 643$$

$500 < Q < 1000$. therefore, we calculate;

$$K_2(Q_2) = c_2(Q_2)D + \frac{AD}{Q_2} + ic_2(Q_2)\left(\frac{Q_2}{2}\right)$$

$$K_2(643) = \left(0.58 + \frac{10}{643}\right)800 + \frac{20 \times 800}{643} + 0.2\left(0.58 + \frac{10}{643}\right)\left(\frac{643}{2}\right) = \$539.63$$

Step 2: Set j = 2 + 1 = 3 ≤ m, so go to step 1

Step 1: $Q_3 \cong 845$ which is not in the interval (1000, ∞], so we set $K_3(Q_3) = \infty$

Step 2: Set j = 3 + 1 = 4 ≥ m, so go to step 3

Step 3: $K_2(Q_2) = \text{Min}_{j=1,m}\ K_j(Q_j) = \text{Min}\ \{\ \infty,\ 539.62,\ \infty\ \}$; then $Q^* = Q_2 = 643$ and $K^*(Q^*) = K_2(Q_2) = \$539.62$

Therefore, if Goldpoint were to place an order with ElectroTech, each order would be 643 units, and the average annual cost is $539.62. this cost is more expensive compared with 1000 units and an average annual cost of $520 for purchasing from Southern Electronic. Obviously, Southern Electronic should be preferred, not only because of the cost advantage but also because of the convenience of placing fewer orders per year as a consequence of the larger order quantity. In figure 6-12 we show the three cost curves of the incremental discount case. Compare it with figure 6-11 for all-units discount.

TABLE 6-3 $C(Q)$ Average unit cost

J	q_j	$C_j(Q)$	$C(q_j)$
1	500	0.60	300
2	1000	$\frac{300+0.58(Q-500)}{Q} = \frac{10}{Q} + 0.58$	590
3	>1000	$\frac{590+0.56(Q-1000)}{Q} = \frac{30}{Q} + 0.56$	-

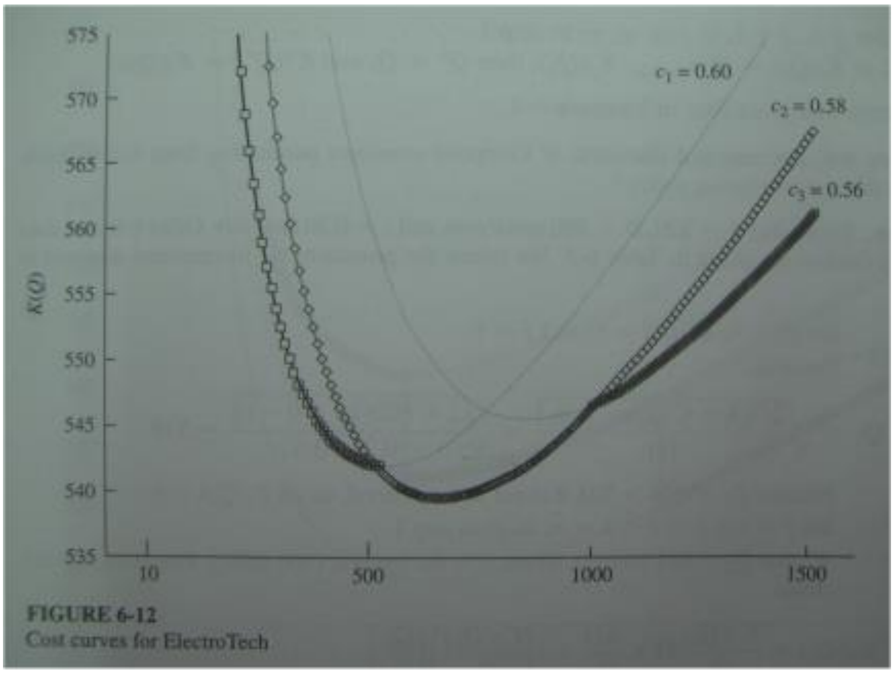

Figure 6.12. Cost curves for ElectroTech

Easy way of finding $Q^*, K^*(Q^*)$

If we want to use computer programming for finding $Q^*, K^*(Q^*)$, that procedures will be beneficial, but for calculating by hand, we introduce better and easier way:

. Let

q_j= the upper limit of the *j*th price break interval
c_j= the cost of a unit in the *j*th price break interval [q_{j-1}, q_j]
Q_j= the EOQ quantity, calculated using c_j
Q_j^*= the best order quantity in interval *j*
Q^*= the optimal order quantity over all prices
$K_j(Q)$ = the cost of *Q* units in interval *j*
$K_j(Q_j)$ = the cost of EOQ units in interval *j*
$K_j({Q_j}^*)$ = the minimum cost in interval *j*
$K^*(Q)^*$= the minimum cost over all prices
$c_j(Q)$= the cost to purchase *Q* units in interval *j*
$c(q_{j-1})$= total cost at break point j-1

Procedure:

- Find Q_j for each interval
- If Q_j is valid ($q_{j-1} < Q_j < q_j$)→ $Q_j^* = Q_j$
- If $Q_j < q_{j-1} \rightarrow Q_j^* = q_{j-1}$
- If $Q_j > q_j \rightarrow Q_j^* = q_j$
- Calculate $K({Q_j}^*)$ for each Q_j^*, minimum $K({Q_j}^*) = K^*(Q^*)$, $Q^* = {Q_j}^*$ *related to* minimum $K({Q_j}^*)$

In this procedure we should define formula of finding Q_j and $K(Q_j^*)$

All units discount

$$Q_j = \sqrt{\frac{2AD}{ic_j}}, \qquad K(Q_j^*) = c_j D + \frac{AD}{Q_j^*} + ic_j(\frac{Q_j^*}{2})$$

Incremental Discount

$$Q_j = \sqrt{\frac{2(A + c(q_{j-1}) - c_j(q_{j-1}))D}{ic_j}}, \qquad K_j(Q_j^*) = c_j(Q_j^*)D + \frac{AD}{Q_j^*} + ic_j(Q_j^*)(\frac{Q_j^*}{2})$$

Gold point example

- **All units discount**

$$Q_j = \sqrt{\frac{2AD}{ic_j}}$$

$$= \begin{cases} Q_1 = \sqrt{\frac{2AD}{ic_1}} = \sqrt{\frac{2*20*800}{0.2*0.6}} = 516.3978, Q_1 > q_1 \rightarrow Q_1^* = 500\, unit \\ Q_2 = \sqrt{\frac{2AD}{ic_2}} = \sqrt{\frac{2*20*800}{0.2*0.58}} = 525.22, q_1 < Q_2 < q_2 \rightarrow Q_2^* = 525.22\, unit \\ Q_3 = \sqrt{\frac{2AD}{ic_3}} = \sqrt{\frac{2*20*800}{0.2*0.56}} = 534.5225, Q_3 > q_3 \rightarrow Q_3^* = 1000\, unit \end{cases}$$

$$K(Q^*) = c_j D + \frac{AD}{Q_j^*} + ic_j\left(\frac{Q_j^*}{2}\right)$$

$$= \begin{cases} K(Q_1^*) = c_1 D + \frac{AD}{Q_1^*} + ic_1\left(\frac{Q_1^*}{2}\right) = 0.6*800 + \frac{20*800}{500} + 0.2*0.6*\frac{500}{2} = \$542 \\ K(Q_2^*) = c_2 D + \frac{AD}{Q_2^*} + ic_2\left(\frac{Q_2^*}{2}\right) = 0.58*800 + \frac{20*800}{525.22} + 0.2*0.58*\frac{525.22}{2} = \$524.92 \\ K(Q_3^*) = c_3 D + \frac{AD}{Q_3^*} + ic_3\left(\frac{Q_3^*}{2}\right) = 0.56*800 + \frac{20*800}{1000} + 0.2*0.56*\frac{1000}{2} = \$520 \end{cases}$$

Minimum $K\left({Q_j}^*\right) = K({Q_3}^*) = K^*(Q^*) = \$520$

$Q^* = {Q_3}^* = 1000\, unit$

- **Incremental Discount**

$$Q_j = \sqrt{\frac{2(A + c\left(q_{j-1}\right) - c_j(q_{j-1}))D}{ic_j}}$$

$$= \begin{cases} Q_1 = \sqrt{\frac{2\left(A + c(q_0) - c_1(q_0)\right)D}{ic_1}} = \sqrt{\frac{2(20+0-0)800}{0.2*0.6}} = \\ 516.3978, Q_1 > q_1 \rightarrow Q_1^* = 500\, unit \\ Q_2 = \sqrt{\frac{2\left(A + c(q_1) - c_2(q_1)\right)D}{ic_2}} = \sqrt{\frac{2(20+300-0.58*500)800}{0.2*0.58}} = \\ 643.2675, q_1 < Q_2 < q_2 \rightarrow Q_2^* = 643.2675\, unit \\ Q_3 = \sqrt{\frac{2\left(A + c(q_2) - c_3(q_2)\right)D}{ic_3}} = \sqrt{\frac{2(20+590-0.56*1000)800}{0.2*0.56}} = \\ 845.15, Q_3 < q_3 \rightarrow Q_3^* = 1000\, unit \end{cases}$$

$$K_j(Q_j^*) = c_j(Q_j^*)D + \frac{AD}{Q_j^*} + ic_j(Q_j^*)(\frac{Q_j^*}{2})$$

$$= \begin{cases} K(Q_1^*) = c_1(Q_1^*)D + \frac{AD}{Q_1^*} + ic_1(Q_1^*)\left(\frac{Q_1^*}{2}\right) = \\ 0.6 * 800 + \frac{20 * 800}{500} + 0.2 * 0.6 * \frac{500}{2} = \$542 \\ K(Q_2^*) = c_2(Q_2^*)D + \frac{AD}{Q_2^*} + ic_2(Q_2^*)\left(\frac{Q_2^*}{2}\right) = \\ (\frac{10}{643.2675} + 0.58) * 800 + \frac{20 * 800}{643.2675} + 0.2 * (\frac{10}{643.2675} + 0.58) * \frac{643.2675}{2} = \$539.6031 \\ K(Q_3^*) = c_3(Q_3^*)D + \frac{AD}{Q_3^*} + ic_3(Q_3^*)\left(\frac{Q_3^*}{2}\right) = \\ (\frac{10}{1000} + 0.58) * 800 + \frac{20 * 800}{1000} + 0.2 * (\frac{10}{1000} + 0.58) * \frac{1000}{2} = \$547 \end{cases}$$

Minimum $K(Q_j{}^*) = K(Q_2{}^*) = K^*(Q^*) = \$539.6031$

$Q^* = Q_2{}^* = 643.2675 \cong 644\ unit$

Conclusion: (In Gold print example) all units discount model is better than incremental model (because of higher value of order and lower value of total average cost)

4 CONTROL DECISIONS

We have introduced a variety of models, policies, and approaches to various aspects of inventory systems. Now we focus on managing and controlling multi-item inventory systems. Multi-item systems may have 300, 3000, 30000, or even 300000 items! We still want to minimize cost and maximize service.

4.1 Pareto Analysis

Pareto analysis, a tool to separate “important” from “nonimportant”, is a useful technique for allocating management effort. It is named after the Italian economist Villfredo Pareto, who studied the distribution of wealth in Milan in the eighteenth century. He noted that a large portion of the wealth was owned by a small segment of the population. The same Pareto principle applies to many other situations; the few have great importance, and the many have little importance. Inventory systems typically have a few items that account for large annual dollar usage (or sales). This feature allows a trade-off between investment and control-an important element in maintaining low cost and high service.

The Pareto principle was first applied to inventory systems by Dickie (1951) of General Electric. He called it **ABC analysis**: A items are few “important” items, and C items are the many “unimportant” items. B items fall between A and C terms. In industry, Pareto analysis is known as ABC analysis. For clarity, we call the tool ABC and the theory Pareto.

4.1.1. The ABC Curve the ABC curve ranks the inventory items in descending order of annual dollar usage (or sales). The ranking in tabular form is called distribution by value; Table 6.18 is a typical example. We can plot the percentage of ranked items to total items against the corresponding cumulative percent of total dollar value represented by that percent of the ranked items. Plot 1 in figure 6-26 is the plot corresponding to table 6-18. In principle, we can classify the ranked items into three groups:

A = "high dollar usage" items
B = "medium dollar usage" items
C = "low dollar usage" items

Typically, ABC curves show that group A items are about 20 percent of the ranked items and 80 percent of the total dollar usage. This is sometimes called the "20-80" rule. That these two numbers total 100 is coincidence.
In more detail, the procedure for preparing ABC curves is

Step 1: Tabulate the inventory items in descending order of annual dollar usage per items. The annual dollar usage is the product of the unit cost and the annual number of units used.
Step 2: Evaluate the cumulative activity by starting at the top of the list and accumulating item activity downwards.

Step 3: Work downwards and calculate:

- Cumulative percent of items based on the total number of items
- Cumulative percent of dollar usage, based on the annual total dollar usage

Step 4: Plot the ABC curve cumulative percent of dollar usage as a function of cumulative percent of items.
Table 6-18 shows the above steps for an inventory system with 29,073 items and annual usage of over $20,000,000. Obviously, it is impossible to show all items, so selected values are tabulated. The graphical description of the distribution by value-the ABC curve-is shown in Figure 6-26 (curve 1)

Table 6.18. Distribution by value

Rank	% of active items	Annual dollar usage	Cumulative annual dollar usage	Cumulative % of total usage
1	0.0034	292,150	292,150	1.43
2	0.0069	225,549	517,699	2.53
3	0.0103	153,418	671,117	3.28
4	0.0138	149,797	820,915	4.01
5	0.0172	146,697	967,614	4.73
6	0.0207	135,362	1,102,976	5.40
7	0.0241	131,011	1,233,987	6.04
8	0.0276	130,760	1,364,857	6.68
9	0.0310	124,702	1,489,559	7.29
10	0.0344	122,959	1,612,518	7.89
11-25	0.0861	63,672	2,907,815	14.23
26-77	0.265	28,900	5,113,576	25.03
78-119	0.410	21,205	6,144,048	30.08
120-248	0.854	12,387	8,170,675	40.00
249-461	1.59	7,418	10,214,450	50.00
462-820	2.82	4,464	12,254,380	60.00
821-1435	4.94	2,559	14,298,174	70.00
1436-2543	8.76	1,338	16,339,170	80.00
2544-5061	17.43	491	18,381,088	90.00
5062-8151	28.07	219	19,402,217	95.00
8152-9197	31.67	172	19,606,515	96.00
9198-10,565	36.38	129	19,810,651	97.00
10,566-12,496	43.03	86	20,014,900	98.00
12,497-15,650	53.89	47	20,219,132	99.00
15,651-16,279	56.06	42	20,247,133	99.13
16,280-18,001	61.99	30	20,307,996	99.43
18,002-20,443	70.40	18	20,364,329	99.71
20,444-24,009	82.68	7	20,407,231	99.92
14,010-29,037	1.00	0	20,423,347	100.00

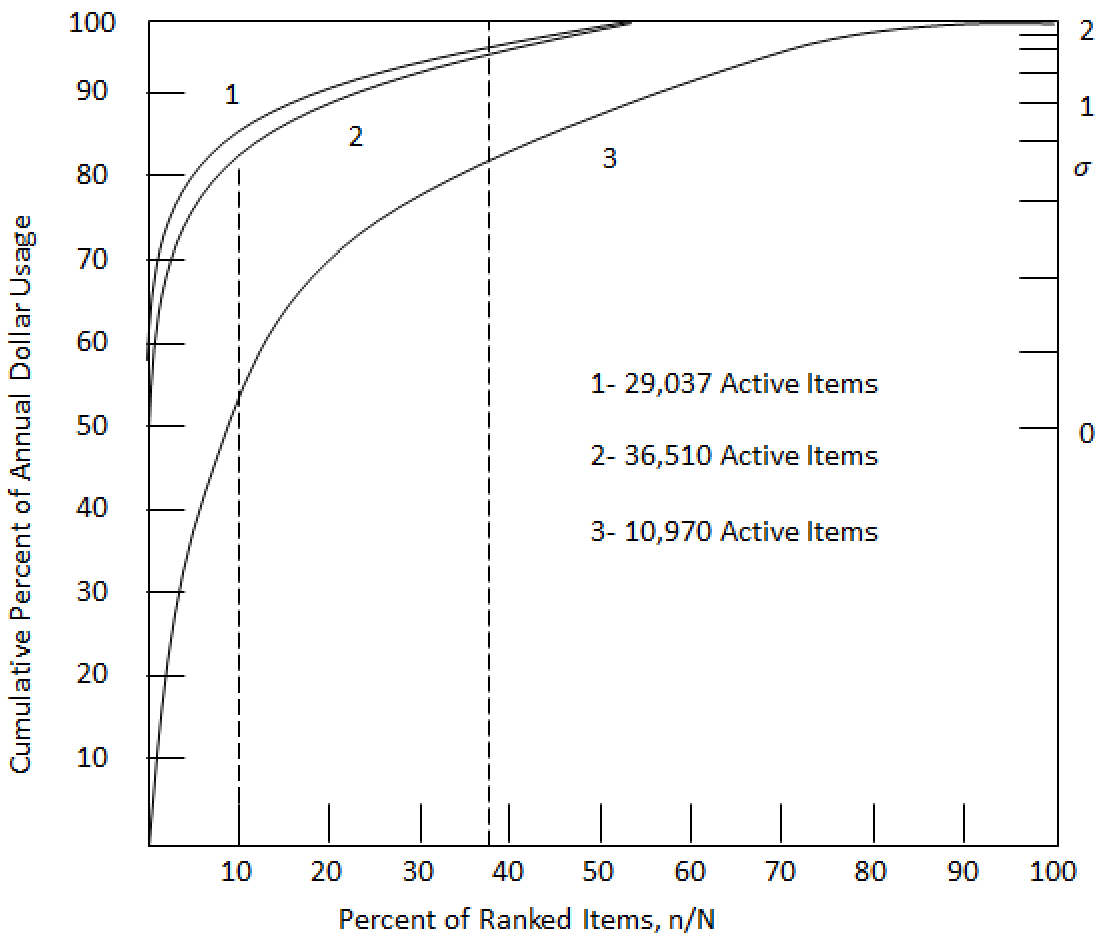

Figure 6-26. ABC curves for three companies

Some observations about ABC curve:

- There is no fixed convention as to which items are in the A, B, or C group. This is usually done by “eyeballing” the curve. Group A is roughly where the ABC curve starts bending, B is at the end of this bending, and C includes the rest of the items.
- Figure 5.1 shows two other curves (2, 3). These curves are less “steep”, and the distinction between A and B is more difficult.
- Typically, the steeper the ABC curve is, the more separation power it has. Separation power is the ability to differentiate between the groups. That is, fewer items will account for higher value; e.g. 15 percent of the items account for 90 percent of the value.
- Unit cost is not the reason for placing an item in the A group. A cheap item with high annual usage can be classified as an A item, and vice versa.

This discussion shows how Pareto principle helps allocate management efforts. Group A, which has most of the investment in inventory, is managed closely. Group C items get little management attention; they are worth less effort.

Example 6-12: ABC is a production company. They use 10 different items in their production. The list of these items and some related data are given in the following table. Apply the ABC analysis and find the classes of the items.

Item Number	101	102	103	104	105	106	107	108	109	110
Unit COST	5	11	15	8	7	16	20	4	9	12
Annual Demand	48000	2000	300	800	4800	1200	18000	300	5000	500

	Percentage of items	**Percentage value of annual usage**	
Class A items	About 20%	About 80%	Close day to day control
Class B items	About 30%	About 15%	Regular review
Class C items	About 50%	About 5%	Infrequent review

Step 1 Calculate the total spending per year

Item number	**Unit cost**	**Annual demand**	**Total cost per year**
101	5	48,000	240,000
102	11	2,000	22,000
103	15	300	4,500
104	8	800	6,400
105	7	4,800	33,600
106	16	1,200	19,200
107	20	18,000	360,000
108	4	300	1,200
109	9	5,000	45,000
110	12	500	6,000
Total usage			737,900

Total cost per year: Unit cost * total cost per year

Step 2: Calculate the usage of item in total usage

Item number	Unit cost	Annual demand	Total cost per year	Usage as a % of total usage
101	5	48,000	240,000	32,5%
102	11	2,000	22,000	3%
103	15	300	4,500	0,6%
104	8	800	6,400	0,9%
105	7	4,800	33,600	4,6%
106	16	1,200	19,200	2,6%
107	20	18,000	360,000	48,8%
108	4	300	1,200	0,2%
109	9	5,000	45,000	6,1%
110	12	500	6,000	0,8%
Total usage			737,900	100%

Usage as a % of total usage = usage of item/total usage

Step 3: Sort the items by usage

Item number	Cumulative % of items	Unit cost	Annual demand	Total cost per year	Usage as a % of total usage	Cumulative % of total
107	10%	20	18,000	360,000	48,8%	48,8%
101	20%	5	48,000	240,000	32,5%	81,3%
109	30%	9	5,000	45,000	6,1%	87,4%
105	40%	7	4,800	33,600	4,6%	92%
102	50%	11	2,000	22,000	3,0%	94,9%
106	60%	16	1,200	19,200	2,6%	97,5%
104	70%	8	800	6,400	0,9%	98,4%
110	80%	12	500	6,000	0,8%	99,2%
103	90%	15	300	4,500	0,6%	99,8%
108	100%	4	300	1,200	0,2%	100%
Total usage				737,900	100%	

Step 4: Results of calculation

Category	Items	Percentage of items	Percentage usage (%)	Action
Class A	107, 101	20%	81.6%	Close control
Class B	109, 105, 102	30%	13.3%	Regular review
Class C	106, 104, 110, 103, 108	50%	4.1%	Infrequent review

Example 6-13: ABC is a production company. They use 7 different items in their production. The list of these items and some related data are given in the following table. Apply the ABC analysis and find the classes of the items.

Item Number	101	102	103	104	105	106	107
Unit COST	80	0.9	3	0.2	0.3	0.1	2
Annual Demand	75	150,000	500	18,000	3,000	20,000	10,000

Step 1 Calculate the total spending per year

Item number	Annual quantity used	Unit value	Usage per year
1	75	80	6,000
2	150,000	0,9	135,000
3	500	3,0	1,500
4	18,000	0,20	3,600
5	3,000	0,30	900
6	20,000	0,10	2,000
7	10,000	2	20,000
Total usage			169,000

Step 2: Calculate the usage of item in total usage

Item number	Annual quantity used	Unit value	Usage per year	Percentage in total usage (%)
1	75	80	6,000	3,51%
2	150,000	0,9	135,000	79,8%
3	500	3,0	1,500	0,87%
4	18,000	0,20	3,600	2,1%
5	3,000	0,30	900	0,53%
6	20,000	0,10	2,000	1,18%
7	10,000	2	20,000	11,8%
Total usage			169,000	

Step 3: Sort the items by usage

Item number	Cumulative % of items	Annual quantity used	Unit value	Usage per year	Percentage in total usage (%)	Cumulative % of total
2	14%	150,000	0,9	135,000	79,8%	79,8%
7	29%	10,000	2	20,000	11,8%	91,6%
1	42%	75	80	6,000	3,51%	95,11%
4	56%	18,000	0,20	3,600	2,1%	97,21%
6	71%	20,000	0,10	2,000	1,18%	98,39%
3	84%	500	3,0	1,500	0,87%	99,46%
5	100%	3,000	0,30	900	0,53%	100%
Total usage				169,000		

Step 4: Results of calculation

Cathegory	Items	Percentage of items	Percentage of usage (%)	Action
Class A items	2	14%	79,8%	Close control
Class B items	7, 1	38%	15,31%	Regular review
Class C items	4, 6, 3, 5	58%	4,89%	Infrequent review