# MATH103 Mathematics for Business and Economics – I

The Mathematical Finance, Compound Interest

# Compound Interest

In these sections we will learn about of the more practical uses of mathematics that is, the mathematics of finance. This is concerned with how mathematics is used to calculate interest and finance charges.

#### <u>Terminology</u>

The **principal** (*P*) represents the amount of money initially put into an investment such as a savings account or a loan. (This is sometimes referred to as the **present value** (*PV*) of the investment.

The amount of money earned by the investment is called the interest (*I*). This represents the profit for the person investing their money.

The **rate** (*r*) is the percentage of the principal earned on the investment over a certain time period. The rate is usually given on a yearly basis unless it is otherwise stated.

The **term** (*t*) is the length of time the investment is to last. It is important the time units for the term match the time units for the rate. In other words if the rate is on yearly basis the term must be measured in years.

The future value (EU) of

the set is the total value of all money in

## **Compound Interest**

Many interest investment will pay a portion of the interest that has been earned of certain regular fixed time periods. This type of interest is called **compound interest** and the time period it is paid over is called a **compounding period**. There is a definite advantage to this type of investment in that it allows the interest to begin earning interest.

There are many different ways to name compounding periods. The chart below shows several of the most common. Keep in mind the prefix "bi" means 2 and the prefix "semi" means ½. What you want to be able to find is the number of compounding periods in 1 year. (Some of the most common

(	pnes are in rec Compouding Period	Number of Periods in 1 year	Compouding Period	Number of Periods in 1 year
	annually	1	biweekly	26
	semiannually	2	weekly	52
	quarterly	4	semiweekly	104
	bimonthly	6	daily	365
	monthly	12	semidaily	730

# **Compound Interest**

- P = principal
- S = the compound amount(Future Value FV) (i.e., the amount of money we have after n time periods)
- t = number of years
- n = number of interest periods per year
- r = annual interest rate, compounded n țimes per year
  - *n* = interest rate *per period*
- k = nt = total number of interest periods $FV = S = P\left(1 + \frac{r}{n}\right)^{nt} = P(1+i)^{k}$  I = S - P

#### <u>Example</u>

A grandfather invests \$10,000 in a trust fund for his grandson's college education when he is born. The fund pays 4% compounded monthly. How much will he have for college on his 18<sup>th</sup> birthday.

**Solution:** The questions wants to know the Compaound amount of this investment. P = 10,000, t=18, n=12r=4%  $k = 12 \cdot 18 = 216$   $i = \frac{0.04}{12}$  $S = P(1+i)^k$ I = S - P $=10,000\left(1+\frac{0.04}{12}\right)^{216}$ = 20,519.70 - 10,000=10,519.7020,519.70 **Compound Interest** 

### Example

A person wants to invest a certain amount of money now in an account that pays 6.4% compounded quarterly so that they have \$30,000 in 5 years. How much money should they invest?

**Solution**: The question wants to know the Principal (present value) for the investment. S = 30,000, r=6,4%,

$$t=5, n=4 \\ k=4 \cdot 5 = 20 \quad i = \frac{0.064}{4}$$
$$P = \frac{S}{(1+i)^{k}}$$
$$= \frac{30,000}{(1+\frac{.064}{4})^{20}}$$
$$= 21.839.70$$

I = S - P= 30,000 - 21,839.70 = 8,160.30

Compound Interest

Future value

- Example: Find the compound amount if \$2900 is deposited at 5% interest for 10 years if interest is compounded daily.
- Solution:

$$S = P(1 + \frac{i}{365})^{365n}$$
  
= \$2900(1 +  $\frac{5\%}{365})^{3650}$   
= \$4781.13

- Find the amount to which \$1500 will grow if deposited in a bank at 5.75% interest compounded quarterly for 5 years.
- Solution: Use the compound interest formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Substitute P = 1500, r = 0.0575, n = 4 and t = 5 to obtain

$$A = 1500 \left( 1 + \frac{0.0575}{4} \right)^{(4)(5)}$$

=\$1995.55

- Example: \$70,000 is invested at 8% interest, compounded annually. After time t, in years, it grows to the amount A given by the function shown below.
- Determine the amount of time after which there will be \$560,000 in the account.
- Solution:

$$S = \$70,000(1.08)^t$$

How long will it take money to double if compounded monthly at 4 % interest?

- Compound interest formula
  Replace A by 2P (double the amount)
- 3. Substitute values for r and m
- 4. Divide both sides by P
- Take In of both sides
- 6. Property of logarithms

7. Solve for *t* and evaluate expression

Solution:  $A = P\left(1 + \frac{r}{m}\right)^{m}$  $2P = P\left(1 + \frac{0.04}{12}\right)^{12t}$  $2 = (1.003333...)^{12t}$  $\ln 2 = \ln \left( (1.003333...)^{12t} \right)$  $\ln 2 = 12t \ln(1.00333...)$  $\frac{\ln 2}{12\ln(1.00333...)} = t \to t = 17.36$