

MATH103

Mathematics for Business and Economics – I

Linear Equations, Fractional Equations and
Radical Equations, Absolute Value Equality

Linear Equations

Equations

An **equation** is a statement that two expressions are equal. They are separated by the **equality sign**, “=”.

EXAMPLE 1 Examples of Equations

a. $x + 2 = 3$

b. $x^2 + 3x + 2 = 0$

c. $\frac{y}{y - 4} = 6$

d. $w = 7 - z$

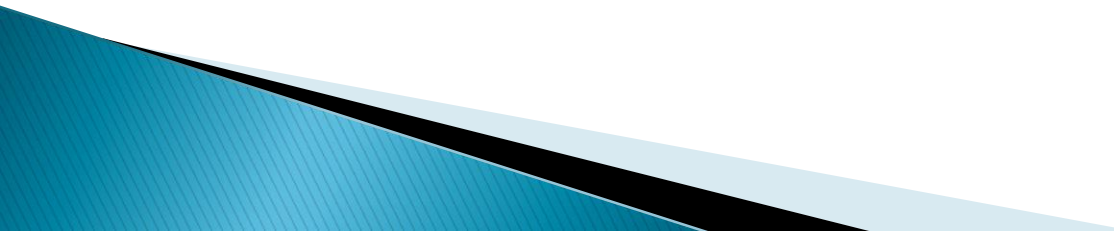
Equivalent Equations

1. Adding (subtracting) the same polynomial³ to (from) both sides of an equation, where the polynomial is in the same variable as that occurring in the equation.

For example, if $-5x = 5 - 6x$, then adding $6x$ to both sides gives the equivalent equation $-5x + 6x = 5 - 6x + 6x$, or $x = 5$.

2. Multiplying (dividing) both sides of an equation by the same nonzero constant.

For example, if $10x = 5$, then dividing both sides by 10 gives the equivalent equation $\frac{10x}{10} = \frac{5}{10}$, or $x = \frac{1}{2}$.

4. Multiplying both sides of an equation by an expression involving the variable;
 5. Dividing both sides of an equation by an expression involving the variable;
- 

LINEAR EQUATIONS

A **linear equation in one variable**, such as x , is an equation that can be written in the *standard form*

$$ax + b = 0,$$

where a and b are real numbers with $a \neq 0$.

FINDING THE SOLUTION: Procedure for solving linear equations in one variable

Step 1 Eliminate Fractions. Multiply both sides of the equation by the least common denominator (LCD) of all the fractions.

Step 2 Simplify. Simplify both sides of the equation by removing parentheses and other grouping symbols (if any) and combining like terms.

FINDING THE SOLUTION: Procedure for solving linear equations in one variable

Step 3 Isolate the Variable Term. Add appropriate expressions to both sides, so that when both sides are simplified, the terms containing the variable are on one side and all constant terms are on the other side.

Step 4 Combine Terms. Combine terms containing the variable to obtain one term that contains the variable as a factor.

FINDING THE SOLUTION: Procedure for solving linear equations in one variable

Step 5 Isolate the Variable. Divide both sides by the coefficient of the variable to obtain the solution.

Step 6 Check the Solution. Substitute the solution into the original equation.

EXAMPLE 1

Solving a Linear Equation

$$\text{Solve: } 6x - [3x - 2(x - 2)] = 11.$$

Solution

$$\text{Step 2} \quad 6x - [3x - 2(x - 2)] = 11$$

$$6x - [3x - 2x + 4] = 11$$

$$6x - 3x + 2x - 4 = 11$$

$$5x - 4 = 11$$

$$\text{Step 3} \quad 5x - 4 + 4 = 11 + 4$$

$$\text{Step 4} \quad 5x = 15$$

EXAMPLE 1

Solving a Linear Equation

$$\text{Solve: } 6x - [3x - 2(x - 2)] = 11.$$

Solution continued

$$\text{Step 5} \quad \frac{5x}{5} = \frac{15}{5}$$

$$x = 3$$

$$\text{Step 6 Check: } 6(3) - [3(3) - 2((3) - 2)] = 11$$

$$18 - [9 - 2] = 11$$

$$18 - 7 = 11$$

The solution set is $\{3\}$.

EXAMPLE 2

Solving a Linear Equation

Solve: $1 - 5y + 2(y + 7) = 2y + 5(3 - y)$.

Solution

Step 2 $1 - 5y + 2(y + 7) = 2y + 5(3 - y)$

$$1 - 5y + 2y + 14 = 2y + 15 - 5y$$

$$15 - 3y = 15 - 3y$$

Step 3 $15 - 3y + 3y = 15 - 3y + 3y$

$$15 = 15$$

$$0 = 0$$

EXAMPLE 2**Solving a Linear Equation**

Solve: $1 - 5y + 2(y + 7) = 2y + 5(3 - y)$.

Solution continued

$0 = 0$ is equivalent to the original equation.

The equation $0 = 0$ is always true and its solution set is the set of real numbers. So the solution set to the original equation is the set of real numbers. The original equation is an identity.

EXAMPLE 3

Solving a Linear Equation

Solve: $-7 + 3x - 9x = 12x - 5$

Solution:

$$-7 + 7 - 6x = 12x - 5 + 7$$

$$-6x - 12x = 12x - 12x + 2$$

$$-18x = 2$$

$$x = -\frac{1}{9}$$

The solution set is $\left\{ -\frac{1}{9} \right\}$.

EXAMPLE 4

Solving a Linear Equation

Solve $\frac{x+2}{2} - \frac{x}{3} = 5$

Solution: Since the LCD of 2 and 3 is 6, we multiply both sides of the equation by 6 to clear of fractions.

Cancel the 6 with the 2 to obtain a factor of 3, and cancel the 6 with the 3 to obtain a factor of 2.

Distribute the 3.

Combine like terms.

The solution set is $\{ 24 \}$.

$$6\left(\frac{x+2}{2} - \frac{x}{3}\right) = 6 \cdot 5$$

$$3(x+2) - 2x = 30$$

$$3x + 6 - 2x = 30$$

$$x + 6 = 30$$

$$x = 24$$

EXAMPLE 5**Solving a Linear Equation**

Solve: $6 - (4 + x) = 8x - 2(3x + 5)$

Solution:

Step 1: Does not apply

Step 2: $6 - 4 - x = 8x - 6x - 10$
 $-x + 2 = 2x - 10$

Step 3: $-x + (-2x) + 2 + (-2) = 2x + (-2x) - 10 + (-2)$
 $-3x = -12$

Step 4: $(-\frac{1}{3})(-3)x = (-\frac{1}{3})(-12)$

$$x = 4$$

Step 5: $6 - (4 + 4) = 8(4) - 2(3(4) + 5)$
 $6 - 8 = 32 - 34$
 $-2 = -2$

The solution set is $\{4\}$.

EXAMPLE 6

Solving a Linear Equation

Solve:
$$\frac{x+1}{2} + \frac{x+3}{4} = \frac{1}{2}$$

Solution:

Step 1:
$$4\left(\frac{x+1}{2} + \frac{x+3}{4}\right) = 4\left(\frac{1}{2}\right)$$

Step 2:
$$4\left(\frac{x+1}{2}\right) + 4\left(\frac{x+3}{4}\right) = 4\left(\frac{1}{2}\right)$$

$$2(x+1) + (x+3) = 2$$

$$2x + 2 + x + 3 = 2$$

Step 3:
$$3x + 5 = 2$$

$$3x + 5 + (-5) = 2 + (-5)$$

$$3x = -3$$

Step 4:
$$\left(\frac{1}{3}\right) 3x = \left(\frac{1}{3}\right)(-3)$$

$$x = -1$$

Step 5:

The solution set is $\{-1\}$.

Check:
$$\frac{-1+1}{2} + \frac{-1+3}{4} = \frac{1}{2} \qquad \frac{2}{4} = \frac{1}{2} \qquad \frac{1}{2} = \frac{1}{2}$$

Solve $5x - 6 = 3x$.

Solution: We begin by getting the terms involving x on one side and the constant on the other. Then we solve for x by the appropriate mathematical operation. We have

$$5x - 6 = 3x$$

$$5x - 6 + (-3x) = 3x + (-3x) \quad (\text{adding } -3x \text{ to both sides})$$

$$2x - 6 = 0 \quad (\text{simplifying, that is, Operation 3})$$

$$2x - 6 + 6 = 0 + 6 \quad (\text{adding 6 to both sides})$$

$$2x = 6 \quad (\text{simplifying})$$

$$\frac{2x}{2} = \frac{6}{2} \quad (\text{dividing both sides by 2})$$

$$x = 3$$

the solution set is $\{3\}$.

EXAMPLE 8

Solving a Linear Equation

Solve $2(p + 4) = 7p + 2$.

Solution: First, we remove parentheses. Then we collect similar terms and solve. We have

$$2(p + 4) = 7p + 2$$

$$2p + 8 = 7p + 2 \quad (\text{distributive property})$$

$$2p = 7p - 6 \quad (\text{subtracting 8 from both sides})$$

$$-5p = -6 \quad (\text{subtracting } 7p \text{ from both sides})$$

$$p = \frac{-6}{-5} \quad (\text{dividing both sides by } -5)$$

$$p = \frac{6}{5}$$

Solution set $\left\{ \frac{6}{5} \right\}$

EXAMPLE 8

Solving a Linear Equation

$$\text{Solve } \frac{7x + 3}{2} - \frac{9x - 8}{4} = 6.$$

Solution: We first clear the equation of fractions by multiplying *both* sides by the least common denominator (LCD),⁴ which is 4. Then we use various algebraic operations to obtain a solution. Thus,

$$4 \left(\frac{7x + 3}{2} - \frac{9x - 8}{4} \right) = 4(6)$$

$$4 \cdot \frac{7x + 3}{2} - 4 \cdot \frac{9x - 8}{4} = 24 \quad (\text{distributive property})$$

$$2(7x + 3) - (9x - 8) = 24 \quad (\text{simplifying})$$

$$14x + 6 - 9x + 8 = 24 \quad (\text{distributive property})$$

$$5x + 14 = 24 \quad (\text{simplifying})$$

$$5x = 10 \quad (\text{subtracting 14 from both sides})$$

$$x = 2 \quad (\text{dividing both sides by 5})$$

Solution set $\{2\}$

Fractional Equations

A **fractional equation** is an equation in which an unknown is in a denominator. We illustrate that solving such a nonlinear equation may lead to a linear equation.

EXAMPLE 8 Solving a Fractional Equation

Solve $\frac{5}{x-4} = \frac{6}{x-3}$.

Multiplying both sides by the LCD, $(x-4)(x-3)$, we have

$$(x-4)(x-3) \left(\frac{5}{x-4} \right) = (x-4)(x-3) \left(\frac{6}{x-3} \right)$$

$$5(x-3) = 6(x-4) \quad (\text{linear equation})$$

$$5x - 15 = 6x - 24$$

$$9 = x$$

Solution set $\{9\}$

EXAMPLE 9 Solving Fractional Equations

a. *Solve* $\frac{3x + 4}{x + 2} - \frac{3x - 5}{x - 4} = \frac{12}{x^2 - 2x - 8}$.

Solution: Observing the denominators and noting that

$$x^2 - 2x - 8 = (x + 2)(x - 4)$$

we conclude that the LCD is $(x + 2)(x - 4)$. Multiplying both sides by the LCD, we have

$$(x + 2)(x - 4) \left(\frac{3x + 4}{x + 2} - \frac{3x - 5}{x - 4} \right) = (x + 2)(x - 4) \cdot \frac{12}{(x + 2)(x - 4)}$$

$$(x - 4)(3x + 4) - (x + 2)(3x - 5) = 12$$

$$3x^2 - 8x - 16 - (3x^2 + x - 10) = 12$$

$$3x^2 - 8x - 16 - 3x^2 - x + 10 = 12$$

$$-9x - 6 = 12$$

$$-9x = 18$$

$$x = -2$$

Solution set = \emptyset

b. Solve $\frac{4}{x-5} = 0$.

Solution: The only way a fraction can equal zero is if the numerator is 0 but its denominator is not. Since the numerator, 4, is never 0, the solution set is \emptyset .

Radical Equations

A **radical equation** is one in which an unknown occurs in a radicand. The next two examples illustrate the techniques employed to solve such equations.

EXAMPLE 11 Solving a Radical Equation

Solve $\sqrt{x^2 + 33} - x = 3$.

We begin by isolating the radical on one side.

$$\sqrt{x^2 + 33} = x + 3$$

Then we square both sides and solve using standard techniques. Thus.

$$x^2 + 33 = (x + 3)^2 \quad (\text{squaring both sides})$$

$$x^2 + 33 = x^2 + 6x + 9$$

$$24 = 6x$$

$$4 = x$$

Solution set $\{4\}$

EXAMPLE 12 Solving a Radical Equation

Solve $\sqrt{y-3} - \sqrt{y} = -3$.

$$\sqrt{y-3} = \sqrt{y} - 3$$

$$y - 3 = y - 6\sqrt{y} + 9 \quad (\text{squaring both sides})$$

$$6\sqrt{y} = 12$$

$$\sqrt{y} = 2$$

$$y = 4 \quad (\text{squaring both sides})$$

Solution set = \emptyset

EXAMPLE 4

Solving Equations Involving Radicals

Solve: $x = \sqrt{6x - x^3}$

Solution

Since $(\sqrt{a})^2 = a$, we raise both sides to power 2.

$$x^2 = (\sqrt{6x - x^3})^2$$

$$x^2 = 6x - x^3$$

$$x^3 + x^2 - 6x = 0$$

$$x(x^2 + x - 6) = 0$$

$$x(x + 3)(x - 2) = 0$$

EXAMPLE 4

Solving Equations Involving Radicals

Solution continued

$$x = 0 \quad \text{or} \quad x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = 0 \quad \text{or} \quad x = -3 \quad \text{or} \quad x = 2$$

Check each solution.

$$-3 \stackrel{?}{=} \sqrt{6(-3) - (-3)^3}$$

$$-3 \neq 3$$

$$0 \stackrel{?}{=} \sqrt{6(0) - (0)^3}$$

$$0 = 0 \quad \checkmark$$

$$2 \stackrel{?}{=} \sqrt{6(2) - (2)^3}$$

$$2 = 2 \quad \checkmark$$

-3 is an extraneous solution.

The solution set is $\{0, 2\}$.

Solve: $\sqrt{2x + 1} + 1 = x$

Solution

Step 1 Isolate the radical on one side.

$$\sqrt{2x + 1} = x - 1$$

Step 2 Square both sides and simplify.

$$\left(\sqrt{2x + 1}\right)^2 = (x - 1)^2$$

$$2x + 1 = x^2 - 2x + 1$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

EXAMPLE 5

Solving Equations Involving Radicals

Solution continued

Step 3 Set each factor = 0.

$$x = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 0 \quad \text{or} \quad x = 4$$

Step 4 Check.

$$\sqrt{2(0)+1} + 1 \stackrel{?}{=} 0$$

$$2 \neq 0$$

$$\sqrt{2(4)+1} + 1 \stackrel{?}{=} 4$$

$$4 = 4 \quad \checkmark$$

0 is an extraneous solution.

The solution set is $\{4\}$.

EXAMPLE 6**Solving an Equation Involving Two Radicals**

Solve: $\sqrt{2x - 1} - \sqrt{x - 1} = 1$

Solution

Step 1 Isolate one of the radicals.

$$\sqrt{2x - 1} = 1 + \sqrt{x - 1}$$

Step 2 Square both sides and simplify.

$$\left(\sqrt{2x - 1}\right)^2 = \left(1 + \sqrt{x - 1}\right)^2$$

$$2x - 1 = 1 + 2\sqrt{x - 1} + x - 1$$

$$2x - 1 = 2\sqrt{x - 1} + x$$

Solution continued

Step 3 Repeat the process - isolate the radical, square both sides, simplify and factor.

$$x - 1 = 2\sqrt{x - 1}$$

$$(x - 1)^2 = (2\sqrt{x - 1})^2$$

$$x^2 - 2x + 1 = 4(x - 1)$$

$$x^2 - 6x + 5 = 0$$

$$(x - 5)(x - 1) = 0$$

EXAMPLE 6

Solving an Equation Involving Two Radicals

Solution continued

Step 4 Set each factor = 0.

$$x - 5 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = 5 \quad \text{or} \quad x = 1$$

Step 5 Check.

$$\begin{array}{l} \sqrt{2(5)-1} - \sqrt{5-1} \stackrel{?}{=} 1 \qquad \sqrt{2(1)-1} - \sqrt{1-1} \stackrel{?}{=} 1 \\ 3 - 2 = 1 \quad \checkmark \qquad \qquad \qquad 1 - 0 = 1 \quad \checkmark \end{array}$$

The solution set is $\{1, 5\}$.

ABSOLUTE VALUE EQUALITY

The absolute value of a real number x , written $|x|$, is defined as

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\text{Ex: } |3| = 3, \quad |-8| = 8, \quad |-2| = 2$$

$$\text{Ex: Solve } |x - 3| = 2$$

$$|x - 3| = 2$$

$$x - 3 = 2, \quad x - 3 = -2$$

$$x = 5, \quad x = 1$$

$$S = \{1, 5\}$$

Ex: Solve $|7 - 3x| = 5$

$$|7 - 3x| = 5$$

$$7 - 3x = 5, \quad 7 - 3x = -5$$

$$-3x = -2, \quad -3x = -12$$

$$x = \frac{2}{3}, \quad x = 4$$

$$S = \left\{ \frac{2}{3}, 4 \right\}$$

Ex: Solve $|x - 4| = -3$, the absolute value of expression always positive, so $S = \emptyset$

Ex: Solve $|x - 8| = 5$

$$|x - 8| = 5$$

$$x - 8 = 5, \quad x - 8 = -5$$

$$x = 13, \quad x = 3$$

$$S = \{3, 13\}$$

Ex: Solve $\left|\frac{4}{x}\right| = 8$

$$\left|\frac{4}{x}\right| = 8$$

$$\frac{4}{x} = 8, \quad \frac{4}{x} = -8$$

$$x = \frac{1}{2}, \quad x = -\frac{1}{2}$$

$$S = \{-1/2, 1/2\}$$

Ex: Solve $|7x + 3| = x$

$$|7x + 3| = x$$

$$7x + 3 = x, \quad 7x + 3 = -x$$

$$x = -\frac{1}{2}, \quad x = -\frac{3}{8}$$

$$S = \emptyset$$

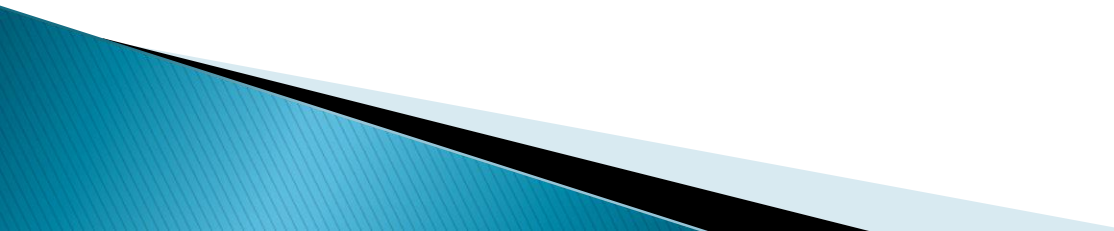
QUADRATIC EQUATION

A quadratic equation in the variable x is an equation equivalent to the equation

$$ax^2 + bx + c = 0,$$

where a , b , and c are real numbers and $a \neq 0$.

A quadratic equation is also called a second-degree equation.



THE ZERO-PRODUCT PROPERTY

Let A and B be two algebraic expressions.

Then $AB = 0$ if and only if $A = 0$ or $B = 0$.

SOLVING A QUADRATIC EQUATION BY FACTORING

- Step 1** Write the given equation in standard form so that one side is 0.
- Step 2** Factor the nonzero side of the equation from Step 1.
- Step 3** Set each factor obtained in Step 2 equal to 0.
- Step 4** Solve the resulting equations in Step 3.
- Step 5** Check the solutions obtained in Step 4 in the original equation.

EXAMPLE 1**Solving a Quadratic Equation by Factoring.**

Solve by factoring: $2x^2 + 5x = 3$.

Solution

$$2x^2 + 5x = 3$$

$$2x^2 + 5x - 3 = 0$$

$$(2x - 1)(x + 3) = 0$$

$$2x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$x + 3 = 0$$

$$x = -3$$

The solutions check in the original equation.

The solution set is

$$\left\{ \frac{1}{2}, -3 \right\}.$$

EXAMPLE 2

Solving a Quadratic Equation by Factoring.

Solve by factoring: $3t^2 = 2t$.

Solution

$$3t^2 = 2t$$

$$3t^2 - 2t = 0$$

$$t(3t - 2) = 0$$

$$t = 0 \quad | \quad 3t - 2 = 0$$

$$3t = 2$$

$$t = \frac{2}{3}$$

The solutions check in the original equation.

The solution set is

$$\left\{ 0, \frac{2}{3} \right\}.$$

Solve by factoring $x^2 + x - 12 = 0$

Solution: The left side factors easily:

$$(x - 3)(x + 4) = 0$$

Whenever the product of two or more quantities is zero, at least one of the quantities *must* be zero. This means that either

$$x - 3 = 0 \quad \text{or} \quad x + 4 = 0$$

Solving these gives $x = 3$ and $x = -4$. Thus, the roots of the original equation are 3 and -4 , and the solution set is $\{3, -4\}$.

b. Solve $6w^2 = 5w$.

Solution: We write the equation as

$$6w^2 - 5w = 0$$

so that one side is 0. Factoring gives

$$w(6w - 5) = 0$$

Setting each factor equal to 0, we have

$$w = 0 \quad \text{or} \quad 6w - 5 = 0$$

$$w = 0 \quad \text{or} \quad 6w = 5$$

Solution set $\{0, 5\}$

EXAMPLE 3 Solving a Higher Degree Equation by Factoring

a. Solve $4x - 4x^3 = 0$.

Solution: This is called a *third-degree equation*. We proceed to solve it as follows:

$$4x - 4x^3 = 0$$

$$4x(1 - x^2) = 0 \quad (\text{factoring})$$

$$4x(1 - x)(1 + x) = 0 \quad (\text{factoring})$$

Setting each factor equal to 0 gives $4 = 0$ (impossible), $x = 0$, $1 - x = 0$, or $1 + x = 0$. Thus,

$$\text{Solution set } \{0, 1, -1\}$$

EXAMPLE 3

Solving a Quadratic Equation by Factoring.

Solve by factoring: $x^2 + 16 = 8x$.

Solution

Step 1 $x^2 + 16 = 8x$

$$x^2 - 8x + 16 = 0$$

Step 2 $(x - 4)(x - 4) = 0$

Step 3 $x - 4 = 0 \quad | \quad x - 4 = 0$

Step 4 $x = 4 \quad | \quad x = 4$

Step 5 Check the solution in original equation.

$$(4)^2 + 16 = 8(4)$$

$$32 = 32$$

The solution set is $\{4\}$.

EXAMPLE 2 Solving a Quadratic Equation by Factoring

Solve $(3x - 4)(x + 1) = -2$.

Solution: We first multiply the factors on the left side:

$$3x^2 - x - 4 = -2$$

Rewriting this equation so that 0 appears on one side, we have

$$3x^2 - x - 2 = 0$$

$$(3x + 2)(x - 1) = 0$$

$$\text{Solution set } \left\{ \frac{-2}{3}, 1 \right\}$$

THE SQUARE ROOT PROPERTY

Suppose u is any algebraic expression and $d \geq 0$.

$$\text{If } u^2 = d, \text{ then } u = \pm\sqrt{d}.$$

EXAMPLE 5 **Solution by Factoring**

Solve $x^2 = 3$.

$$x = \mp\sqrt{3}$$

Solution set $\{\sqrt{3}, -\sqrt{3}\}$

EXAMPLE 4

Solving an Equation by the Square Root Method

Solve: $(x - 3)^2 = 5$.

Solution

$$(x - 3)^2 = 5$$

$$u^2 = 5$$

$$u = \pm\sqrt{5}$$

$$x - 3 = \pm\sqrt{5}$$

$$x = 3 \pm \sqrt{5}$$

The solution set is $\{3 + \sqrt{5}, 3 - \sqrt{5}\}$.

Quadratic Formula

Solving quadratic equations by factoring can be quite difficult, as is evident by trying that method on $0.7x^2 - \sqrt{2}x - 8\sqrt{5} = 0$. However, there is a formula called the *quadratic formula* that gives the roots of any quadratic equation.

Quadratic Formula

The roots of the quadratic equation $ax^2 + bx + c = 0$, where a , b , and c are constants and $a \neq 0$, are given by

$$\Delta = b^2 - 4ac$$

if $\Delta > 0$ then there are two roots $x_1 = \frac{-b + \sqrt{\Delta}}{2a}$ and $x_2 = \frac{-b - \sqrt{\Delta}}{2a}$

if $\Delta = 0$ then there are two equal roots $x_1 = x_2 = \frac{-b}{2a}$

if $\Delta < 0$ then there are no real roots in \mathbb{R}

EXAMPLE 6 **A Quadratic Equation with Two Real Roots**

Solve $4x^2 - 17x + 15 = 0$ by the quadratic formula.

$$\Delta = b^2 - 4ac = (-17)^2 - 4 \cdot 4 \cdot 15 = 49 > 0$$

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-(-17) + \sqrt{49}}{2 \cdot 4} = \frac{24}{8} = 3$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-(-17) - \sqrt{49}}{2 \cdot 4} = \frac{10}{8} = \frac{5}{4}$$

$$\text{Solution set } \left\{ 3, \frac{5}{4} \right\}$$

EXAMPLE 7 A Quadratic Equation with One Real Root

Solve $2 + 6\sqrt{2}y + 9y^2 = 0$ by the quadratic formula.

Solution $\Delta = b^2 - 4ac = (6\sqrt{2})^2 - 4 \cdot 9 \cdot 2 = 0$

$$x_1 = x_2 = \frac{-b}{2a} = \frac{-6\sqrt{2}}{2 \cdot 9} = -\frac{\sqrt{2}}{3}$$

$$\text{Solution set } \left\{ -\frac{\sqrt{2}}{3} \right\}$$

EXAMPLE 8 A Quadratic Equation with No Real Solution

Solve $z^2 + z + 1 = 0$ by the quadratic formula.

Solution $\Delta = b^2 - 4ac = (1)^2 - 4 \cdot 1 \cdot 1 = -3 < 0$

The equation has no roots