MATH103

Mathematics for Business and Economics – I

Linear Inequality Systems and Linear Programming

Graphing linear Inequalities in 2 Variables

Linear Inequalities

- A **linear inequality in two variables** is an inequality that can be written in the form

$$Ax + By < C,$$

where A, B, and C are real numbers and A and B are not both zero. The symbol $<$ may be replaced with $\leq$, $>$, or $\geq$.

The **solution set** of an inequality is the set of all ordered pairs that make it true. The **graph of an inequality** represents its solution set.

Checking Solutions

- An ordered pair (x,y) is a solution if it makes the inequality true.
- Are the following solutions to:
- $3x + 2y \geq 2$
- (0,0) (2,-1) (0,2)

$3(0) + 2(0) \geq 2$

$0 \geq 2$

Not a solution

$3(2) + 2(-1) \geq 2$

$4 \geq 2$

Is a solution

$3(0) + 2(2) \geq 2$

$4 \geq 2$

Is a solution

To sketch the graph of a linear inequality:

1. Replace the inequality symbol with an equals sign and graph this related equation. If the inequality symbol is < or >, draw the line dashed(- - - - -). If the inequality symbol is ≤ or ≥, draw the line solid(——).
2. This line separates the coordinate plane into 2 half-planes. In one half-plane – all of the points are solutions of the inequality. In the other half-plane - no point is a solution
3. Pick a test point in one of the half planes determined by the line in step 1. Use the values of x and y to determine if the test point satisfies the inequality.
4. If the test point satisfies the inequality, the half plane including the test point, therefore Shade the half-plane that has the solutions to the inequality. If not, Shade the other half plane that has the solutions to the inequality.

The graph of an inequality is the graph of all the solutions of the inequality

- Graph the inequality $3x + 2y \geq 2$
- $y \geq -3/2x + 1$ (put into slope intercept to graph easier)
- Graph the line that is the boundary of 2 half planes
- Before you connect the dots check to see if the line should be **solid** or **dashed**
- $y \geq -3/2x + 1$

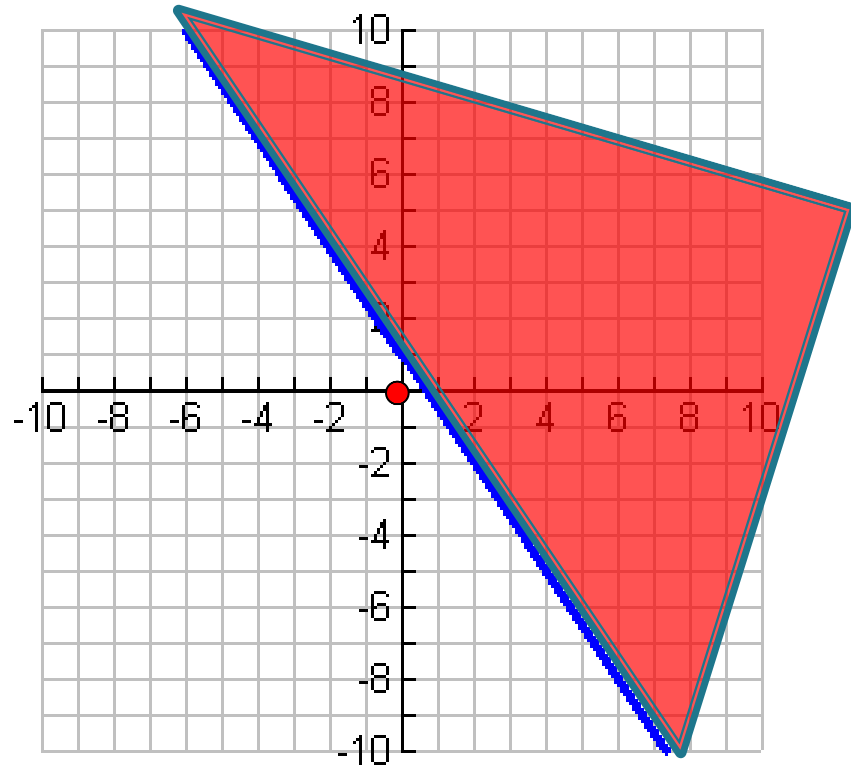
Step 1: graph the boundary
(the line is solid $\geq$)

Step 2: test a point NOT On the line

(0,0) is always The easiest if it's Not on the line!!

$3(0) + 2(0) \geq 2$

$0 \geq 2$ Not a solution



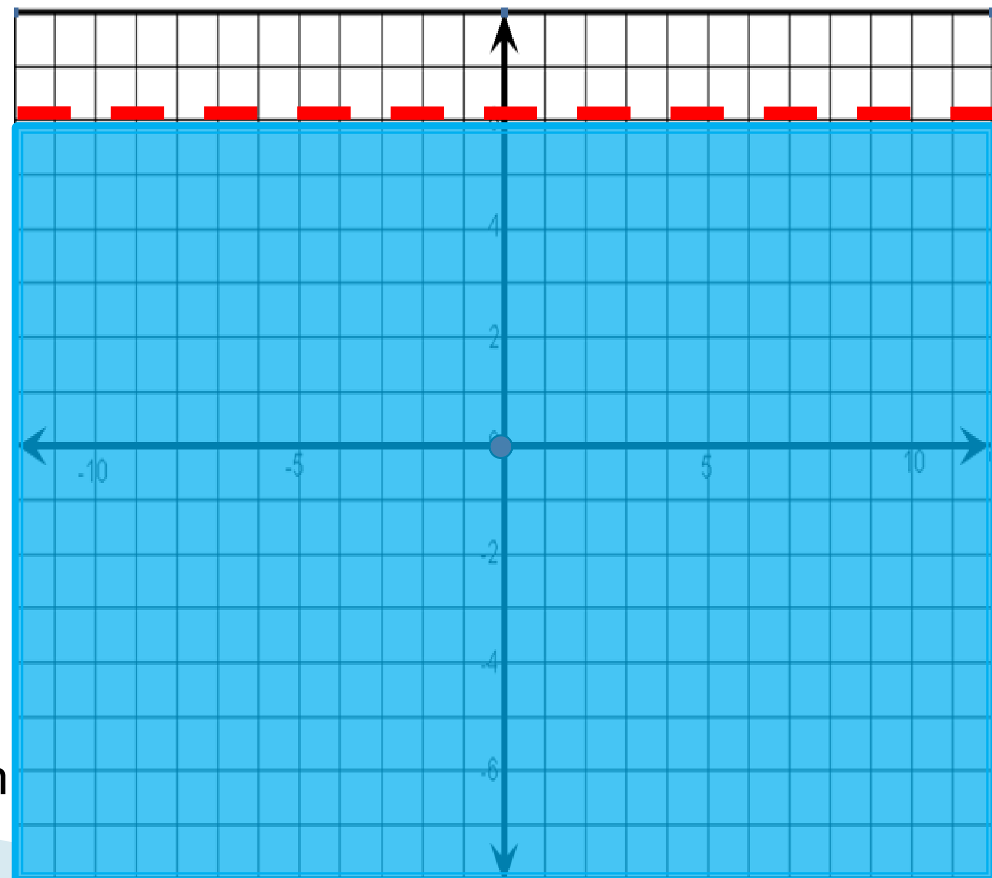
So shade the other side of the line!!

- Example : Graph the inequality y < 6
- Graph the line that is the boundary of 2 half planes
- Before you connect the dots check to see if the line should be **<u>solid</u>** or **<u>dashed</u>** y < 6

Step 1: graph the boundary
(the line is Dashed <)

Step 2: test a point NOT On the line
(0,0) is always The easiest if it's Not on the line!!
0 < 6 true its a solution

So shade the test point side of the lin



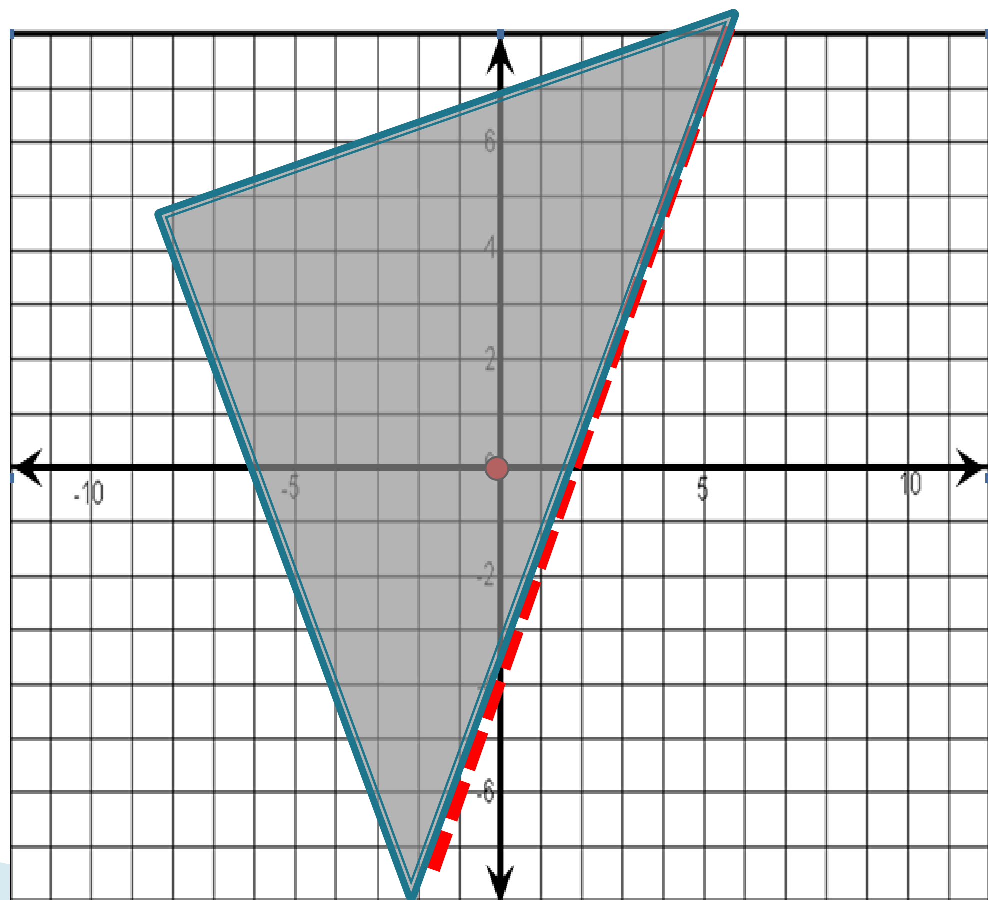
- Example : Graph the inequality $4x - 2y < 8$
- $y < -2x + 4$ (put into slope intercept to graph easier)
- Graph the line that is the boundary of 2 half planes
- Before you connect the dots check to see if the line should be **solid** or **dashed**
- $y < -2x + 4$

Step 1: graph the boundary
(the line is Dashed <)

Step 2: test a point NOT On the line
(0,0) is always The easiest if it's Not on the line!!

$4(0) - 2(0) < 8$

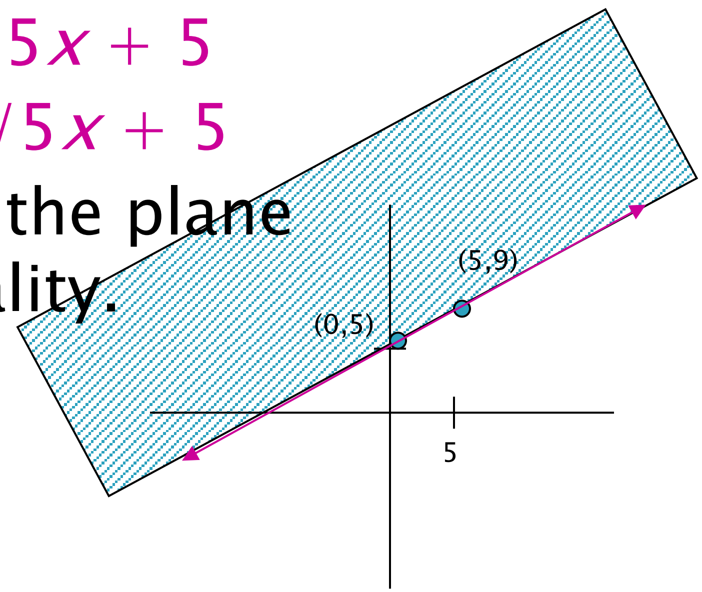
$0 < 8$ is a solution



So shade the other side of the line!!

Graph the inequality: $4x - 5y + 25 \geq 0$

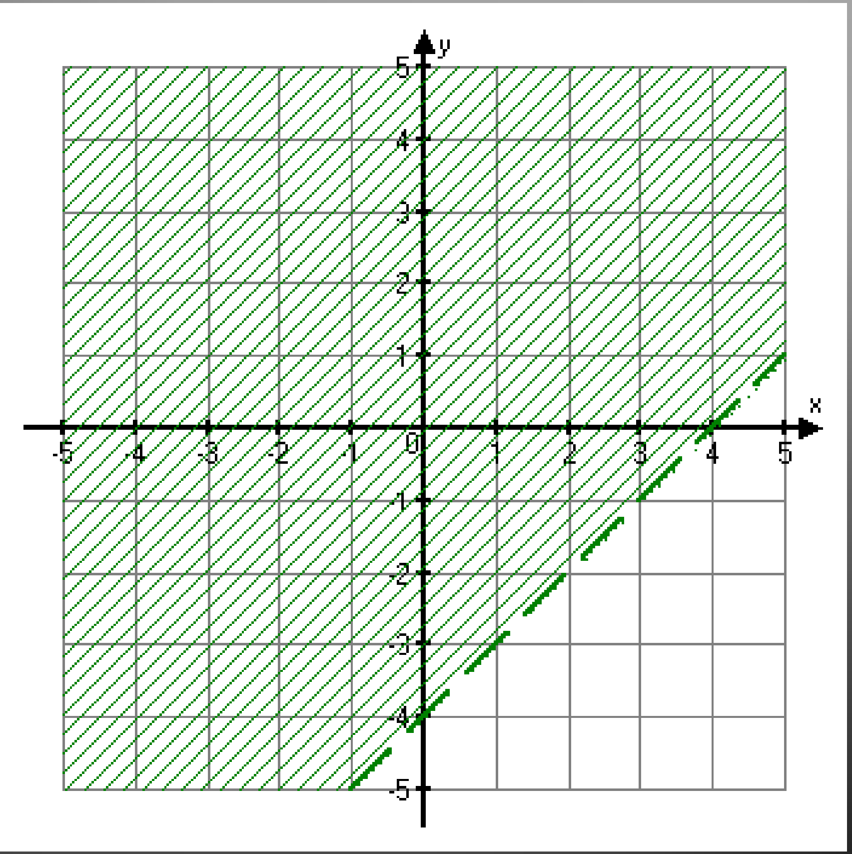
- ✓ Put inequality in standard form (watch the signs!)
- ✓ $y \leq 4/5x + 5$
- ✓ Draw the graph of $y = 4/5x + 5$
- ✓ Cross out the portion of the plane not satisfying the inequality.



Example

- Graph $y > x - 4$.
- We begin by graphing the **related equation** $y = x - 4$. We use a dashed line because the inequality symbol is $>$. This indicates that the line itself is not in the solution set.
- Determine which half-plane satisfies the inequality.

- $y > x - 4$
  $0 \,?\, 0 - 4$
  $0 > -4$ True



Example

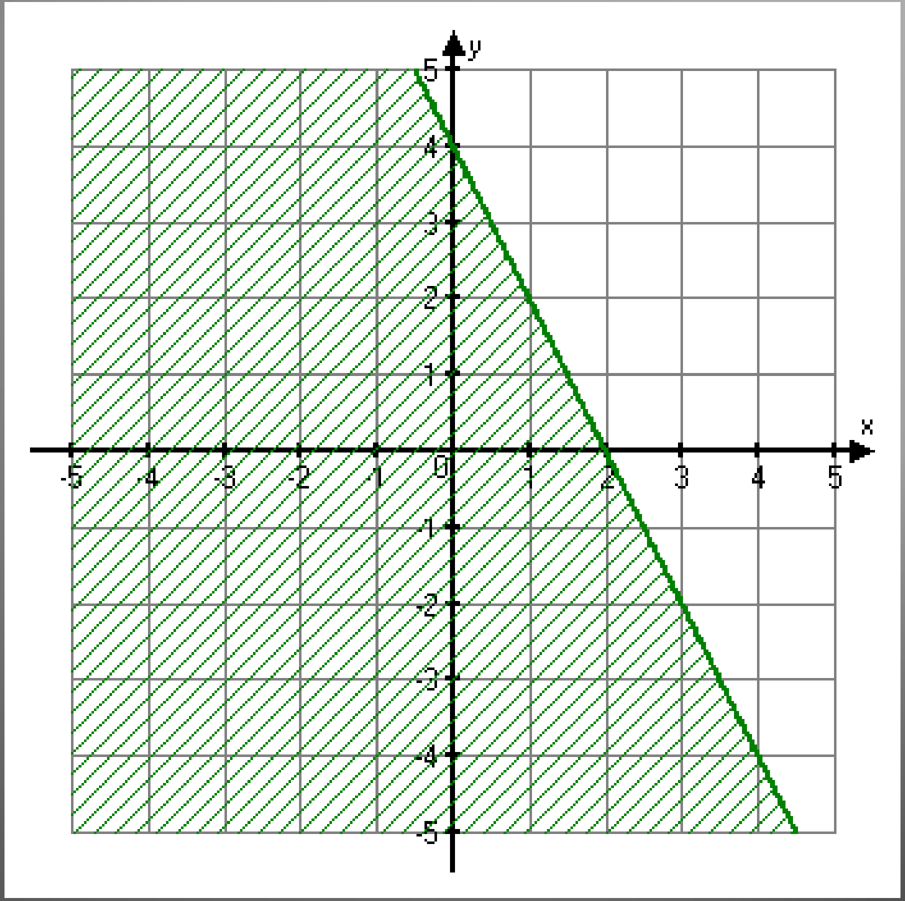
- Graph: $4x + 2y \leq 8$
- 1. Graph the related equation, using a solid line.
- 2. Determine which half-plane to shade.

$$4x + 2y \leq 8$$

$$4(0) + 2(0) \quad ? \; 8$$

$$0 \leq 8$$

We shade the region containing (0, 0).



Example

- Graph $x > 2$ on a plane.

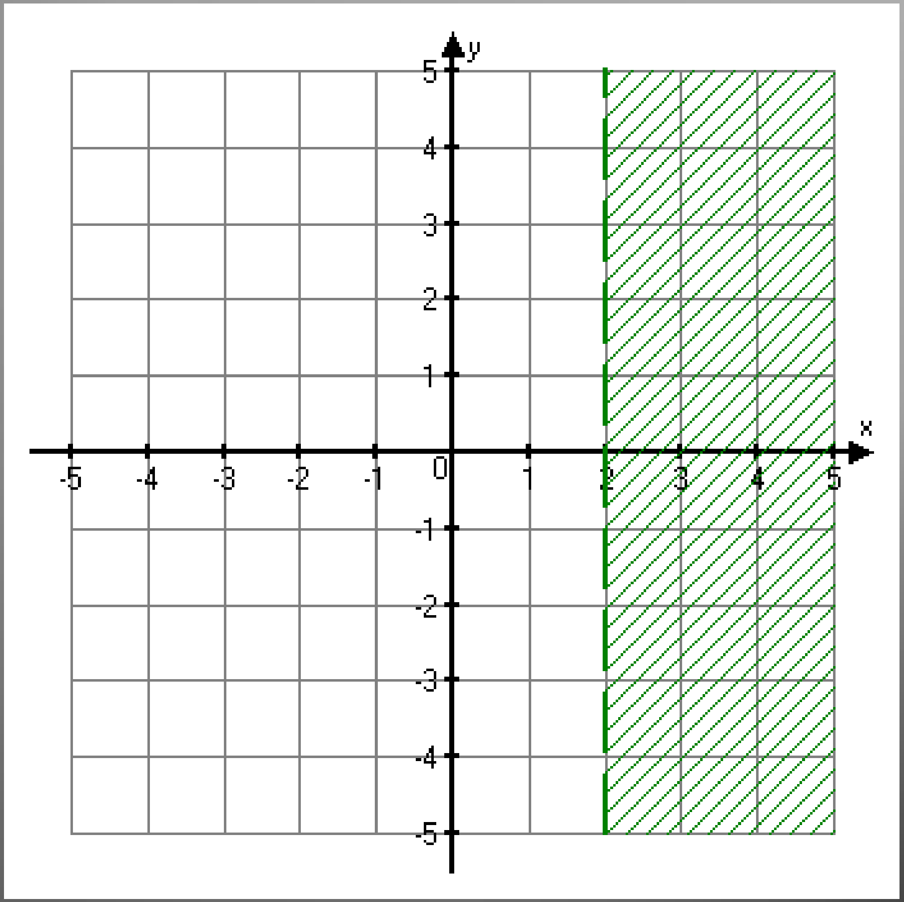
1. Graph the related equation.

2. Pick a test point (0, 0).

   $x > 2$

   $0 > 2$ False

   Because (0, 0) is not a solution, we shade the half-plane that does not contain that point.



Example

- Graph $y \leq 2$ on a plane.

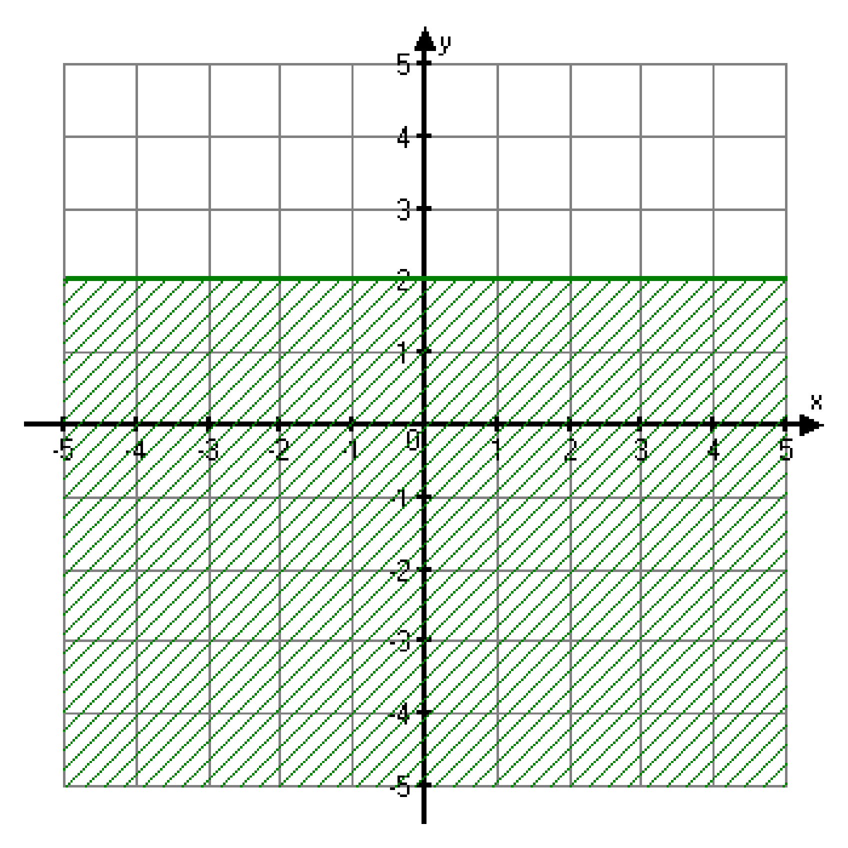
1. Graph the related equation.

2. Select a test point (0, 0).

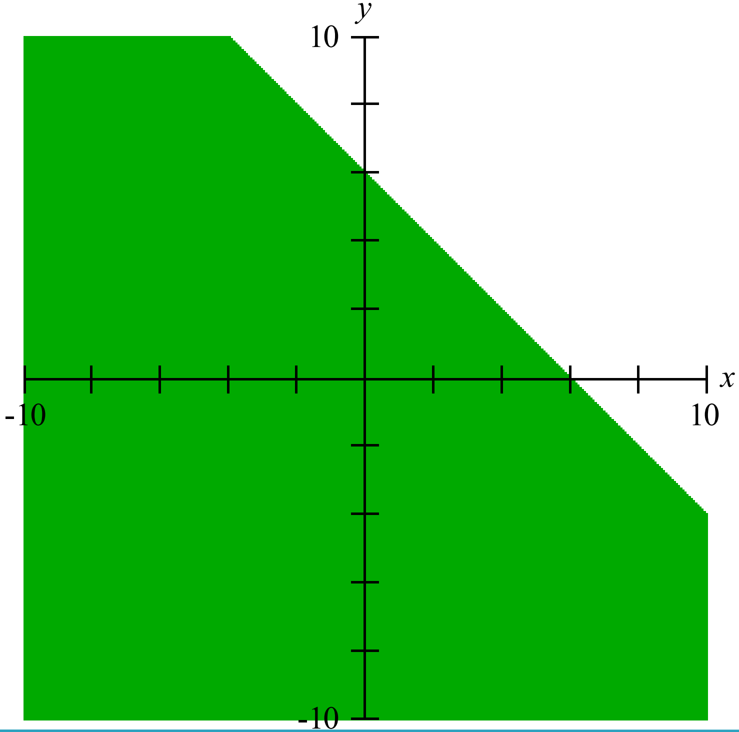
   $y \leq 2$

   $0 \leq 2$ True

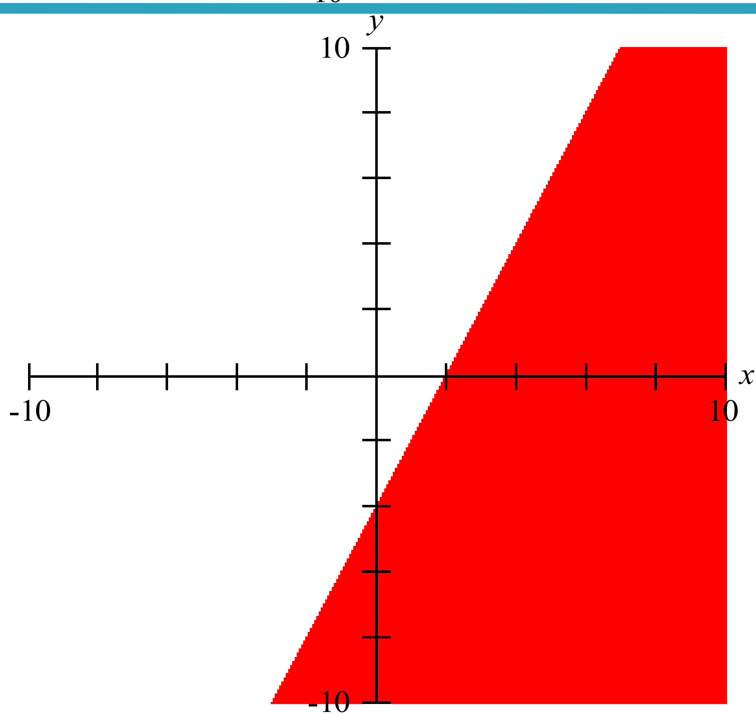
   Because (0, 0) is a solution, we shade the region containing that point.



- Example: Graph the inequality
  $x + y \le 6$



- Example: Graph the inequality
  $2x - y > 4$.



Graphing Systems of Linear Inequalities

1. Draw graph of each the equation created by replacing the inequality with “=”. Use dotted line for equations corresponding to strict inequalities.
2. Find the half planes that satisfy each inequality.
3. The intersection of the half planes is the solution set for the system of equations.
4. If there is no intersection of the half planes, then the solution set for the system of equations is the empty set.

EXAMPLE 1 Graphing a System of Two Inequalities

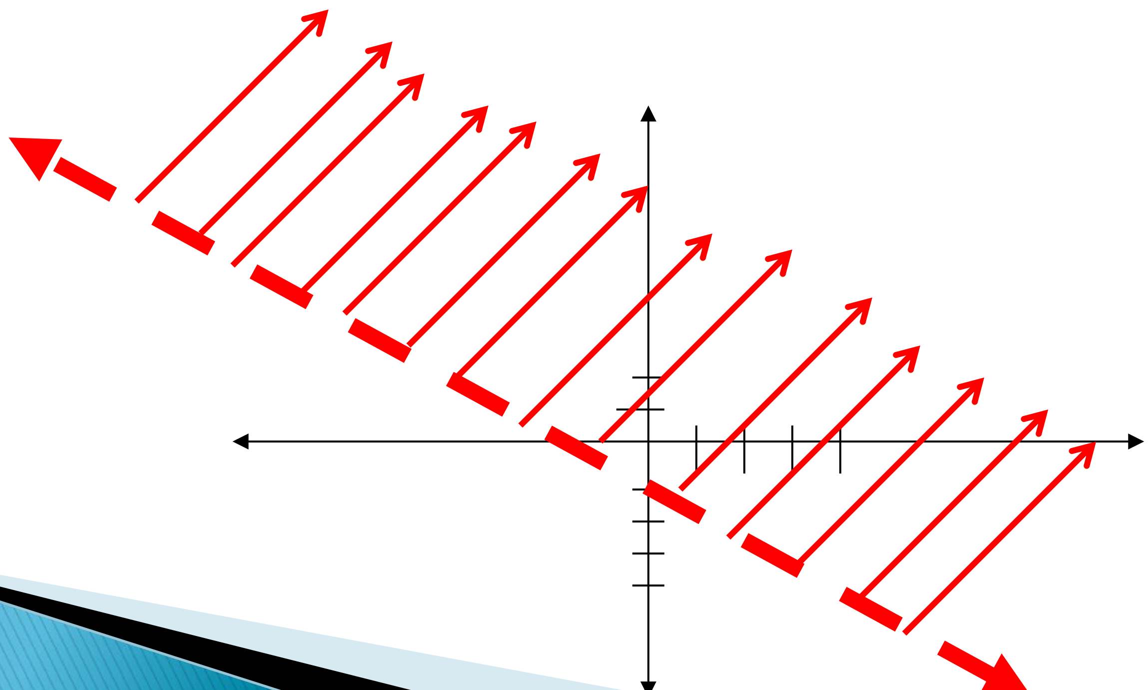
Graph the solution set of the system of inequalities:

$3x + 4y > -4$

$x + 2y < 2$

$a: \ 3x + 4y > -4$

$a: \ y > -\frac{3}{4}x - 1$

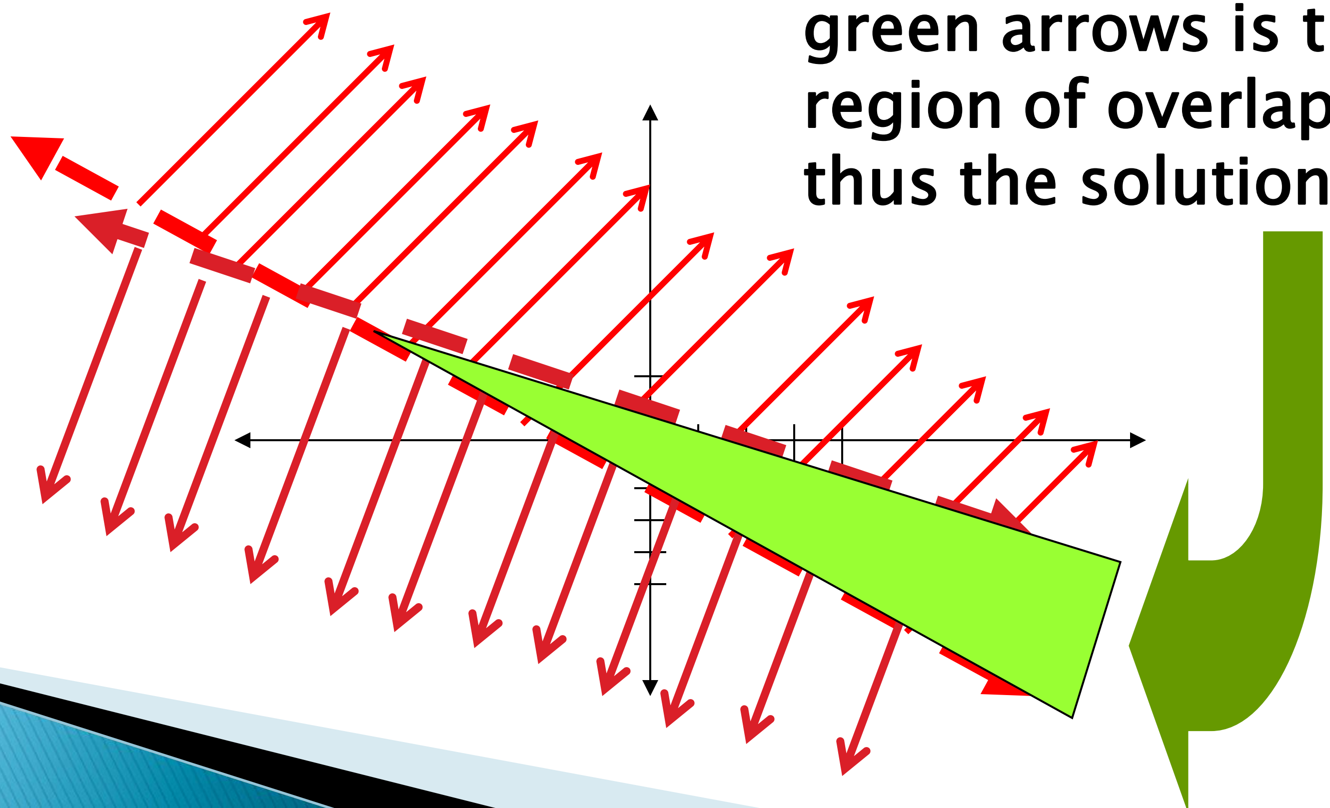


a: $3x + 4y > -4$

b: $x + 2y < 2$

$$a: \; y > -\frac{3}{4}x - 1$$

$$b: \; y < -\frac{1}{2}x + 1$$



The area between the green arrows is the region of overlap and thus the solution.

EXAMPLE 2 | Graphing a System of Two Inequalities

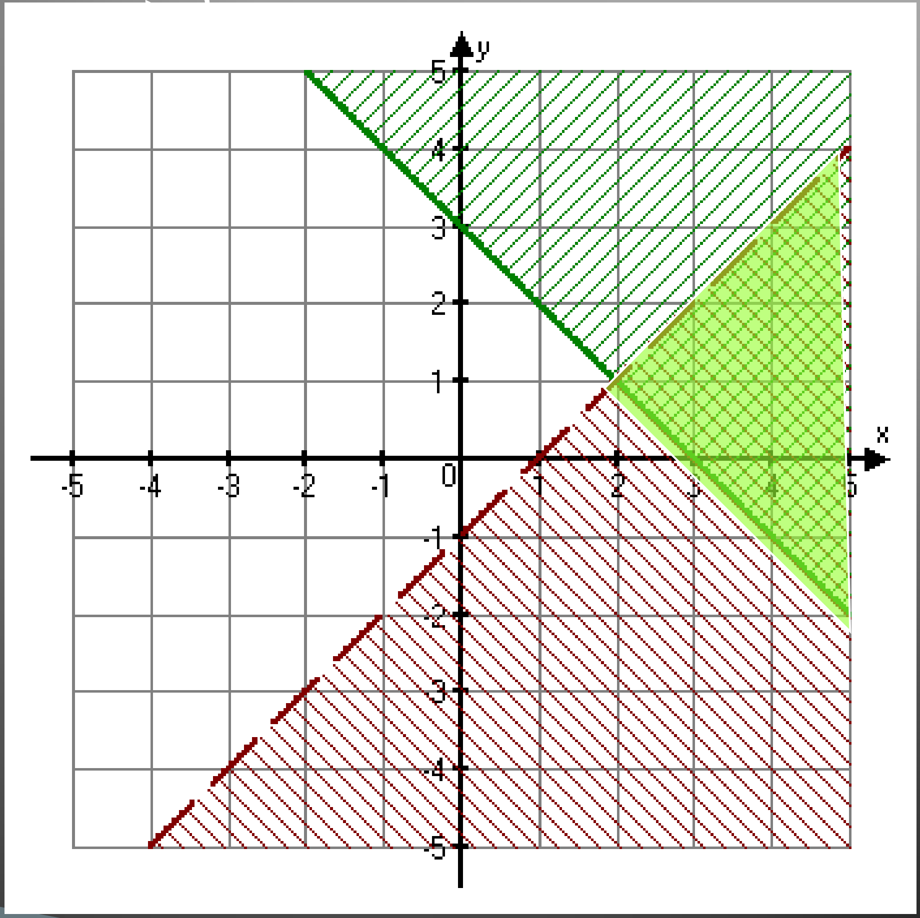
- Graph the solution set of the system.

$$x + y \geq 3$$

$$x - y > 1$$

- First, we graph $x + y \geq 3$ using a solid line. Choose a test point (0, 0) and shade the correct plane.
- Next, we graph $x - y > 1$ using a dashed line. Choose a test point and shade the correct plane.

The solution set of the system of equations is the region shaded both red and green, including part of the line $x + y \geq 3$



EXAMPLE 3 Graphing a System of Two Inequalities

Graph the solution set of the system of inequalities:

$$\begin{cases} 2x + 3y > 6 & (1) \\ y - x \le 0 & (2) \end{cases}$$

Solution

Step 1 $2x + 3y = 6$

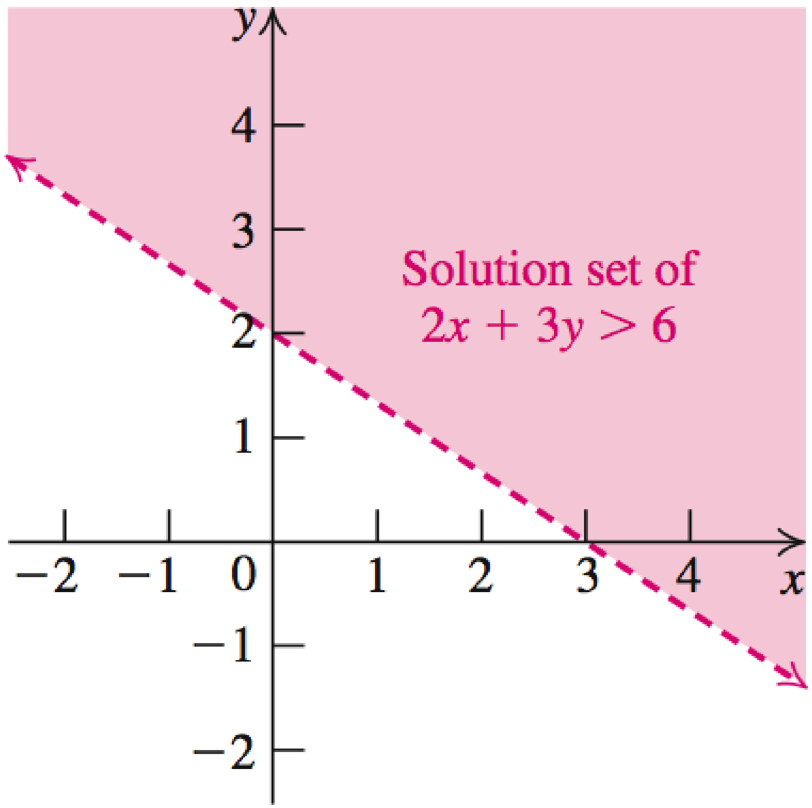
Step 2 Sketch as a dashed line by joining the points (0, 2) and (3, 0).

Step 3 Test (0, 0). 2(0) + 3(0) > 6 is a false statement.

Step 4 Shade the solution set.

EXAMPLE 3 Graphing a System of Two Inequalities

Solution continued



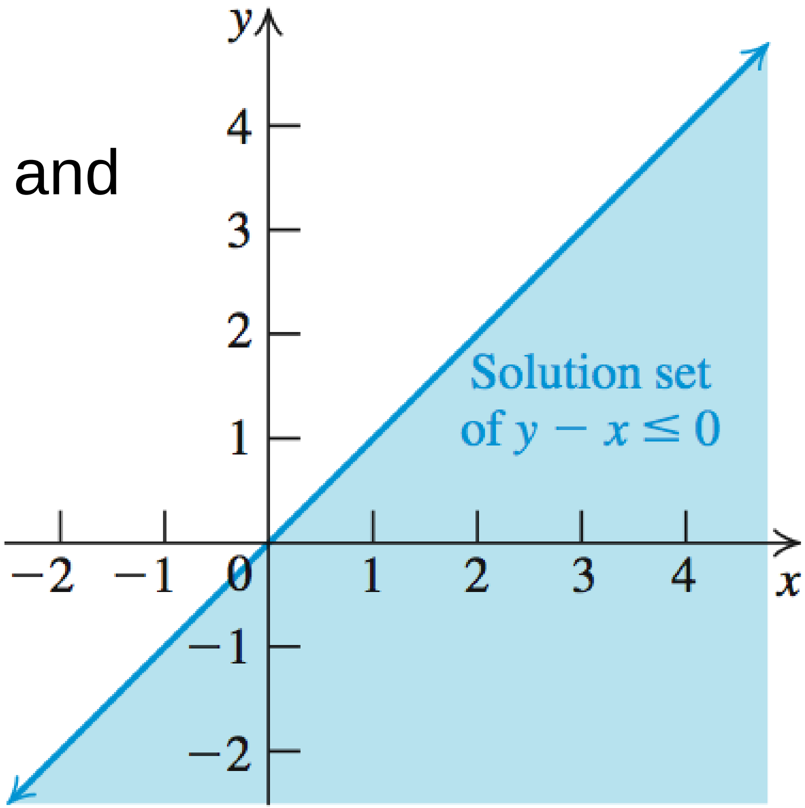
Now graph the second inequality.

EXAMPLE 3 Graphing a System of Two Inequalities

Solution continued

Step 1 $y - x = 0$

Step 2 Sketch as a solid line by joining the points (0, 0) and (1, 1).



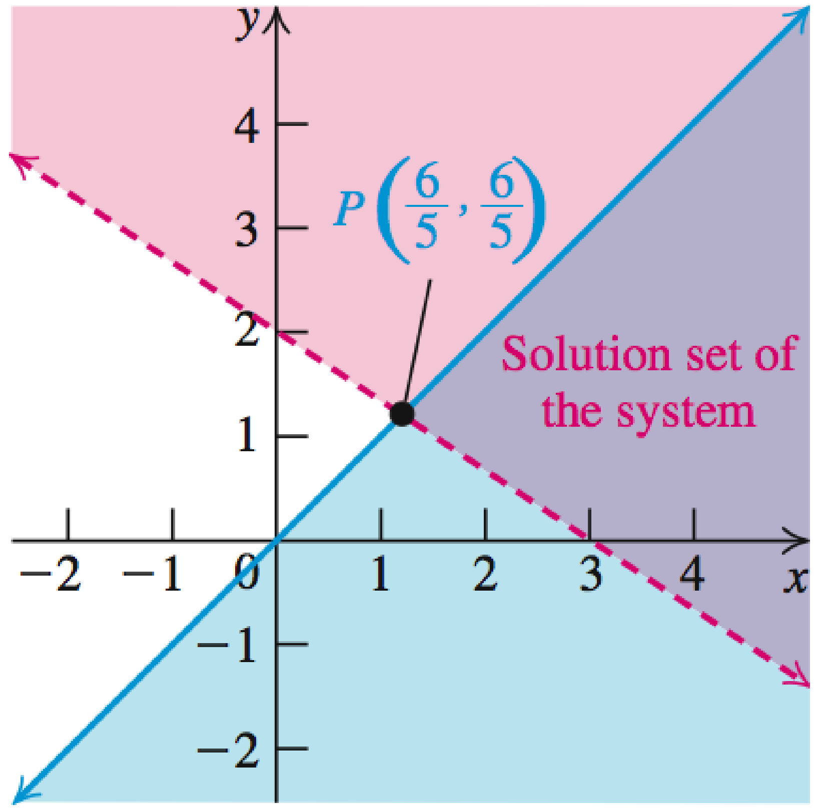
Step 3 Test (1, 0). 2(0) – 3(1) > 6 is a false statement.

Step 4 Shade the solution set.

EXAMPLE 3 Graphing a System of Two Inequalities

Solution continued

The graph of the solution set of inequalities (1) and (2) is the region where the shading overlaps.

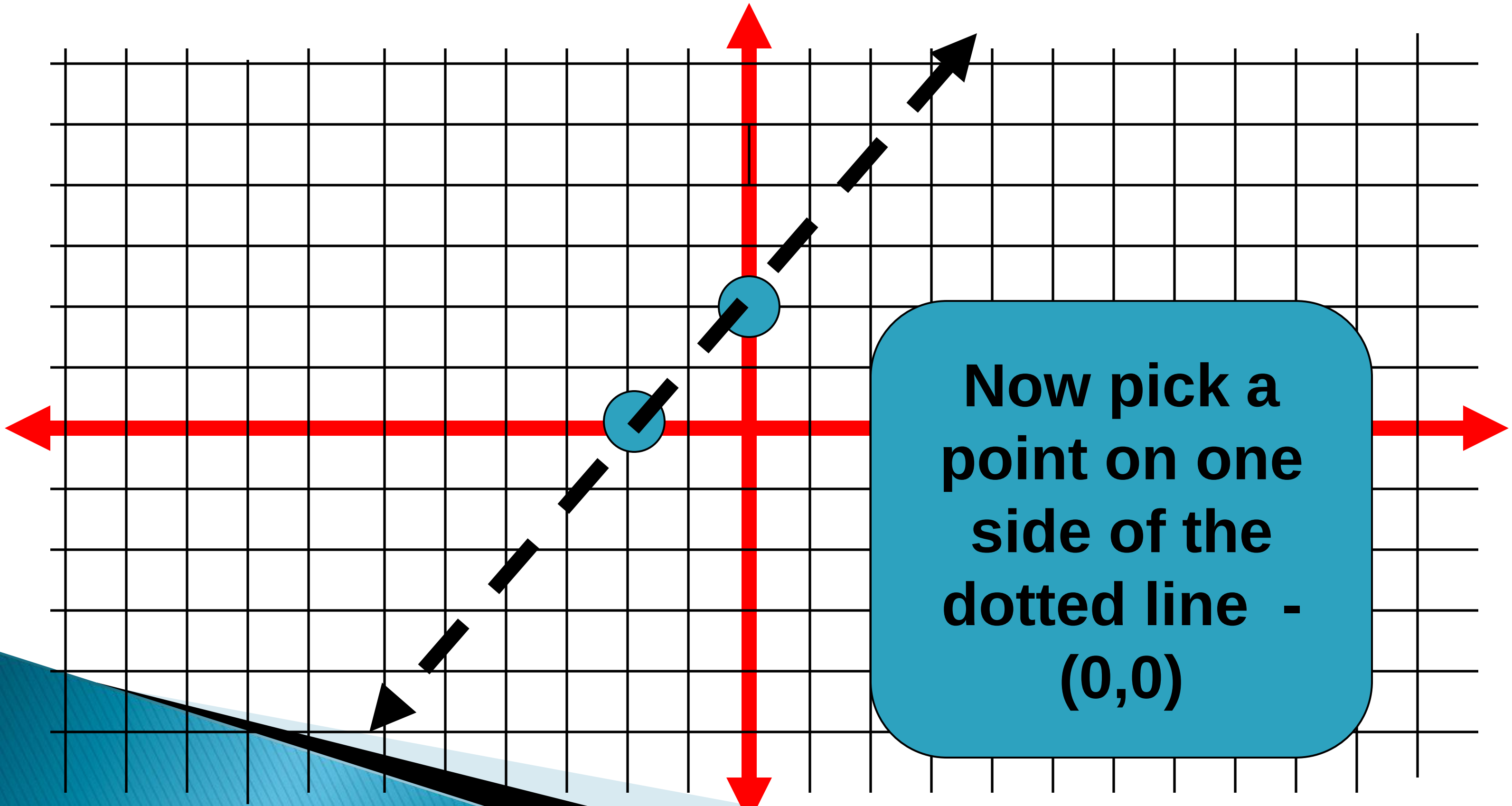


- Graph the solution set of the system of inequalities:

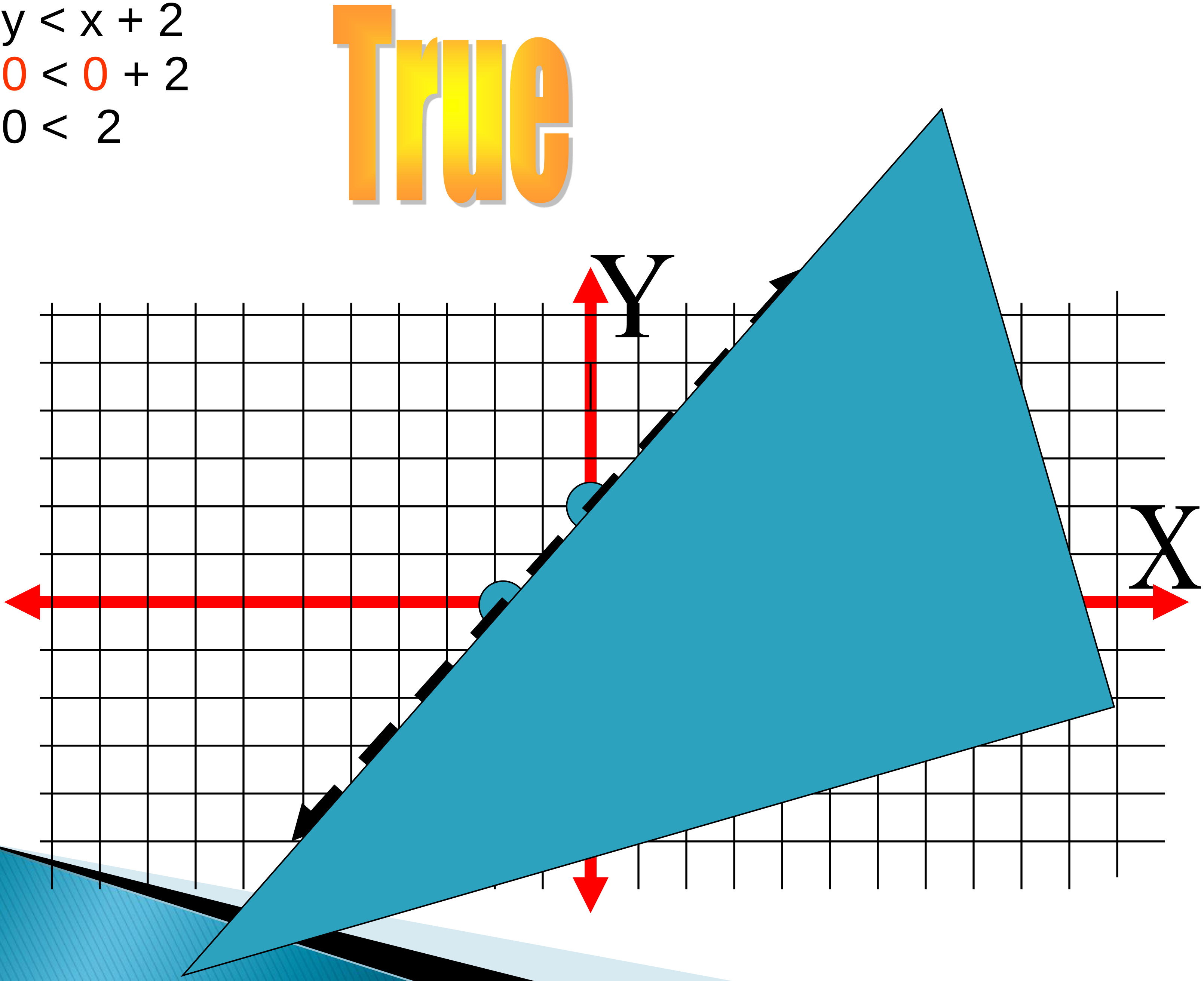
$y < x + 2$

$y > -1/2x + 5$

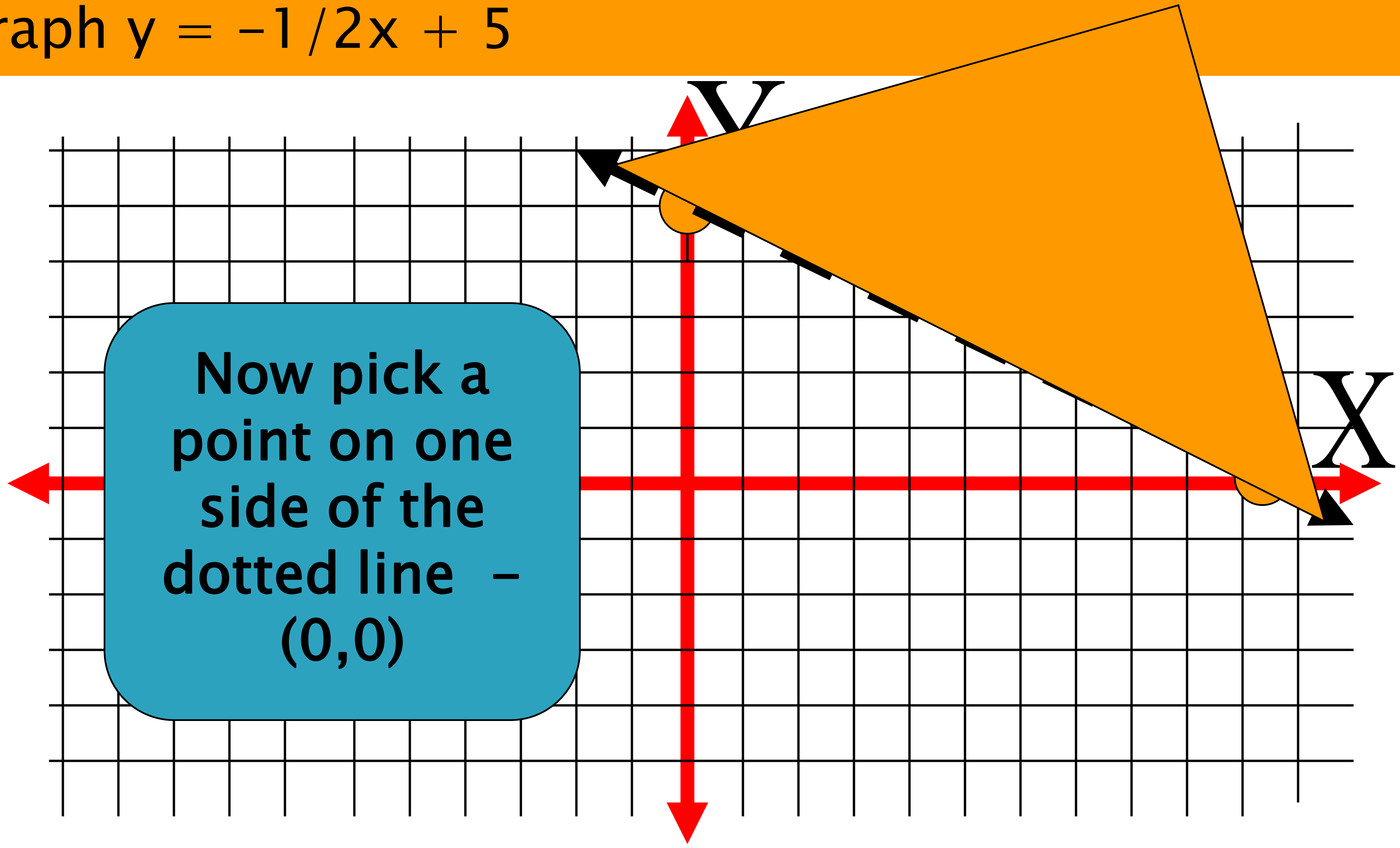
Solution: Graph y = x + 2



- Substitute (0,0) in for x and y
- y < x + 2
- 0 < 0 + 2
- 0 < 2



Graph y = −1/2x + 5



- Substitute (0,0) in for x and y
- y > -1/2x + 5
- 0 > -1/2(0) + 5
- 0 > 5

False

Linear Programming

- In many applications, we want to find a maximum or minimum value. Linear programming can tell us how to do this.
- **Constraints** are expressed as inequalities. The solution set of the system of inequalities made up of the constraints contains all the **feasible solutions** of a linear programming problem.
- The function that we want to maximize or minimize is called the **objective function.**

Linear Programming Procedure

- To find the maximum or minimum value of a linear objective function subject to a set of constraints:

1. Graph the region of feasible solutions.
2. Determine the coordinates of the vertices of the region.
3. Evaluate the objective function at each vertex. The largest and smallest of those values are the maximum and minimum values of the function, respectively.

Given an objective function and a system of inequalities representing constraints: (a) graph the system, (b) find the value of the function at each corner and (c) use the results to maximize the objective function.

Objective function $z = 3x + 2y$

Constraints $x \geq 0, y \geq 0$

$2x + y \leq 8$

$x + y \geq 4$

Example

- A tray of corn muffins requires 4 cups of milk and 3 cups of wheat flour. A tray of pumpkin muffins requires 2 cups of milk and 3 cups of wheat flour. There are 16 cups of milk and 15 cups of wheat flour available, and the baker makes \$3 per tray profit on corn muffins and \$2 per tray profit on pumpkin muffins. How many trays of each should the baker make in order to maximize profits?

  Solution: We let $x =$ the number of corn muffins and $y =$ the number of pumpkin muffins. Then the profit P is given by the function $P = 3x + 2y$.

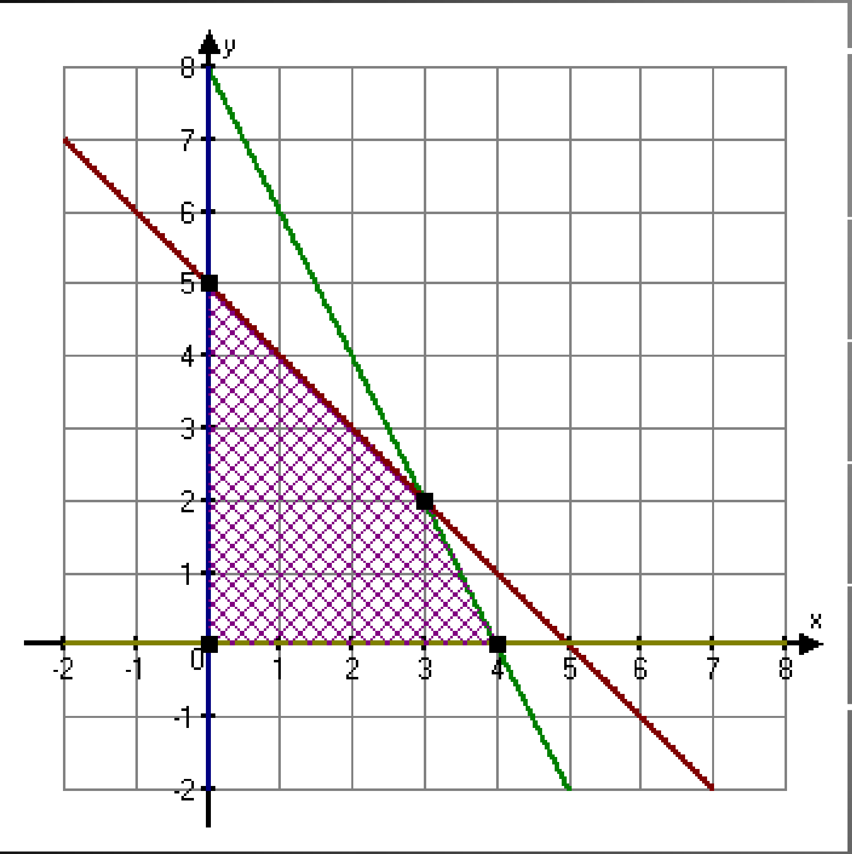
Example continued

- We know that x corn muffins require 4 cups of milk and y pumpkin muffins require 2 cups of milk. Since there are no more than 16 cups of milk, we have one constraint. $4x + 2y \leq 16$
- Similarly, the corn muffins require 3 cups of wheat flour and the pumpkin muffins 3 cups of wheat flour. There are no more than 15 cups of flour available, so we have a second constraint.
- $3x + 3y \leq 15$
- We also know $x \geq 0$ and $y \geq 0$ because the baker cannot make a negative number of either muffin.

Example continued

- Thus we want to maximize the objective function
  $P = 3x + 2y$ subject to the constraints
  $4x + 2y \leq 16,$
  $3x + 3y \leq 15,$
  $x \geq 0,$
  $y \geq 0.$
  We graph the system of inequalities and determine the vertices. Next, we evaluate the objective function P at each vertex.

Example continued



Vertices	Profit $P = 3x + 2y$
(0, 0)	$P = 3(0) + 2(0) = 0$
(4, 0)	$P = 3(4) + 2(0) = 12$
(0, 5)	$P = 3(0) + 2(5) = 10$
(3, 2)	$P = 3(3) + 2(2) = 13$

Maximum

The baker will make a maximum profit when 3 trays of corn muffins and 2 trays of pumpkin muffins are produced.