MATH 104
THE SOLUTIONS OF THE ASSIGNMENT

Question19. (Page 275) Solve **AX = B** if $A^{-1} = \begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and write a system.

Solution:

$X = A^{-1}B$,

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$x_1 = 2*1 + 2*4 = 10$
$x_2 = 8*2 + 4*1 = 20$

For finding the system, we use $\left(A^{-1}\right)^{-1} = A$,
$\left|A^{-1}\right| = \begin{vmatrix} 1 & 2 \\ 8 & 1 \end{vmatrix} = 1 - 16 = -15$,
$\left(A^{-1}\right)^{-1} = \dfrac{1}{-15}\begin{pmatrix} 1 & -2 \\ -8 & 1 \end{pmatrix} = \begin{pmatrix} -1/15 & 2/15 \\ 8/15 & -1/15 \end{pmatrix}$

The system is $Ax = B$, $\begin{pmatrix} -1/15 & 2/15 \\ 8/15 & -1/15 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\begin{matrix} -x_1 + 2x_2 = 30 \\ 8x_1 - x_2 = 60 \end{matrix}$

Question 39. (Page 286) Solve for x, if $\begin{vmatrix} x & -2 \\ 7 & 7-x \end{vmatrix} = 26$.

Solution:

$x(7-x) - (-2)(7) = 26$
$7x - x^2 + 14 = 26$
$x^2 - 7x + 12 = 0$
$(x-3)(x-4) = 0$
So $x = 3$ or $x = 4$.

Question 40. (Page 286) Solve for x, if $\begin{vmatrix} 3 & x & 2x \\ 0 & x & 99 \\ 0 & 0 & x-1 \end{vmatrix} = 60$

Solution:

$\begin{vmatrix} 3 & x & 2x \\ 0 & x & 99 \\ 0 & 0 & x-1 \end{vmatrix} = 60$ All entries below the main diagonal are zeros.

$$(3)(x)(x-1) = 60$$
$$3(x^2 - x) = 60$$

Thus, $x^2 - x = 20$, So $x = 5$ or $x = -4$.

$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$

Question 35. (Page 297) Solve the systems by using Cramer's rule.

$$\begin{cases} 3x - y = 1 \\ 2x + 3y = 8 \end{cases}$$

Solution:

$$\begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$$

A= $\begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}$ = (3*3)-(2*(-1))=9+2=11

$A_x = \begin{vmatrix} 1 & -1 \\ 8 & 3 \end{vmatrix} = 11$ $A_y = \begin{vmatrix} 3 & 1 \\ 2 & 8 \end{vmatrix} = 22$

$x = \frac{A_x}{A} = \frac{11}{11} = 1$, $y = \frac{A_y}{A} = \frac{22}{11} = 2$

Question 36. (Page 297) Solve the system by using Cramer`s rule.

$$\begin{cases} x+2y-z=0 \\ y+4z=0 \\ x+2y+2z=0 \end{cases}$$

Solution:

$$A=\begin{vmatrix} 1 & 2 & -1 \\ 0 & 1 & 4 \\ 1 & 2 & 2 \end{vmatrix}\begin{matrix} 1 & 2 \\ 0 & 1 \\ 1 & 2 \end{matrix}$$ = (1*1*2) + (2*4*1) + (-1*0*2)-(-1*1*1)-(1*4*2)-(2*0*2)

A= 2+8+0+1-8-0=3

$$A_x=\begin{vmatrix} 0 & 2 & -1 \\ 0 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix}\begin{matrix} 0 & 2 \\ 0 & 1 \\ 0 & 2 \end{matrix}$$ = (0*1*2) + (2*4*0) + (-1*0*2)-(-1*1*0)-(0*4*2)-(2*0*2)

A_x = 0+0+0+0-0-0 =0

$$A_y=\begin{vmatrix} 1 & 0 & -1 \\ 0 & 0 & 4 \\ 1 & 0 & 2 \end{vmatrix}\begin{matrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{matrix}$$ = (1*0*2) + (0*4*1) + (-1*0*0)-(-1*0*1)-(1*4*0)-(0*0*2)

A_y = 0+0+0-0-0-0 =0

$$A_z=\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 0 \end{vmatrix}\begin{matrix} 1 & 2 \\ 0 & 1 \\ 1 & 2 \end{matrix}$$ = (1*1*0) + (2*0*1) + (0*0*2)-(0*1*1)-(1*0*2)-(2*0*0)

A_z = 0+0+0-0-0-0 =0

$x=\dfrac{A_x}{A}=\dfrac{0}{3}=0$, $y=\dfrac{A_y}{A}=\dfrac{0}{3}=0$, $z=\dfrac{A_z}{A}=\dfrac{0}{3}=0$

Question 24. (page 712) Given the demand equation $q^2(1+p)^2 = p$, determine the point elasticity of demand when p=9.

Solution:

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q}\frac{dq}{dp},$$ we differentiate implicitly for $\frac{dq}{dp}$.

$$q^2(1+p)^2 = p$$
$$q^2 2(1+p)(1) + (1+p^2)(2q\frac{dq}{dp}) = 1$$
$$2q^2(1+p) + 2q(1+p)^2\frac{dq}{dp} = 1$$

Thus $\frac{dq}{dp} = \frac{1-2q^2(1+p)}{2q(1+p)^2}$.

Hence

$$\eta = \frac{q^2(1+p)^2}{q}.\frac{1-2q^2(1+p)}{2q(1+p)^2} = \frac{1-2q^2(1+p)}{2}$$

If p=9, we fine q from the given equation:

$$q^2(1+9)^2 = 9$$
$$q^2 = \frac{9}{100}$$
$$q = \frac{3}{10}$$

Since q>0. Thus $\eta_{p=9} = \frac{1-2\left(\frac{3}{10}\right)^2(1+9)}{2} = -0,4$

$|\eta| = 0.4 < 1$ which is inelastic

Question 25. (page 712) The demand equation for a product is $q = \frac{60}{p} + \ln(65 - p^3)$.

a) Determine the point elasticity of demand when p=4, and classify the demand as elastic, inelastic, or of unit elasticity at this price level.
b) If the price is lowered by 2% (from $4, 00 to $3,92), use the answer to part (a) to estimate the corresponding percentage change in quantity sold.
c) Will the changes in part (b) result in an increase or decrease in revenue? Explain.

Solution:

a) $q = \frac{60}{p} + \ln(65 - p^3)$.

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q}\frac{dq}{dp} = \frac{p}{q}\left[-\frac{60}{p^2} - \frac{3p^2}{65 - p^3}\right]$$

If p=4, then $q = \frac{60}{4} + \ln 1 = 15$, so $\eta = \frac{4}{15}\left[-\frac{60}{16} - \frac{3(16)}{65-64}\right] = -\frac{207}{15} = -13,8$.

$|\eta| = 13.8 > 1$ which is elastic.

%1 increase in price p, results 13.8% decrease in demand.

b)The percentage change in q is (-2). (-13, 8) =27, 6%, so q increases by approximately 27, 6%.

c)Lowering the price increases revenue because demand is elastic.

Question3. *(page 718)* **Revenue**
The demand function for a monopolist's product is

$$p = \sqrt{600 - q}$$

If the monopolist wants to produce at least 100 units, but not more than 300 units, how many units should be produced to maximize total revenue?

Solution:

The revenue function is $R(q) = pq = q\sqrt{600-q}$ and $100 \le q \le 300$

$$R'(q) = \sqrt{600-q} - \frac{1}{2}(600-q)^{-1/2} q = \sqrt{600-q} - \frac{1}{2}\frac{q}{\sqrt{600-q}} = 0$$

$R'(q) = 2(600-q) - q = 0$, $R'(q) = 1200 - 3q = 0$, $q = 400$ units, but
$q = 400 \notin [100, 300]$

So, there is no critical values on $[100,300]$. Since $R'(q) > 0$ on $[100,300]$ means $R(q)$ is increasing on $[100,300]$.

$$R(100) = pq = 100\sqrt{600-100} = 100\sqrt{500}$$

$$R(300) = pq = 300\sqrt{600-300} = 300\sqrt{300}$$

So, $R(q)$ must have a maximum at $q = 300$.

Question4. *(page 718)* **Average Cost**

If $C = 0.001q^2 + 5q + 100$ is a cost function, find the average cost function. At what level of production q is there a minimum average cost.

Solution:

The average cost is $\overline{C} = \frac{C}{q} = \frac{0.001q^2 + 5q + 100}{q} = 0.001q + 5 + \frac{100}{q}$

$\overline{C}' = \frac{d\overline{C}}{dq} = 0.001 - \frac{100}{q^2} = 0$, $\frac{d\overline{C}}{dq} = 0.001q^2 - 100 = 0$, $q = \pm 100$, but $q = -100$ is not possible.

So , $q = 100$.

From the second derivative test $\overline{C}''(q) = \frac{200}{q^3}$, $\overline{C}''(100) = \frac{200}{(100)^3} > 0$, Relative minimum

So, when $q = 100$, the average cost is minimum.

Question5. *(page 718)* **Profit**
The demand function for a monopolist's product is

$$p = 400 - 2q$$

and the average cost per unit for producing q units is

$$\overline{C} = q + 160 + \frac{2000}{q}$$

Where p and $\overline{c}$ are in dollars per unit. Find the maximum profit that the monopolist can achieve.

Solution:

The revenue function is $R(q) = pq = (400 - 2q)q = 400q - 2q^2$ and

The cost function is $C(q) = \overline{C}q = q^2 + 160q + 2000$

For the maximum profit, $MR = MC$

$MR = R'(q) = 400 - 4q$

$MC = 2q + 160$

$400 - 4q = 2q + 160$, $6q = 240$, $\boxed{q = 40}$

When $q = 40$, the profit is maximum.

The profit function is

$P(q) = R(q) - C(q) = 400q - 2q^2 - q^2 - 160q - 2000 = -3q^2 + 240q - 2000$

And the maximum profit is $P(40) = -3(40)^2 + 240(40) - 2000 = 3600$

Question 18. (Page 719) Given demand equation $p=\frac{500}{q}$; q=200. Determine whether demand is elastic, is inelastic, or has unit elasticity for the indicated value of q.

Solution:

$$p=\frac{500}{q}$$

$$\eta=\frac{\frac{p}{q}}{\frac{dp}{dq}}=\frac{\frac{\frac{500}{q}}{q}}{-\frac{500}{q^2}}=\frac{500}{q}\frac{1}{q}\left(-\frac{q^2}{500}\right)=\frac{500}{q^2}\left(-\frac{q^2}{500}\right)=-1$$

Since $|\eta|=1$, demand has unit elasticity when q=200.

Determine the following indefinite integrals. *(page 749)*

Question1. $\int\frac{2x^4+3x^3-x^2}{x^3}dx$

Solution:

$$\int\frac{2x^4+3x^3-x^2}{x^3}dx=\int\frac{2x^4}{x^3}dx+\int\frac{3x^3}{x^3}dx-\int\frac{x^2}{x^3}dx$$

$$=\int 2x\,dx+\int 3\,dx-\int\frac{1}{x}dx$$

$$=x^2+3x-\ln x+c$$

Question10. $\int\left(e^x+x^e+ex+\frac{e}{x}\right)dx$

Solution:

$$\int\left(e^x+x^e+ex+\frac{e}{x}\right)dx=\int e^x dx+\int x^e dx+\int ex\,dx+\int\frac{e}{x}dx$$

$$=e^x+\frac{x^{e+1}}{e+1}+e\frac{x^2}{2}+e\ln x+c$$

Evaluate the following definite integrals. (page 771)

Quetion7. $\int_{2}^{3}\left(y^{2}-2y+1\right)dy$

Solution:

$$\int_{2}^{3}\left(y^{2}-2y+1\right)dy=\left[\frac{y^{3}}{3}-y^{2}+y\right]\Bigg|_{2}^{3}$$

$$=\left[\frac{y^{3}}{3}-y^{2}+y\right]\Bigg|_{2}^{3}=\left(\frac{27}{3}-9+3\right)-\left(\frac{8}{3}-4+2\right)=3+\frac{10}{3}=\frac{19}{3}$$

Quetion20. $\int_{1}^{3}\left(x+3\right)^{3}dx$

Solution:

$$\int_{1}^{3}\left(x+3\right)^{3}dx=\int_{1}^{3}\left(x^{3}+3x^{2}+9x+27\right)dx$$

$$=\left(\frac{x^{4}}{4}+x^{3}+\frac{9x^{2}}{2}+27x\right)\Bigg|_{1}^{3}$$

$$=\left(\frac{81}{4}+27+\frac{81}{2}+81\right)-\left(\frac{1}{4}-1+\frac{9}{2}+27\right)=\frac{675}{4}-\frac{123}{4}=\frac{552}{4}=138$$

Question 23. $\int_0^1 e^5 dx$

Solution:

$$\int_0^1 e^5 dx = e^5 x\Big|_0^1 = e^5 - 0 = e^5$$

Question 10. (Page 789) The demand equation for a product $(p+20)(q+10)=800$ and the supply equation is $q=2p+30=0$.

a)Verify, by substitution, that market equilibrium occurs when $p=20, q=10$.

b) Determine consumer's surplus under market equilibrium.

Solution:

a-) $(20+20)(10+10)=800$
$800=800$

$$10-2(20)+30=0$$
$$10-40+30=0$$

b-) $(p+20)(q+10)=800, \quad p+20=\frac{800}{q+10}, \quad p=\frac{800}{q+10}-20$

$$CS=\int_0^{10}\left[\left(\frac{800}{q+10}-20\right)-20\right]dq=\left[800\ln(q+10)-40q\right]\Big|_0^{10}$$
$$=800\ln(20)-400-(800\ln 10)=800\ln(2)-400$$

Question 59. (Page 792) **Marginal Revenue**

If marginal revenue is given by $\frac{dr}{dq}=100-\frac{3}{2}\sqrt{2q}$, determine the corresponding demand equation.

Solution:

$$r=\int\left(100-\frac{3}{2}\sqrt{2q}\right)dq=\int 100dq-\frac{3}{2}\sqrt{2}\int q^{\frac{1}{2}}dq=100q-\sqrt{2}q^{\frac{3}{2}}+C$$

$when \quad q=0, \quad then \quad r=0. Thus \quad 0=0-0+C \quad so \quad C=0$.

$Hence \quad r=100q-\sqrt{2}q^{\frac{3}{2}}$. $Since \quad r=p.q \quad then \quad p=\frac{r}{q}=100-\sqrt{2}q^{\frac{1}{2}}=100-\sqrt{2q}$

$Thus \quad p=100-\sqrt{2q}$.

Question 60. (Page 792) **Marginal Cost**

If marginal cost is given by $\frac{dc}{dq} = q^2 + 7q + 6$, and fixed costs are 2500, determine the total cost of producing six units. Assume that costs are in dollars.

Solution:

$$c = \int \left(q^2 + 7q + 6\right) dq = \frac{q^3}{3} + \frac{7}{2}q^2 + 6q + C\ .$$

$$When \quad q = 0,\ then\ \ c = 2500. Thus \quad 2500 = 0 + 0 + 0 + C\ \ so\ \ C = 2500$$

$$c = \frac{q^3}{3} + \frac{7}{2}q^2 + 6q + 2500. When \quad q = 6, then\ c = \$2734\ .$$

Question 61. (Page 792) **Marginal Revenue**

A manufacturer's marginal-revenue function is $\frac{dr}{dq} = 275 - q - 0.3q^2$. If r is in dollars, find the increase in the manufacturer's total revenue if production is increased from 10 to 20.

Solution:

$$\int_{10}^{20} \left(275 - q - 0.3q^2\right) dq = \left(275q - \frac{q^2}{2} - \frac{0.3q^3}{3}\right)\Bigg|_{10}^{20} = \$1900\ .$$

Question 26. (Page 914**)** **Profit**

A monopolist sells two competitive products, A and B, for which the demand functions are

$$q_A = 1 - 2p_A + 4p_B, \quad q_B = 11 + 2p_A - 6p_B$$

If the constant average cost of producing a unit of A is 4 and a unit of B is 1, how many units of A and B should be sold to maximize the monopolist's profit?

Solution:

Revenue from $A = p_A q_A$. Revenue from $B = p_B q_B$.

Total cost of producing q_A units of A and q_B units of B is $4q_A + 1q_B$.

Total Profit=Total Revenue-Total Cost

$$P = p_A q_A + p_B q_B - (4q_A + q_B)$$

$$P = -2p_A^2 - 6p_B^2 + 6p_A p_B + 7p_A + p_B - 15$$

$$\begin{cases} \dfrac{\partial P}{\partial p_A} = -4p_A + 6p_B + 7 = 0 \\ \dfrac{\partial P}{\partial p_B} = 6p_A - 12p_B + 1 = 0 \end{cases}$$

Critical point : $p_A = \dfrac{15}{2}, \quad p_B = \dfrac{23}{6}$

$\dfrac{\partial^2 P}{\partial p_A^2} = -4 < 0, \quad \dfrac{\partial^2 P}{\partial p_B^2} = -12 < 0, \quad \dfrac{\partial^2 P}{\partial p_B \partial p_A} = 6.$ At $p_A = \dfrac{15}{2}$ and $p_B = \dfrac{23}{6}$ then

$D\left(\dfrac{15}{2}, \dfrac{23}{6}\right) = (-4)(-12) - (6)^2 = 12 > 0$ thus relative maximum.

When $p_A = \dfrac{15}{2}$ and $p_B = \dfrac{23}{6}$ then $q_A = \dfrac{4}{3}$ and $q_B = 3$.

Thus to maximize profit $q_A = \dfrac{4}{3}, \quad q_B = 3$.

Question 15. (Page 922) **Maximizing Output**

The production function for a firm is $f(l,k)=12l+20k-l^2-2k^2$. The cost to the firm of l and k is 4 and 8 per unit, respectively. If the firm wants the total cost of input to be 88, find the greatest output possible, subject to this budget constraint.

Solution:

We maximize $f(l,k)=12l+20k-l^2-2k^2$ subject to the constraint $4l+8k=88$.

$F(l,k,\lambda)=12l+20k-l^2-2k^2-\lambda(4l+8k-88)$

$$\begin{cases} F_l=12-2l-4\lambda=0 & (1) \\ F_k=20-4k-8\lambda=0 & (2) \\ F_\lambda=-4l-8k+88=0 & (3) \end{cases}$$

Eliminate λ from (1) and (2) $\Rightarrow$ $k=l-1$. Substitute $k=l-1$ into (3) yields $l=8,\quad k=7$. Therefore the greatest output is $f(8,7)=74$ units.