Math 104 – Mathematics for Business and Economics II *28 March 2008*

QUIZ II

Duration 40 minutes

Name		*Student No*	
Surname		*Group*	

For the following questions ***show all your work clearly*** to find the answer.

Question 1. (10 *pts.*) Find the following limits.

a. $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x - 1} = \lim\limits_{x \to 1} \dfrac{(x+2)(x-1)}{x-1} = \lim\limits_{x \to 1}(x+2) = 3$

b. $\lim\limits_{x \to 0} \dfrac{(x+2)^2 - 4}{x} = \lim\limits_{x \to 0} \dfrac{x^2 + 4x + 4 - 4}{x} = \lim\limits_{x \to 0} \dfrac{x^2 + 4x}{x} = \lim\limits_{x \to 0}(x+4) = 4$

Question 2. (*10 pts.*) Find the derivative of the following functions.

a. $f(x) = x^{1/2}\left(x^2 + x - 4\right)$

$$f'(x) = \frac{1}{2}x^{-1/2}\left(x^2 + x - 4\right) + (2x+1)x^{1/2}$$

b. $f(x) = \dfrac{6x^2}{x^2 - 2x + 1} + e^{3x^2}$

$$f'(x) = \frac{12x\left(x^2 - 2x + 1\right) - (2x-2)6x^2}{\left(x^2 - 2x + 1\right)^2} + 6xe^{3x^2}$$

c. $f(x) = \ln\left(x^3 + 4\right)$

$$f'(x) = \frac{3x^2}{x^3 + 4}$$

Question 3. (10 *pts.*) Find the equation of the tangent line touching the curve $y = f(x) = 2x^3 + 6x^2 + 10$ at $x = 1$.

$$y' = f'(x) = 6x^2 + 12x$$
$$m = f'(1) = 6(1)^2 + 12(1) = 18$$
$$x_1 = 1, \quad y_1 = 2(1)^3 + 6(1)^2 + 10 = 18$$

The equation of the tangent line is

$$y - y_1 = m(x - x_1)$$
$$y - 18 = 18(x - 1)$$
$$y = 18x$$

Question 4. (20 *pts.*) Find all critical points and their nature (relative maximum, relative minimum and inflection points) of $y = f(x) = 2x^3 - x^4$.

$y' = f'(x) = 6x^2 - 4x^3 = 2x^2(3 - 2x) = 0$, the critical points are $x = 0, x = \frac{3}{2}$.

From the First Derivative Test,

x		0		3/2	
$2x^2$	+	○	+		+
$3-2x$	+		+	○	-
$f'(x)$	+		+		-
$f(x)$	↗		↗		↘

When $x = \frac{3}{2}$, then $y = f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^4 = \frac{27}{16}$.

The relative maximum point is $\left(\frac{3}{2}, \frac{27}{16}\right)$.

$y'' = f''(x) = 12x - 12x^2 = 12x(1 - x) = 0$, $x = 0, x = 1$

x		0		1	
$12x$	-	○	+		+
$1-x$	+		+	○	-
$f''(x)$	-		+		-
$f(x)$	∩		∪		∩

When $x = 0$, then $f(0) = 0$. When $x = 1$, then $f(1) = 1$.

The inflection points are $(0,0)$ and $(1,1)$.